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THE RESPONSE OF NONLINEAR MULTI-STORY STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION

by

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THE RESPONSE OF NONLINEAR MULTI-STORY STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION

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ABSTRACT

The dynamic responses of a 20-story nonlinear structural frame representative of a modern high rise building are analyzed with the aid of a digital computer. Related analytical studies of continuous systems are carried out. Quantitative information is provided on the importance of a wide range of modes to the various responses of a multi-story structure during an earthquake. The effect of yielding on the response is observed. The magnitude of the structural responses are compared with common measurements of earthquake strength.

At the ends of each girder and column of the structural frame are yield hinges which have bilinear bending moment-rotation hysteretic characteristics. Two beam models having such characteristics are studied; one of these models can treat curvilinear hysteretic behavior. Three definitions of ductility factor are discussed, one of which is applicable to both bilinear and curvilinear hysteresis loops. In the computer program, the frame is subjected to the time history of an earthquake accelerogram, the equations of motion are stepwise integrated, and the various structural responses - displacement, bending moments, incurred yielding, etc. - are determined.

The agreement between the response parameters resulting from excitation by seven different earthquake ground motions indicates that these response characteristics are determined more by the properties of the structure than by the earthquake. These results throw some light on extreme value statistics of the response of yielding structures subjected to earthquakes. The characteristic patterns observed in the computed responses of the nonlinear structure can be related to analytical studies of linear elastic, shear-type, uniform and tapered continuous cantilever beams.

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CHAPTER I

INTRODUCTION

Until recently, comparatively little has been known about the response of multi-story structures subjected to earthquakes. Consequently, in regions where earthquakes have occurred relatively frequently, building codes imposed stringent height limits on structures. In the past few years, however, there has been a significant improvement in the understanding of how structures respond to earthquakes, and, as a result, in some cities height restrictions have been relaxed for structures which are designed on the basis of the dynamic forces, deformation, and yielding which would be induced by a strong earthquake.

A common type of high-rise building being constructed today (1967) uses glass and light-weight walls extensively, relying on large columns and girders for support. The investigation described in this report was undertaken in order to provide additional information on how this kind of multi-story structure responds to earthquake excitation.

The initial work on problems encountered in designing earthquake resistant multi-story structures was done during the 1920's and early 1930's by K. Muto[3] and K. Suyehiro[4] of Japan and M. A. Biot[5,6] and J. R. Freeman[7] in the United States. Although the equations of motion for earthquake excited multi-story structures were well known, the time history solutions of the responses of multi-story structures could not be obtained because neither analog

nor digital computers were available to make the extensive calculations necessary.

Through the years the one degree of freedom system has received extensive study by several investigators using analytical techniques and, as they become available, analog and digital computers. Among them are D. E. Hudson[8], G. W. Housner[9], T. K. Caughey[10], G. V. Berg[11], S. S. Thomaides[11], J. Penzien[12], W. D. Iwan[13], Paul C. Jennings[14], C. V. Chelapati[15], and L. D. Lutes[16]. In addition to direct application to the one degree of freedom system, the results of their investigations also provide significant insight into the understanding of how multi-story structures respond to earthquake excitation.

One early study in multi-story structures appeared in 1938. In that study, L. S. Jacobsen and R. S. Ayre[17] experimentally determined the dynamic shears for at least the first four modes of a sixteen story model of an office building using a decaying sinusoidal base excitation of finite duration. The results indicated that the higher modes are important for shear forces in the upper floors.

With the advent of analog and digital computers, the equations of motion of multi-story structures subjected to earthquake excitation could be solved. The earlier research work on multi-story structures using these tools was performed by T. P. Tung[18], Richard L. Jennings[19,20], N. M. Newmark[18,20,23], R. W. Clough[21], J. A. Blume[22,23], J. E. Goldberg[24], J. L. Bogdanoff[24], Z. L. Moh[24], L. Corning[23], J. Penzien[25] and many others. More recent work of this nature has been carried out by J. I.

Bustamente[26], H. Umemura[27], Y. Osawa[27], A. Shibata[27], and G. V. Berg[28], to mention a few.

A number of researchers have visualized this problem as a wave propagation problem in a continuous cantilever beam. Among them are K. Muto[3,29], H. Suyehiro[4], H. M. Westergaard[30], L. S. Jacobsen[31], K. Kanai[32], G. N. Bycroft[33,34,35], M. J. Murphy[35] and L. W. Harrison[35].

During the five years preceding 1967 a large amount of research on multi-story structures using high capacity-high speed digital computers has been done by R. W. Clough[36,37,38,39,1] and his co-workers E. L. Wilson[37,38,39], I. P. King[37,38], K. L. Benuska[39], and T. Y. Lin & Associates[2].

Many of these authors have noted that several modes can be excited in a multi-story structure and that the "higher" modes must be considered in the interfloor shear force in the upper portion of the structure. In this report, quantitative information is provided on the importance of all modes including the "higher" ones to the various responses of a multi-story structure during an earthquake.

In Chapter II two different models of a nonlinear beam which can be used to represent each girder and column in a structural frame are presented and compared. One beam model can have different curvilinear or bilinear bending moment-rotation hysteresis loops located at the yield joints at each end of the beam. The other model is restricted to having bilinear hysteresis loops at the ends. Three definitions of ductility factor applicable to these beams are given and discussed.

The properties of the class of nonlinear multi-story structures considered in this report are listed in Chapter III. In addition, the damping mechanisms are described and the equations of motion for this system are derived. Using the Wilson-Clough integration technique[40], the equations of motion are put into a form suitable for solution by the 7090/7094 digital computer at the Computer Center of the California Institute of Technology.

As an overall check on the computer program for analyzing the response of nonlinear multi-story structures, several tests on it were carried out which are briefly described in Chapter IV. One of these was a comparison with results for a corresponding system presented in the FHA Study[1].

In Chapter V uniform and tapered, shear-type, linear elastic, continuous cantilever beams are analytically studied in order to determine the importance of "higher modes" to the responses, i.e., displacement, strain, shear force, and total acceleration, at various positions in an earthquake excited structure. "Whiplash" is discussed and a definition suggested. A method of determining "equivalent" modal participation factors for nonlinear structures is presented and examples of the "equivalent" modal contributions to displacements and interfloor displacements (strain) are given.

Chapter VI treats the response results for a nonlinear twenty story structure subjected to earthquake excitation. The effect of yielding is observed by comparing the responses for the nonlinear structure with those for the corresponding linear structure. Time history response plots, displacement envelopes, and ductility factors

for the nonlinear structure subjected to the entire duration of several earthquakes are presented and analyzed. The effects of various modes are observed by comparing time history response plots of the multi-story structure with those for one degree of freedom linear oscillators representing the individual modes. Three common measurements of the strength of an earthquake are given and are compared with the magnitudes of the corresponding structural responses for the earthquakes used.

CHAPTER II

MODELS OF BEAMS WITH NONLINEAR CHARACTERISTICS AND ASSOCIATED DEFINITIONS OF DUCTILITY FACTOR

2.1 Introduction

The primary purpose of this chapter is to describe and compare two different models of a nonlinear beam which can be used to represent each girder and column in a discretized model of a nonlinear tall structure. One beam model has identical bilinear bending moment-end rotation hysteresis loops at the yield joints located at each end of the beam. The other model is more versatile since it can have different curvilinear or bilinear bending moment-end rotation hysteresis loops at each end.

In order to develop concepts and terminology useful in discussing yielding beams, simple yielding systems are studied first. In this study the term "simple yielding system" always refers to a system with only one yield joint. A yield joint is similar to a Coulomb slider, but is more general since it can have a curvilinear force-deflection (or bending moment-rotation) relationship. In particular, certain nomenclature pertaining to hysteresis loops is presented, two different simple yielding systems are studied, and three definitions of ductility factor for these simple systems are given. Two independent models of beams with yield joints at each end are presented as extensions from the simple yielding systems. Three definitions of ductility factor which apply to yielding beams are also discussed.

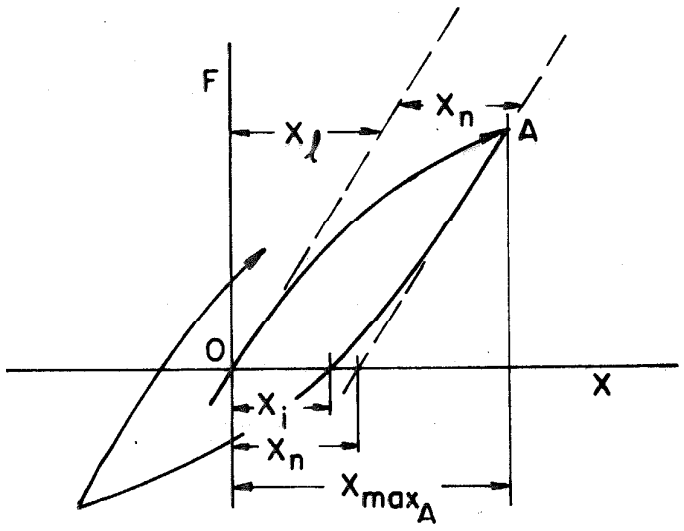

Fig. 2.1 Curvilinear Hysteresis Loop

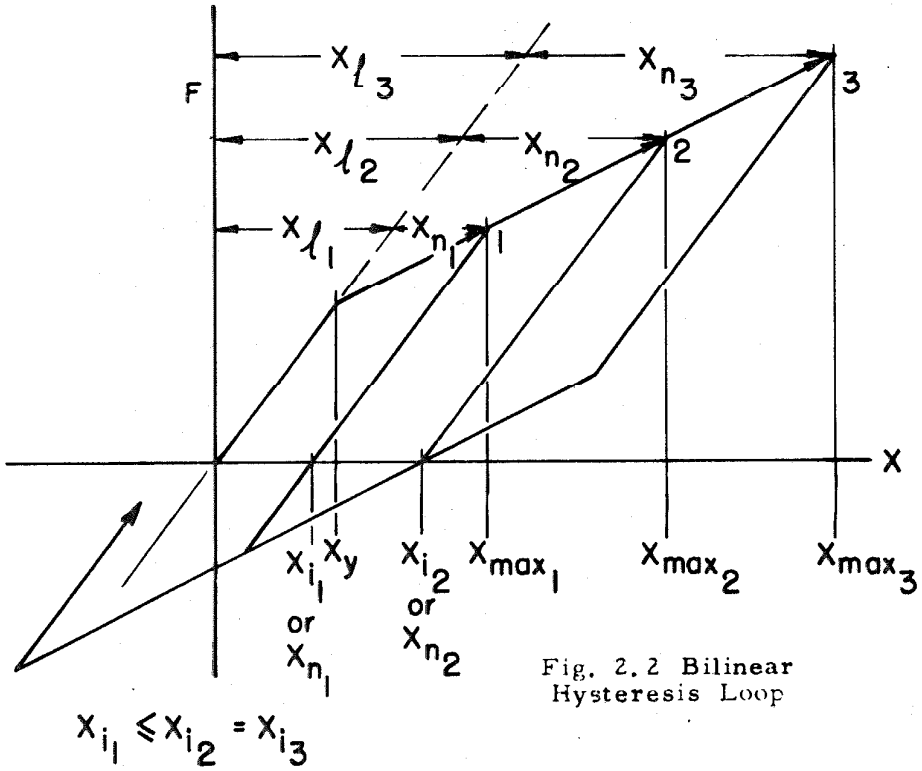

Fig. 2.2 Bilinear Hysteresis Loop

2.2 Nomenclature Pertaining to Hysteresis Loops

The nomenclature pertaining to hysteresis loops as used in this report is defined in this section. A typical curvilinear hysteresis loop is shown in Fig. 2.1 and a typical bilinear hysteresis loop is depicted in Fig. 2.2. For both of these loops, the following nomenclature applies:

$|x|_{max}$ = maximum absolute displacement,

x_y = displacement at yield (yield level),

x_n = nonlinear displacement: departure from the initial tangent (absolute value),

$x_\ell = |x|_{max} - x_n$ = linear displacement: displacement along the initial tangent at the force level of $|x|_{max}$ (absolute value),

x_i = irrecoverable displacement or permanent set (absolute value), and

$x_r = |x|_{max} - x_i$ = recoverable displacement (absolute value).

Hence,

$$x_\ell + x_n = x_r + x_i ,$$

or

$$x_r = x_\ell + x_n - x_i . \tag{2.1}$$

From Fig. 2.1 and Fig. 2.2 it is seen that

$$x_i \leq x_n \quad \text{and} \quad x_\ell \leq x_r . \tag{2.2}$$

Note that for the bilinear hysteresis loop of Fig. 2.2 with

$$|x|_{max} \leq x_{max_2} , \tag{2.3}$$

the irrecoverable displacement is identical to the nonlinear displacement, and the linear displacement is identical to the recoverable displacement:

$$x_i = x_n \quad \text{and} \quad x_\ell = x_r \quad . \tag{2.4}$$

The bilinear hysteresis loop with the maximum displacement restricted by Eq. 2.3 is the one of primary interest in this report.

Looking at Fig. 2.1 and Fig. 2.2, it appears that in many instances the irrecoverable displacement can be much smaller than the corresponding nonlinear displacement. Furthermore, failure is probably more closely related to the nonlinear displacement than to the irrecoverable displacement. Consequently, one definition of ductility based on the nonlinear displacement rather than the irrecoverable displacement is given in the next section.

2.3 Three Definitions of "Ductility Factor" for Simple Yielding Systems

In this report, the term "ductility factor" is used as a measure of the amount of yielding incurred in a system. However, a "ductility factor" has no precise meaning until the method of measuring it has been defined. The purpose of this section is to make three definitions of the term "ductility factor" for the nonlinear spring of the simple yielding system of Fig. 2.3. All three definitions apply to the bilinear hysteresis loop shown in Fig. 2.4 and one of them also applies to the more general curvilinear hysteresis loop of Fig. 2.5.

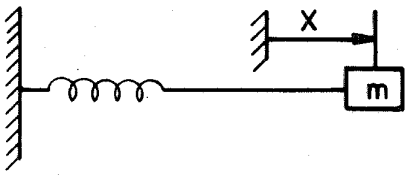

Fig. 2.3 Simple Yielding System With Nonlinear Spring

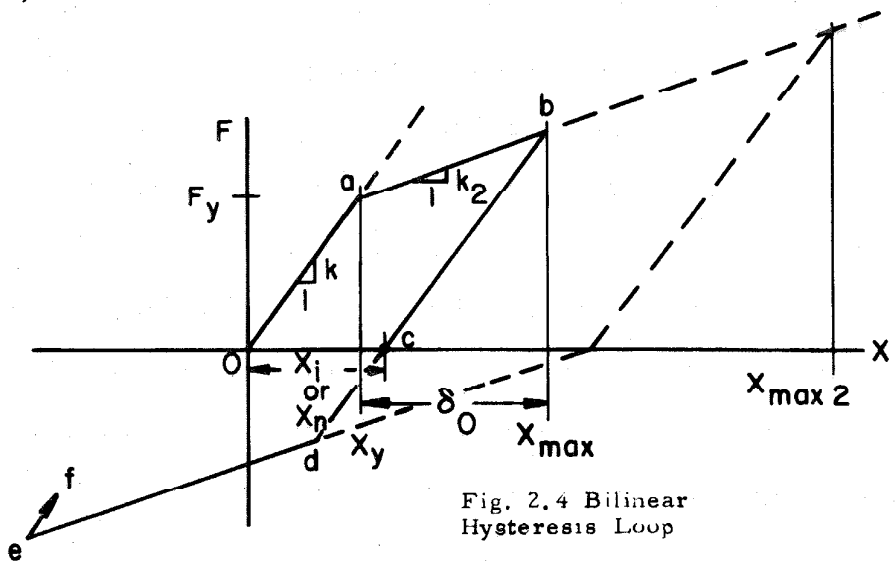

Fig. 2.4 Bilinear Hysteresis Loop

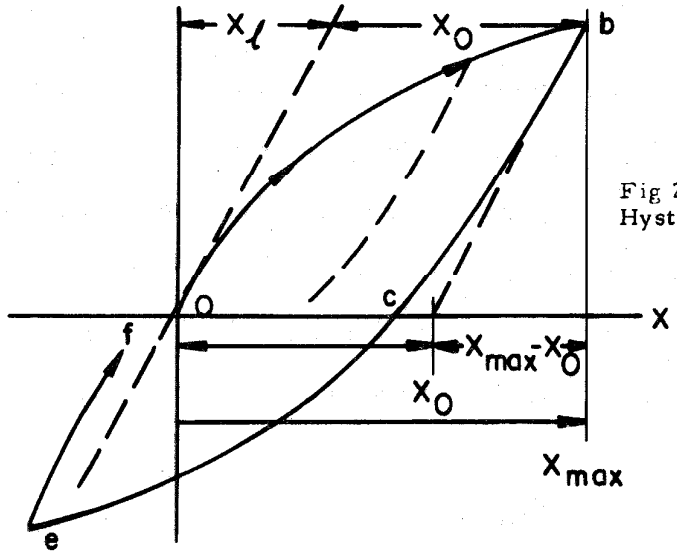

Fig 2.5 Curvilinear Hysteresis Loop

The next section, Section 2.4, treats two systems composed of linear springs and Coulomb sliders which can be used to model a simple yielding system with bilinear hysteretic characteristics. However, for the purposes of this section, it is only necessary to consider the simple yielding system of Figs. 2.3, 2.4, and 2.5.

For the nonlinear spring of the system of Fig. 2.3, one possible hysteresis loop is the path 0, a, b, c, d, e, f in Fig. 2.4. On this path only linear displacement is incurred until the displacement x exceeds x_y at point (a), whereupon it yields. At point (b) the displacement $|x|_{max}$ is composed of the linear displacement, x_y, and additional displacement incurred during yielding, δ_o, where

$$\delta_o = |x|_{max} - x_y \ .$$

Furthermore, using the terminology of Section 2.2, δ_o has both "linear" and "nonlinear" components. Now, if the spring is released so that the force becomes zero, the equilibrium displacement is the irrecoverable displacement x_i, at point (c). Note that for this example, $|x|_{max}$ is less than x_{max_2}; consequently, the nonlinear displacement x_n and irrecoverable displacement x_i are equal:

$$x_n = x_i \quad (\text{if } |x|_{max} \leq x_{max_2}) \ . \tag{2.5}$$

From geometrical considerations, x_n is found from Fig. 2.4 to be

$$x_n = \left(1 - \frac{k_2}{k}\right) \cdot \delta_o \quad . \tag{2.6}$$

Hence, the additional linear displacement incurred in going from point (a) to point (b) is

$$\delta_o - x_n = \left(\frac{k_2}{k}\right) \cdot \delta_o \quad . \tag{2.7}$$

Therefore, the total linear displacement incurred in traversing from (0) to (a) to (b) is

$$x_\ell = x_y + \left(\frac{k_2}{k}\right) \cdot \delta_o \quad . \tag{2.8}$$

The first definition of ductility factor, μ_1, is simply a measure of the maximum absolute displacement at point (b), $|x|_{max}$, (see Fig. 2.4) with respect to the yield displacement, x_y, without regard to the second slope, k_2:

$$\mu_1 = \frac{|x|_{max}}{x_y} \tag{2.9}$$

or

$$\mu_1 = 1 + \frac{\delta_o}{x_y} \quad . \tag{2.10}$$

Although Eq. 2.10 can be used to measure the yielding for any bilinear hysteresis loop, its most appropriate application is to the ideally elasto-plastic hysteresis loop, i.e., a bilinear hysteresis loop with the second slope identically equal to zero, $k_2 \equiv 0$. In this

case, all of the displacement from the yield displacement at point (a) to point (b) is nonlinear:

$$x_n = \delta_o = |x|_{max} - x_y \ , \quad (k_2 \equiv 0) \tag{2.11}$$

and the total linear displacement at point (b) is the yield displacement, x_y:

$$x_\ell = x_y \qquad (k_2 \equiv 0) \ . \tag{2.12}$$

Furthermore, the total energy dissipated, E_D in a complete, symmetric hysteresis loop with $k_2 \equiv 0$ is proportional to $(\mu_1 - 1)$ and is given by

$$E_D = 4F_y \cdot \delta_o \qquad (k_2 \equiv 0) \tag{2.13}$$

where

$$F_y = k \cdot x_y \ . \tag{2.14}$$

The amount of energy dissipated is of interest since, in some cases, dissipating energy by yielding can limit the amplitudes of the response.

Now, for systems with $k_2 \not\equiv 0$, a second definition of ductility factor which measures the nonlinear displacement (instead of the maximum absolute displacement) at point (b) with respect to the yield displacement is

$$\mu_2 = 1 + \frac{x_n}{x_y} \tag{2.15}$$

which, by substituting Eq. 2.6 for x_n, becomes

$$\mu_2 = 1 + \left(1 - \frac{k_2}{k}\right)\frac{\delta_o}{x_y} \quad . \tag{2.16}$$

In this case, the total energy dissipated in a complete symmetric hysteresis loop is proportional to $(\mu_2 - 1)$:

$$E_D = 4F_y \cdot \left(1 - \frac{k_2}{k}\right) \cdot \delta_o \quad . \tag{2.17}$$

For the first two definitions of ductility factor, it is necessary to have a well-defined yield level. However, most curvilinear hysteresis loops do not have an inherently well-defined yield level. Nevertheless, for any hysteresis loop, except one with a vertical initial tangent, the nonlinear and linear displacements are well defined. For these loops a third definition of ductility factor relating the maximum absolute displacement, $|x|_{max}$, at point (b) (see Fig. 2.4) to the linear displacement, x_ℓ, can be made:

$$\mu_3 = \frac{|x|_{max}}{x_\ell}$$

or

$$\mu_3 = 1 + \frac{x_n}{x_\ell} \quad . \tag{2.18}$$

However, the energy dissipated either in a curvilinear or in a bilinear hysteresis loop cannot be directly related to $(\mu_3 - 1)$ as was done above for $(\mu_2 - 1)$ in Eq. (2.17).

When used for the bilinear hysteresis loop of Fig. 2.4, the

third definition can be written in another form. For the maximum absolute displacement in this loop (at point (b)), x_ℓ and x_n are given by Eq. 2.6 and Eq. 2.8, respectively. Substituting these into Eq. 2.18, μ_3 becomes

$$\mu_3 = 1 + \frac{\left(1 - \frac{k_2}{k}\right) \cdot \delta_o}{x_y + \left(\frac{k_2}{k} \cdot \delta_o\right)} \quad . \tag{2.19}$$

In order to show how these three definitions differ for bilinear hysteresis loops, the following numerical examples are considered: Let $x_y = 1$ and $|x|_{max} = 5$ for systems with $k_2/k = 0.05$ and $k_2/k = 0.95$. The various ductility factors are:

Definition	$\frac{k_2}{k} = 0.05$	$\frac{k_2}{k} = 0.95$
$\mu_1 \equiv 1 + \frac{\delta_o}{x_y}$	5.0	5.0
$\mu_2 \equiv 1 + \left(1 - \frac{k_2}{k}\right) \cdot \frac{\delta_o}{x_y}$	4.8	1.2
$\mu_3 \equiv 1 + \frac{\left(1 - \frac{k_2}{k}\right) \cdot \delta_o}{x_y + \left(\frac{k_2}{k} \cdot \delta_o\right)}$	4.17	1.17

(Note that when $k_2 \equiv k = 0$, $\mu_1 = \mu_2 = \mu_3$.) From these examples, one can see that the choice of the definition of ductility factor can make a sizeable difference in the resulting numerical value.

If only bilinear hysteresis loops are of interest, the second definition, μ_2, is a better indication of the amount of yielding

incurred as well as of the energy dissipated than the first, μ_1. On the other hand, if one wishes to compare the ductility factors of curvilinear systems to those of bilinear systems, the third definition, μ_3, is best.

2.4 Two Systems for Modeling a Simple Yielding System With Bilinear Hysteretic Characteristics

This section treats two systems composed of linear springs and Coulomb sliders (see Fig. 2.6 and Fig. 2.7) which are used to model the simple yielding system of Fig. 2.3 with the bilinear hysteresis loop of Fig. 2.4. The purpose of studying these systems is to form a base from which to extend to the more complex problem of nonlinear beams.

As seen in Figs. 2.6, 2.7, 2.8 and 2.9, the primary difference between these two simple yielding systems is the arrangement of the springs and Coulomb sliders. In one, the springs are in series, and in the other, the springs are in parallel. Consequently, the general equations relating forces to displacements as well as those relating the ductility factors to the displacement incurred during yielding have different functional dependencies for the two systems. Concerning the functional dependence of the ductility factor for the springs-in-series system of Fig. 2.6 and Fig. 2.8, the incurred yielding (the amount of slip in the Coulomb slider), δ', is purely the nonlinear* displacement and the corresponding linear* displacement is

*See Section 2.2 for definitions of linear and nonlinear displacements.

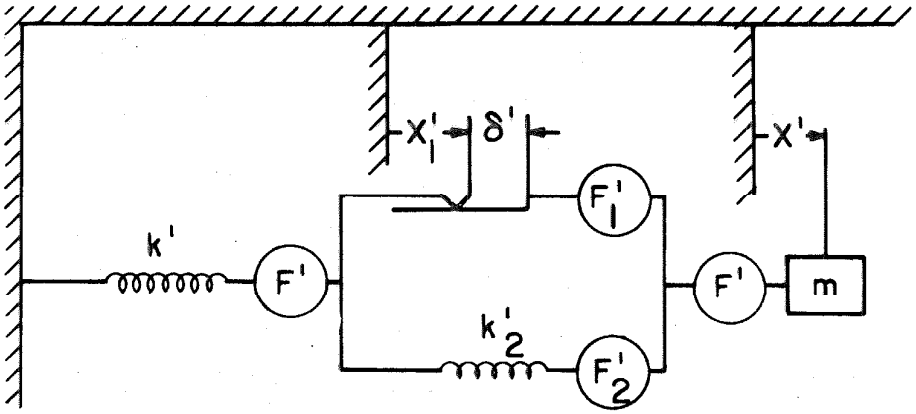

Fig. 2.6 Springs-in-Series

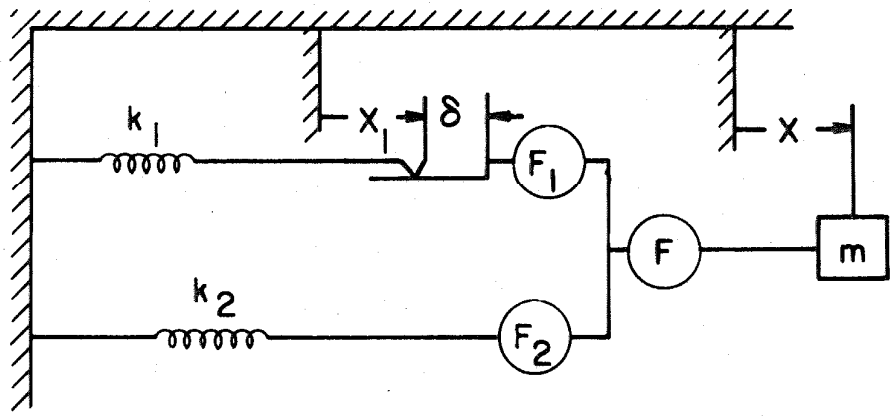

Fig. 2.7 Springs-in-Parallel

SPRINGS-IN-SERIES

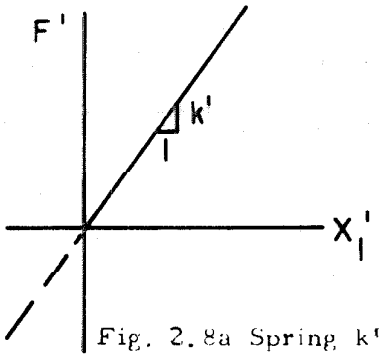

Fig. 2.8a Spring k'

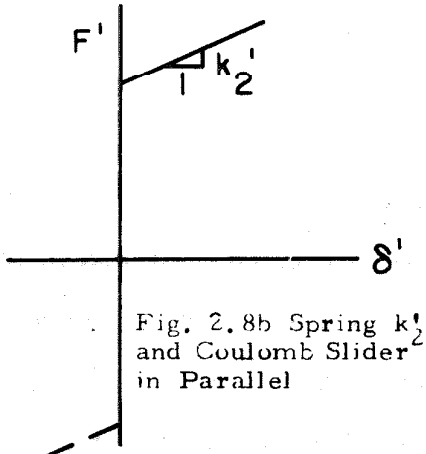

Fig. 2.8b Spring k_2' and Coulomb Slider in Parallel

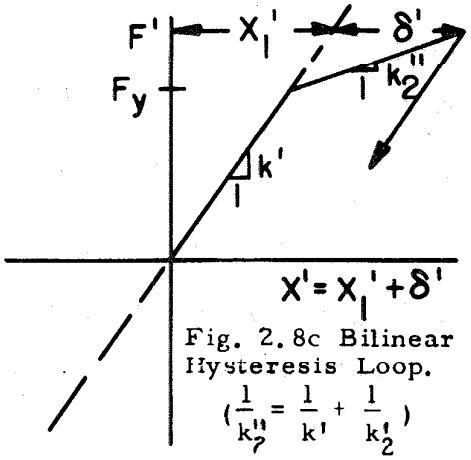

Fig. 2.8c Bilinear Hysteresis Loop.

$$\left(\frac{1}{k_2''} = \frac{1}{k'} + \frac{1}{k_2'}\right)$$

SPRINGS-IN-PARALLEL

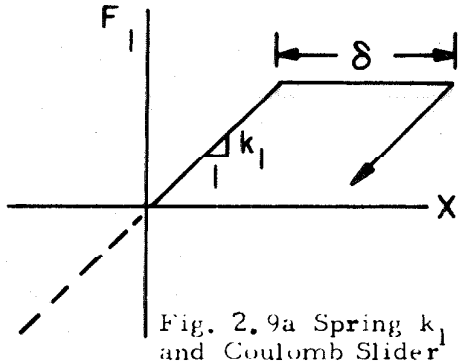

Fig. 2.9a Spring k_1 and Coulomb Slider in Series

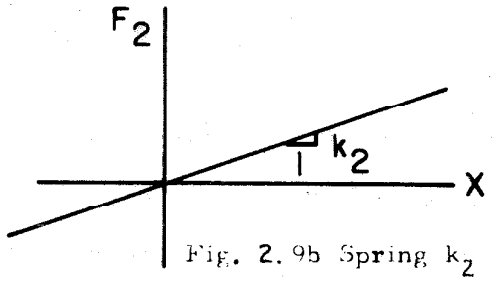

Fig. 2.9b Spring k_2

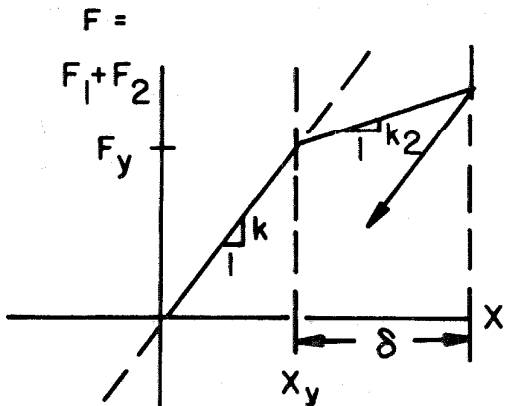

Fig. 2.9c Bilinear Hysteresis Loop.

$(k = k_1 + k_2)$

$$|x|_{max} - \delta' \quad .$$

Since δ', as shown in Fig. 2.8c, corresponds to x_n of Fig. 2.4, the ductility factor for this system by the third definition, Eq. 2.19, is

$$\mu_{3_{series}} = 1 + \frac{\delta'}{|x|_{max} - \delta'}$$

or

$$\mu_{3_{series}} = 1 + \frac{\delta'}{x_1'} \quad . \tag{2.20}$$

In the springs-in-parallel system of Figs. 2.7 and 2.9, the displacement incurred during yielding, δ, is the sum of both the nonlinear and the linear displacements incurred by the entire system beyond the yield point. Since δ of Fig. 2.9c corresponds to δ_o of Fig. 2.4, the ductility factor for this system, by the third definition, Eq. 2.19, is

$$\mu_{3_{parallel}} = 1 + \frac{\left(1 - \frac{k_2}{k}\right)\delta}{x_y + \frac{k_2}{k}\delta} \tag{2.21}$$

where $k = k_1 + k_2$.

In the following vis-à-vis presentation, the different functional dependencies of the forces upon displacements in the two systems are studied.

Springs-in-Series (Fig. 2.6 and Fig. 2.8)

General equations:

$$F' = F'_1 + F'_2;$$

$$x' = x'_1 + \delta' ;$$

in incremental form:

$$\Delta F' = \Delta F'_1 + \Delta F'_2 ; \qquad (2.22.1)$$

$$\Delta x' = \Delta x'_1 + \Delta \delta' . \qquad (2.23.1)$$

For small displacements:

$$F' = k'x' .$$

Spring k':

$$F' = k'x'_1 ;$$

$$\Delta F' = k'\Delta x'_1 . \qquad (2.24.1)$$

Spring k'_2:

$$F'_2 = k'_2\delta' ;$$

$$\Delta F'_2 = k'_2\Delta\delta' . \qquad (2.25.1)$$

Coulomb slider:

If F'_y is the yield limit for the force F', then

$$F'_{1y} = F'_y$$

Springs-in-Parallel (Fig. 2.7 and Fig. 2.9)

General equations:

$$F = F_1 + F_2 ;$$

$$x = x_1 + \delta ;$$

in incremental form:

$$\Delta F = \Delta F_1 + \Delta F_2 ; \qquad (2.22.2)$$

$$\Delta x = \Delta x_1 + \Delta \delta . \qquad (2.23.2)$$

For small displacements:

$$F = kx$$

where

$$k = k_1 + k_2 ;$$

or

$$k_1 \equiv qk, \quad k_2 \equiv pk ,$$

and

$$p + q = 1 .$$

Spring k_1:

$$F_1 = k_1x_1 ;$$

$$\Delta F_1 = k_1\Delta x_1 . \qquad (2.24.2)$$

Spring k_2:

$$F_2 = k_2x ;$$

$$\Delta F_2 = k_2\Delta x . \qquad (2.25.2)$$

Coulomb slider:

If F_y is the yield limit for the force F, then

$$F_{1y} = q \cdot F_y$$

Springs-in-Series, cont.

is the yield limit for the Coulomb slider:

$$|F_1'| \leq F_{1y}' .$$

Yield Criteria:

When

$$|F_1'| < F_{1y}' \quad \text{(linear state)},$$

$$\Delta\delta' = 0 .$$

Hence,

$$\Delta F' = k' \cdot \Delta x' . \qquad (2.26.1)$$

When

$$|F_1'| = F_{1y}' \quad \text{(nonlinear state)}$$

$$\Delta F_1' = 0 .$$

Hence, from Eq. 2.22.1,

$$\Delta F' = \Delta F_2'$$

and with Eq. 2.25.1,

$$\Delta F' = k_2' \cdot \Delta\delta' . \qquad (2.27.1)$$

Also, in this state

$$\Delta F' = k_2'' \cdot \Delta x'$$

where k_2'', as shown in Fig. 2.8c, is given by

$$\frac{1}{k_2''} = \frac{1}{k'} + \frac{1}{k_2'} .$$

Springs-in-Parallel, cont.

is the yield level for the Coulomb slider:

$$|F_1| \leq F_{1y} .$$

Yield Criteria:

When

$$|F_1| < F_{1y} \quad \text{(linear state)},$$

$$\Delta\delta = 0 .$$

Hence,

$$\Delta x_1 = \Delta x, \text{ and}$$

$$\Delta F = k \cdot \Delta x . \qquad (2.26.2)$$

When

$$|F_1| = F_{1y} \quad \text{(nonlinear state)}$$

$$\Delta F_1 = 0 .$$

Hence,

$$\Delta x_1 = 0$$

and

$$\Delta\delta = \Delta x .$$

From Eq. 2.22.2, with $\Delta F_1 = 0$,

$$\Delta F = \Delta F_2 \qquad (2.27.2)$$

and

$$\Delta F = k_2 \Delta x ,$$

i.e., any increase of the force above the yield level is done via spring k_2 only.

In order to make the bilinear spring systems appear the same to mass (m) in Fig. 2.6 and Fig. 2.7, let

$$x' = x \ , \quad (\Delta x' = \Delta x) \ ;$$

and

$$F' = F \ ; \quad (\Delta F' = \Delta F) \ .$$

Then, from Eq. 2.26.1 and Eq. 2.26.2 for the linear state, it is found that

$$k' = k$$

and, correspondingly, for the nonlinear state, it is found that

$$k_2'' = k_2 \ ,$$

Hence, by substitution, an equation for k_2' in terms of k and k_2 arises:

$$\frac{1}{k_2'} = \frac{1}{k_2} - \frac{1}{k} \ .$$

This shows that it is possible to adjust the spring constants of these two simple yielding systems so that they appear the same from the outside in both states of yield. As will be seen in the next section, it is <u>not</u> possible to adjust the spring constants of the two models of nonlinear beams so that they appear the same in all states of yield.

2.5 Two Models of Nonlinear Beams

The two beam models presented in this section have nonlinear hysteretic bending moment-end rotation characteristics; consequently, they are more complicated than the simple yielding systems of the previous sections. It should be noted that the "one-component" beam model, Fig. 2.10, is an extension of the springs-in-series system,

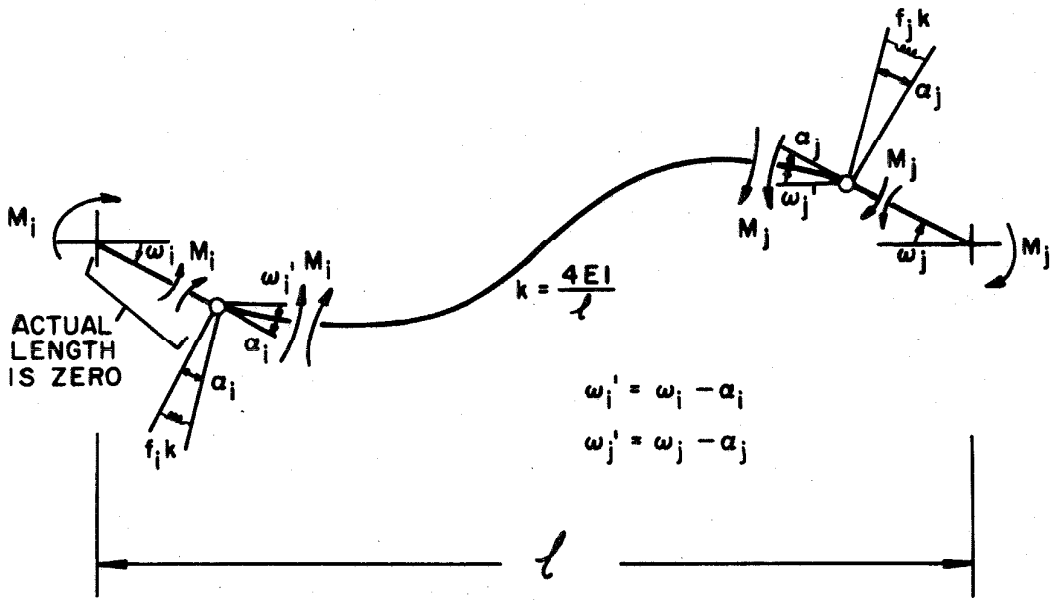

Fig. 2.10 One-component Model of a Nonlinear Beam

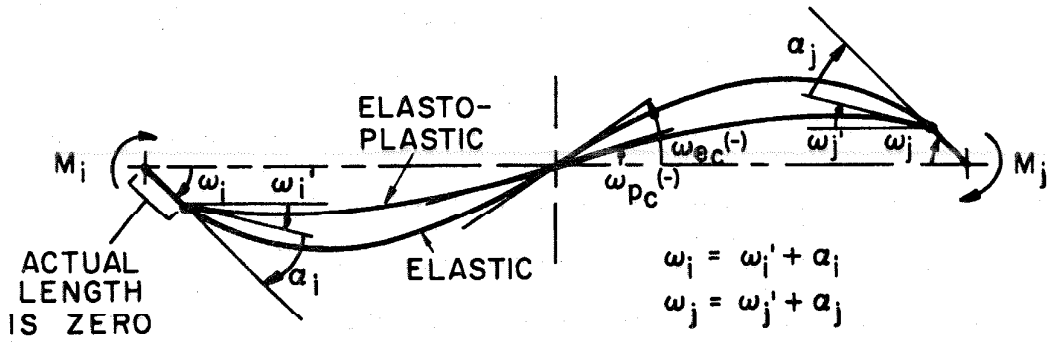

Fig. 2.11a Two-component Model
of a Nonlinear Beam

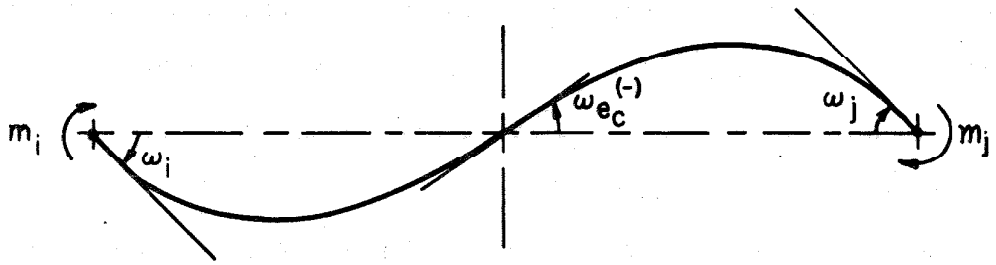

Fig. 2.11b The Linear Component

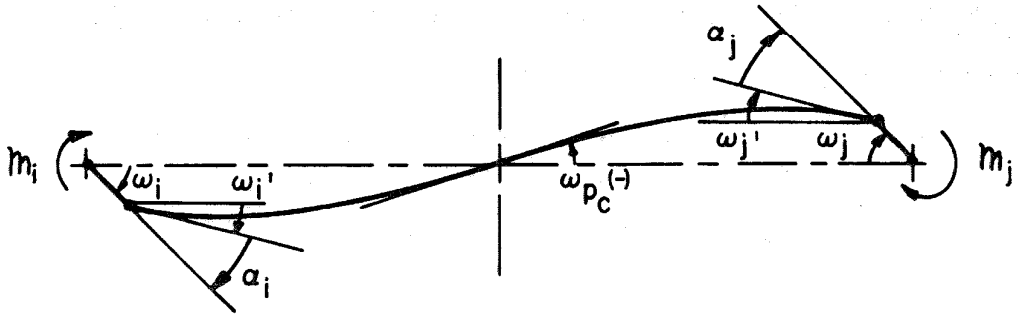

Fig. 2.11c The Ideally Elasto-plastic Component
(Note that the plastic angles α_i and α_j occur only in this component.)

Fig. 2.6 and the "two-component" beam model, Fig. 2.11, is an extension of the springs-in-parallel system, Fig. 2.7.

As seen in Fig. 2.10 and Fig. 2.11, the one-component beam is the more appealing model of a yielding column or girder from the physical point of view. Furthermore, nonlinear (bilinear and curvilinear) hysteresis loops can be used with the one-component beam, the only restriction being that the initial slopes of the hysteresis loops at both ends of the same beam must be equal. On the other hand, the two-component model can have only bilinear hysteresis loops at each end. Although the yield levels can differ, these loops must have identical initial and identical second slopes. For this model, the second slope of the loop is determined by the stiffness, pk, of the linear component of the beam.

Nevertheless, the two-component beam has been used by other investigators* in the study of nonlinear tall structures with results that are very similar to those obtained using the corresponding one-component beam. Consequently, both of these beam models are studied in the following vis-à-vis presentation.

In the linear state both beam models have stiffness

$$k = \frac{4EI}{\ell}$$

where

E is Youngs modulus (lbs/in^2),

*R. W. Clough and K. L. Benuska, FHA Study, reference 1.

I is the area moment of inertia (in^4), and

ℓ is the length of the beam (in) .

The shear deflection in the beam is neglected.

Although the same symbols are used in the moment-rotation equations for both models, some of the definitions differ slightly, particularly for the plastic angles α. As seen in Figs. 2.10 and 2.11, the symbols have the following meanings:

M_i, M_j	bending moments at the ends (i) and (j) (same in both models),
m_i, m_j	bending moments at the ends of the linear component of the two-component model,
$\mathbb{m}_i, \mathbb{m}_j$	bending moments at the ends of the elasto-plastic component of the two-component model,
ω_i, ω_j	end rotations (same in both models),
ω_i', ω_j'	end rotations of central beam in one-component model, or end rotations of central beam of elasto-plastic component in two-component model,
α_i, α_j	incurred plastic angle at the ends in the one-component model (note correspondence to δ' in Fig. 2.8c); or incurred plastic angle at the ends of the elasto-plastic component in the two-component model (note correspondence to δ in Fig. 2.9c).

One-component Model (Fig. 2.10)	Two-component Model (Fig. 2.11)
Stiffness distribution:	Stiffness distribution:
The central beam has stiffness k. The end beams have zero length and are dropped from further consideration.	The stiffness k of the total beam is apportioned to the two components as follows:
	linear component:
	$k_{linear} = p \cdot k$;
	elasto-plastic component:
	$k_{elasto\text{-}plastic} = q \cdot k$;
	where
	$p + q = 1$; (usually $p \approx 0.05$).
Fundamental Bending Moment-End Rotation Equations:	Fundamental Bending Moment-End Rotation Equations:
	linear component:
	$m_i = pk(\omega_i + \frac{1}{2}\omega_j)$
	$m_j = pk(\frac{1}{2}\omega_i + \omega_j)$
	or, in incremental form,
	$\Delta m_i = pk(\Delta\omega_i + \frac{1}{2}\Delta\omega_j)$
	$\Delta m_j = pk(\frac{1}{2}\Delta\omega_i + \Delta\omega_j)$
	elasto-plastic component:
	$\mathfrak{m}_i = qk(\omega_i' + \frac{1}{2}\omega_j')$
	$\mathfrak{m}_j = qk(\frac{1}{2}\omega_i' + \omega_j')$
	where $q = 1 - p$.
	From Fig. 2.11a,
	$\omega_i' = \omega_i - \alpha_i$

One-component Model, cont.

Central beam

$$M_i = k(\omega_i' + \tfrac{1}{2}\omega_j')$$

$$M_j = k(\tfrac{1}{2}\omega_i' + \omega_j') \ .$$

From Fig. 2.10,

$$\omega_i' = \omega_i - \alpha_i$$

and

$$\omega_j' = \omega_j - \alpha_j \ .$$

By substitution, the fundamental bending moment-end rotation equations for this beam model are obtained:

$$\begin{aligned} M_i &= k[\ (\omega_i - \alpha_i) + \tfrac{1}{2}(\omega_j - \alpha_j)] \\ M_j &= k[\tfrac{1}{2}\ (\omega_i - \alpha_i) + (\omega_j - \alpha_j)] \end{aligned} \qquad (2.28.1)$$

or, in incremental form:

$$\Delta M_i = k[\ (\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - \Delta\alpha_j)] \qquad (2.29.1)$$

$$\Delta M_j = k[\tfrac{1}{2}\ (\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j)] \ .$$

Two-component Model, cont.

and

$$\omega_j' = \omega_j - \alpha_j \ .$$

By substitution, then,

$$\mathbb{m}_i = qk[\ (\omega_i - \alpha_i) + \tfrac{1}{2}(\omega_j - \alpha_j)]$$

$$\mathbb{m}_j = qk[\ \tfrac{1}{2}(\omega_i - \alpha_i) + (\omega_j - \alpha_j)]$$

or, in incremental form:

$$\Delta\mathbb{m}_i = qk[\ (\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - \Delta\alpha_j)]$$

$$\Delta\mathbb{m}_j = qk[\ \tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j)]$$

For the total beam:

$$M_i = \mathbb{m}_i + m_i$$

$$M_j = \mathbb{m}_j + m_j \ ;$$

similarly in incremental form.

By addition, the fundamental bending moment-end rotation equations for this beam model are obtained:

$$\begin{aligned} M_i &= k[\ (\omega_i - q\alpha_i) + \tfrac{1}{2}(\omega_j - q\alpha_j)] \\ M_j &= k[\tfrac{1}{2}(\omega_i - q\alpha_i) + (\omega_j - q\alpha_j)] \end{aligned} \qquad (2.28.2)$$

or, in incremental form:

$$\Delta M_i = k[\ (\Delta\omega_i - q\Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - q\Delta\alpha_j)] \qquad (2.29.2)$$

$$\Delta M_j = k[\tfrac{1}{2}\ (\Delta\omega_i - q\Delta\alpha_i) + (\Delta\omega_j - q\Delta\alpha_j)] \ .$$

The purpose of writing these equations for both beam models in incremental form is that in this form it is possible to solve the equations of motion using finite integration techniques assuming that the state of yield remains constant throughout each time increment. As seen above, the incremental bending moments, ΔM, are related to both the incremental rotations, $\Delta\omega$, and incremental plastic angles, $\Delta\alpha$. Now, if the state of yield is known at the beginning of the time increment, it is possible to establish beforehand an equation of the form

$$\Delta\alpha = \Delta\alpha(\Delta\omega_i, \Delta\omega_j)$$

relating the incremental plastic angles to the end rotations. Using these equations it is possible to eliminate the incremental plastic angles from the incremental moment-rotation equations resulting in equations of the form:

$$\Delta M = \Delta M(\Delta\omega_i, \Delta\omega_j)$$

which are valid for each time increment.

The criteria for establishing the state of yield at the time t (at the beginning of a time increment) are based upon the bending moment at time t and the last incremental bending moment prior to time t. These criteria, which are the same for both beam models, are (see Fig. 2.12) the following:

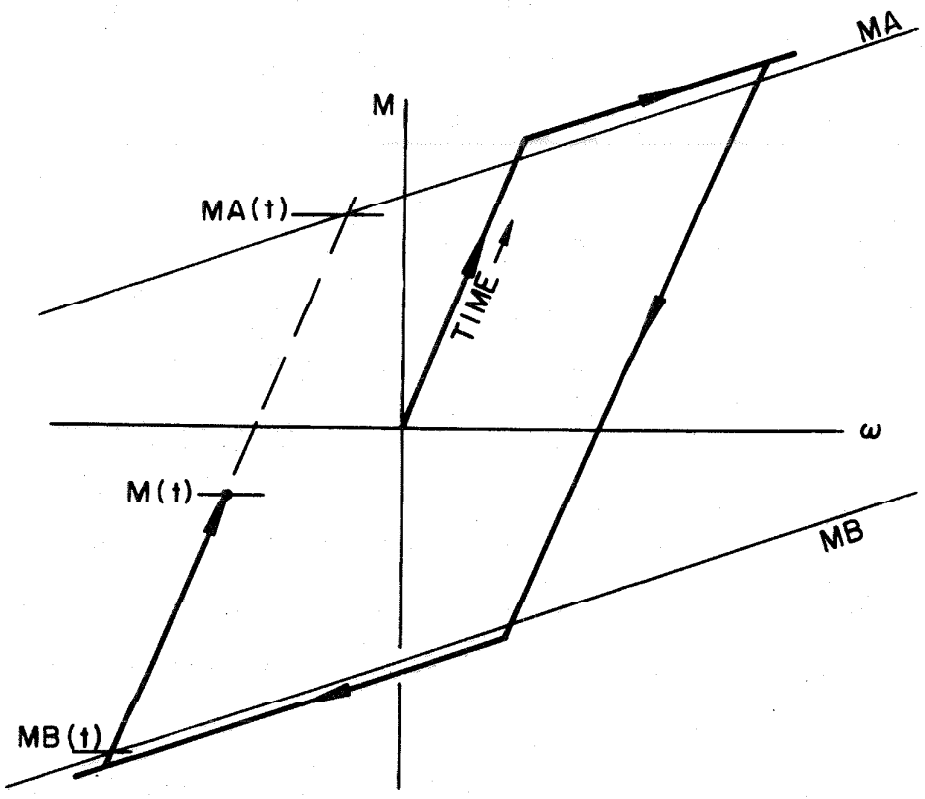

Fig. 2.12 Bilinear Hysteresis Loop Showing "Overshooting" Upon Entering the Nonlinear State of Yield and "Backtracking" Upon Returning to the Linear State

If $MB(t) < M(t) < MA(t)$, then the state of yield is linear;

if $\left\{ \begin{array}{l} M(t) \geq MA(t) \text{ and } \Delta M(t) > 0, \\ \text{or} \\ M(t) \leq MB(t) \text{ and } \Delta M(t) < 0 \end{array} \right\}$ $\left\{ \begin{array}{l} \text{then the state of} \\ \text{yield is nonlinear;} \end{array} \right.$

where

$M(t)$: total bending moment at time t,

$MA(t)$: upper yield bending moment at time t,

$MB(t)$: lower yield bending moment at time t, and

$\Delta M(t)$: the last incremental bending moment prior to time t.

Inherent in this procedure is the phenomena of "overshooting" the yield limit upon entering the nonlinear state and "backtracking" upon returning to the linear state, as shown in Fig. 2.12. This results from the assumption that the state of yield remains constant throughout the time increment.

Although the yield criteria and the general form of the equations relating moments and plastic angles to rotations are the same for both beams, the actual equations for each beam are different as shown below.

One-component Model	Two-component Model
When the state of yield is linear at end (i) and/or end (j), the corresponding incremental plastic angle must be zero:	When the state of yield is linear at end (i) and/or end (j), the corresponding incremental plastic angle must be zero:
at end (i), $\Delta\alpha_i = 0$;	at end (i), $\Delta\alpha_i = 0$;
and/or	and/or

One-component Model, cont.

at end (j), $\Delta\alpha_j = 0$.

When the state of yield is nonlinear at end (i) and/or end (j), the corresponding incremental bending moment is proportional to the incremental plastic angle:

at end (i), $\Delta M_i = f_i k \Delta\alpha_i$;

and/or

at end (j), $\Delta M_j = f_j k \Delta\alpha_j$;

where f_i and f_j are independent. These equations are analogous to Eq. 2.27.1 of the springs-in-series system of the previous section.

Two-component Model, cont.

at end (j), $\Delta\alpha_j = 0$.

When the state of yield is nonlinear at end (i) and/or end (j), the corresponding incremental total bending moment equals the incremental bending moment of the linear component, and the incremental bending moment in the elasto-plastic component is zero:

at end (i), $\Delta M_i = \Delta m_i$,

$\Delta \mathfrak{m}_i = 0$;

and/or

at end (j), $\Delta M_j = \Delta m_j$,

$\Delta \mathfrak{m}_j = 0$.

These equations are analogous to Eq. 2.27.2 of the springs-in-parallel system in the previous section.

Since f_i and f_j are independent, it is possible to use a curvilinear hysteresis loop with the one-component model. The method of determining f_i and f_j is discussed in the next section. On the other hand, for the two-component model, Δm_i and Δm_j are dependent upon the same stiffness parameter (p). Consequently,

this model can have hysteresis loops at the ends with only the two slopes (k) and (pk) although the yield levels may be different.

Since there are two possible states of yield for each yield joint, there are four possible states of yield for a beam. These states, denoted by (a), (b), (c), and (d), are described below.

One-component Model	Two-component Model
State (a) - linear at (i) and (j):	State (a) - linear at (i) and (j):
$\Delta\alpha_i = 0$;	$\Delta\alpha_i = 0$;
$\Delta\alpha_j = 0$.	$\Delta\alpha_j = 0$.
State (b) - nonlinear at (i); linear at (j):	State (b) - nonlinear at (i); linear at (j):
$\Delta M_i = f_i k \Delta\alpha_i$;	$\Delta M_i = \Delta m_i$; $(\Delta\alpha_i \neq 0)$;
$\Delta\alpha_j = 0$.	$\Delta\alpha_j = 0$.
State (c) - linear at (i); nonlinear at (j):	State (c) - linear at (i); nonlinear at (j):
$\Delta\alpha_i = 0$;	$\Delta\alpha_i = 0$;
$\Delta M_j = f_j k \Delta\alpha_j$.	$\Delta M_j = \Delta m_j$; $(\Delta\alpha_j \neq 0)$.
State (d) - nonlinear at (i) and (j):	State (d) - nonlinear at (i) and (j):
$\Delta M_i = f_i k \Delta\alpha_i$;	$\Delta M_i = \Delta m_i$; $(\Delta\alpha_i \neq 0)$;
$\Delta M_j = f_j k \Delta\alpha_j$.	$\Delta M_j = \Delta m_j$; $(\Delta\alpha_j \neq 0)$.
The functional dependencies of the incremental plastic angles upon rotations are as follows:	The functional dependencies of the incremental plastic angles upon rotations are as follows:

One-component Model, cont.

State (a) - linear at (i) and (j):

$\Delta\alpha_i = 0$;

$\Delta\alpha_j = 0$.

State (b) - nonlinear at (i); linear at (j):

from state (b) above and

Eq. 2.29.1,

$k[(\Delta\omega_i - \Delta\alpha_i) + \frac{1}{2}\Delta\omega_j] = f_i k\Delta\alpha_i$.

Hence,

$\Delta\alpha_i = \left(\frac{1}{1+f_i}\right)(\Delta\omega_i + \frac{1}{2}\Delta\omega_j)$;

$\Delta\alpha_j = 0$.

State (c) - linear at (i) ; nonlinear at (j) :

(similar to state (b); replace (i) by (j)):

$\Delta\alpha_i = 0$;

$\Delta\alpha_j = \left(\frac{1}{1+f_j}\right)(\frac{1}{2}\Delta\omega_i + \Delta\omega_j)$

State (d) - nonlinear at (i) and (j):

from state (d) above and

Eq. 2.29.1,

$k[(\Delta\omega_i - \Delta\alpha_i) + \frac{1}{2}(\Delta\omega_j - \Delta\alpha_j)] = f_i k\Delta\alpha_i$

$k[\frac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j)] = f_j k\Delta\alpha_j$

Two-component Model, cont.

State (a) - linear at (i) and (j):

$\Delta\alpha_i = 0$;

$\Delta\alpha_j = 0$.

State (b) - nonlinear at (i); linear at (j):

from state (b) above and

Eq. 2.29.2

$k[(\Delta\omega_i - q\Delta\alpha_i) + \frac{1}{2}\Delta\omega_j] = pk(\Delta\omega_i + \frac{1}{2}\Delta\omega_j)$.

Hence,

$\Delta\alpha_i = (\Delta\omega_i + \frac{1}{2}\Delta\omega_j)$;

$\Delta\alpha_j = 0$.

State (c) - linear at (i) ; nonlinear at (j):

(similar to state (b); replace (i) by (j)):

$\Delta\alpha_i = 0$;

$\Delta\alpha_j = (\frac{1}{2}\Delta\omega_i + \Delta\omega_j)$.

State (d) - nonlinear at (i) and (j):

from state (d) above and

Eq. 2.29.2,

$k[(\Delta\omega_i - q\Delta\alpha_i) + \frac{1}{2}(\Delta\omega_j - q\Delta\alpha_j)] = pk(\Delta\omega_i + \frac{1}{2}\Delta\omega_j)$

$k[\frac{1}{2}(\Delta\omega_i - q\Delta\alpha_i) + (\Delta\omega_j - q\Delta\alpha_j)] = pk(\frac{1}{2}\Delta\omega_i + \Delta\omega_j)$

One-component Model, cont.

rearranging:

$$(1+f_i)\Delta\alpha_i + \tfrac{1}{2}\Delta\alpha_j = \Delta\omega_i + \tfrac{1}{2}\Delta\omega_j ;$$

$$\tfrac{1}{2}\Delta\alpha_i + (1+f_j)\Delta\alpha_j = \tfrac{1}{2}\Delta\omega_i + \Delta\omega_j .$$

Hence,

$$\Delta\alpha_i = \frac{\left(1+\frac{4}{3}f_j\right)\Delta\omega_i + \frac{2}{3}f_j\Delta\omega_j}{1+\frac{4}{3}(f_i+f_j+f_i\cdot f_j)} ;$$

$$\Delta\alpha_j = \frac{\frac{2}{3}f_i\Delta\omega_i + \left(1+\frac{4}{3}f_i\right)\Delta\omega_j}{1+\frac{4}{3}(f_i+f_j+f_i\cdot f_j)} .$$

By substitution, the $\Delta\alpha$'s can be eliminated from the incremental moment-rotation equations:

State (a) - linear at (i) and (j):

$$\Delta M_i = k(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) ;$$

$$\Delta M_j = k(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j) .$$

State (b) - nonlinear at (i); linear at (j):

$$\Delta M_i = \left(\frac{1}{1+f_i}\right)k(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) ;$$

$$\Delta M_j = k\left[\tfrac{1}{2}\left(\frac{1}{1+f_i}\right)\Delta\omega_i + \left(\frac{3+4f_i}{4(1+f_i)}\right)\Delta\omega_j\right].$$

Two-component Model, cont.

rearranging:

$$(\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}(\Delta\omega_j - \Delta\alpha_j) = 0 ;$$

$$\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + (\Delta\omega_j - \Delta\alpha_j) = 0 .$$

Hence,

$$\Delta\alpha_i = \Delta\omega_i ;$$

$$\Delta\alpha_j = \Delta\omega_j .$$

By substitution, the $\Delta\alpha$'s can be eliminated from the incremental moment-rotation equations:

State (a) - linear at (i) and (j):

$$\Delta M_i = k(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) ;$$

$$\Delta M_j = k(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j) .$$

State (b) - nonlinear at (i); linear at (j):

$$\Delta M_i = pk(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j) ;$$

$$\Delta M_j = k\left[\tfrac{1}{2}p\Delta\omega_i + \left(1-\frac{q}{4}\right)\Delta\omega_j\right].$$

One-component Model, cont.

State (c) - linear at (i); nonlinear at (j):

$$\Delta M_i = k\left[\left(\frac{3+4f_j}{4(1+f_j)}\right)\Delta\omega_i + \tfrac{1}{2}\left(\frac{f_j}{1+f_j}\right)\Delta\omega_j\right];$$

$$\Delta M_j = \left(\frac{f_j}{1+f_j}\right)k\left(\tfrac{1}{2}\Delta\omega_i+\Delta\omega_j\right) .$$

State (d) - nonlinear at (i) and (j):

$$\Delta M_i = \frac{f_i k\left[\left(1+\frac{4}{3}f_j\right)\Delta\omega_i + \frac{2}{3}f_j\Delta\omega_j\right]}{1+\frac{4}{3}(f_i+f_j+f_i\cdot f_j)} ;$$

$$\Delta M_j = \frac{f_j k\left[\frac{2}{3}f_i\Delta\omega_i + \left(1+\frac{4}{3}f_i\right)\Delta\omega_j\right]}{1+\frac{4}{3}(f_i+f_j+f_i\cdot f_j)} .$$

(2.30.1)

Two-component Model, cont.

State (c) - linear at (i); nonlinear at (j):

$$\Delta M_i = k\left[\left(1-\frac{q}{4}\right)\Delta\omega_i + \tfrac{1}{2}p\Delta\omega_j\right] ;$$

$$\Delta M_j = pk\left(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j\right) .$$

State (d) - nonlinear at (i) and (j):

$$\Delta M_i = pk\left(\Delta\omega_i + \tfrac{1}{2}\Delta\omega_j\right) ;$$

$$\Delta M_j = pk\left(\tfrac{1}{2}\Delta\omega_i + \Delta\omega_j\right) .$$

(2.30.2)

Since the incremental bending moment-end rotation equations have a regular pattern for all four states of yield, the following matrix equations using the "effective stiffness" parameters S_A, S_B and S_C can be established:

One-component Model

$$\begin{Bmatrix}\Delta M_i\\ \Delta M_j\end{Bmatrix} = \begin{bmatrix}S_A & S_B\\ S_B & S_C\end{bmatrix}\begin{Bmatrix}\Delta\omega_i\\ \Delta\omega_j\end{Bmatrix}$$

(2.31.1)

Two-component Model

$$\begin{Bmatrix}\Delta M_i\\ \Delta M_j\end{Bmatrix} = \begin{bmatrix}S_A & S_B\\ S_B & S_C\end{bmatrix}\begin{Bmatrix}\Delta\omega_i\\ \Delta\omega_j\end{Bmatrix}$$

(2.31.2)

One-component Model, cont.

where

state of yield:	S_A	S_B	S_C
a	k	$\frac{1}{2}k$	k
b	$\frac{f_i k}{(1+f_i)}$	$\frac{1}{2}\frac{f_i k}{(1+f_i)}$	$\frac{(3+4f_i)k}{4(1+f_i)}$
c	$\frac{(3+4f_j)k}{4(1+f_j)}$	$\frac{1}{2}\frac{f_j k}{(1+f_j)}$	$\frac{f_j k}{(1+f_j)}$
d	$\frac{f_i(1+\frac{4}{3}f_j)k}{D}$	$\frac{\frac{2}{3}f_i f_j k}{D}$	$\frac{f_j(1+\frac{4}{3}f_i)k}{D}$

where

$$D = 1 + \frac{4}{3}(f_i + f_j + f_i \cdot f_j)$$

Note that for bilinear hysteresis loops, which are of primary concern here, $f_i = f_j = f$. As a result, some of the effective stiffnesses can be simplified.

Two-component Model, cont.

where

state of yield:	S_A	S_B	S_C
a	k	$\frac{1}{2}k$	k
b	pk	$\frac{1}{2}pk$	$(1-\frac{q}{4})k$
c	$(1-\frac{q}{4})k$	$\frac{1}{2}pk$	pk
d	pk	$\frac{1}{2}pk$	pk

In order to identically match the bilinear hysteretic response of the two beam models, an (f:p) relationship must be found which equates the corresponding incremental moment-rotation equations for each of the four states of yield. Although one (f:p) relationship forms a match between the two beam models for yield states (a), (b),

and (c), and another (f:p) relationship forms a match for yield states (a) and (d), it is not possible to find an (f:p) relationship which forms a match for all four states of yield.

However, in order to compare the response of a nonlinear tall structure using the one-component model to the response of the same nonlinear tall structure using the two-component model, it is necessary to choose the most representative (f:p) relationship. Since it is common in a tall structure for both ends of a girder to rotate symmetrically and to yield at the same time, the (f:p) relationship chosen is the one which matches the incremental moment-rotation equations for yield states (a) and (d). With

$$\Delta M_i = \Delta M_j = \Delta M$$

and

$$\Delta \omega_i = \Delta \omega_j = \Delta \omega \ ,$$

the following moment-rotation equations are obtained for yield state (d). For the one-component model, Eq. 2.30.1 yields

$$\Delta M = \frac{f}{(1 + \frac{2}{3} f)} k\Delta\omega \ ; \qquad (2.32.1)$$

and for the two-component model, Eq. 2.30.2 yields

$$\Delta M = 1.5 \ pk\Delta\omega \ . \qquad (2.32.2)$$

By equating the incremental bending moments and end rotations for the two models, the desired (f:p) relationship is found

$$f = \frac{1.5p}{1 - p} \qquad \text{or} \qquad p = \frac{f}{1.5 + f} \quad .$$

For example, if $p = 0.050$, then $f = 0.079$.

2.6 Hysteresis Loops and Simple Yielding Systems for Nonlinear Beams

In order to draw an exact two-dimensional hysteresis loop for either end of a beam, it is necessary to have the same functional relationship between the bending moment and the curvature for all states of yield. Such a hysteresis loop, Fig. 2.13d, can be established for end (i) of the one-component model of Fig. 2.13a by defining the "simple yielding beam" shown in Fig. 2.13b having the following properties:

M_i	the bending moment at end (i) of the one-component model,
α_i	the plastic angle at end (i) of the one-component model,
ω_{ii}	the rotation of the simple yielding beam having bending moment M_i and plastic angle α_i,
$\omega'_{ii} = \omega_{ii} - \alpha_i$	an angle representing the curvature at end (i) of the central beam of the one-component model, and
$\omega_{ji} = \omega_i - \omega_{ii}$	the angle representing the cross-effect of end (j) on end (i) where ω_i is the end rotation of the one-component beam.

A similar "simple yielding beam" can be defined for end (j) of the one-component model having the above properties but with (j) and (i) interchanged. By using the simple yielding beam for end (i),

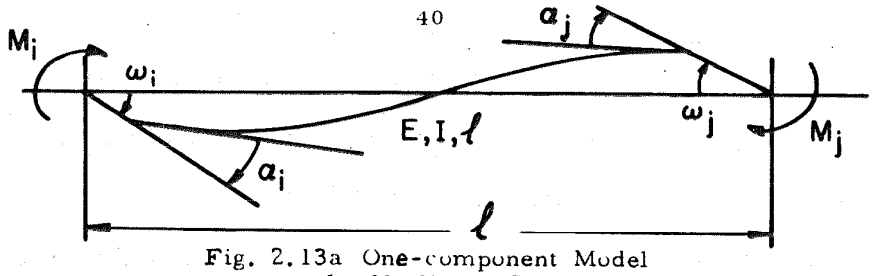

Fig. 2.13a One-component Model of a Nonlinear Beam

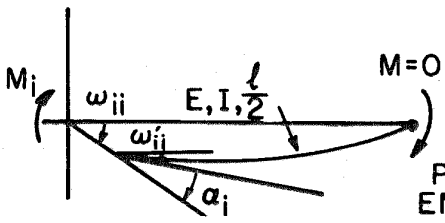

Fig. 2.13b Simple Yielding Beam for End (i)

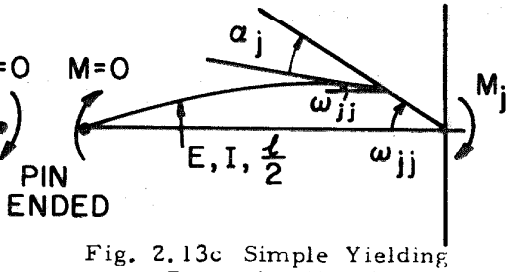

Fig. 2.13c Simple Yielding Beam for End (j)

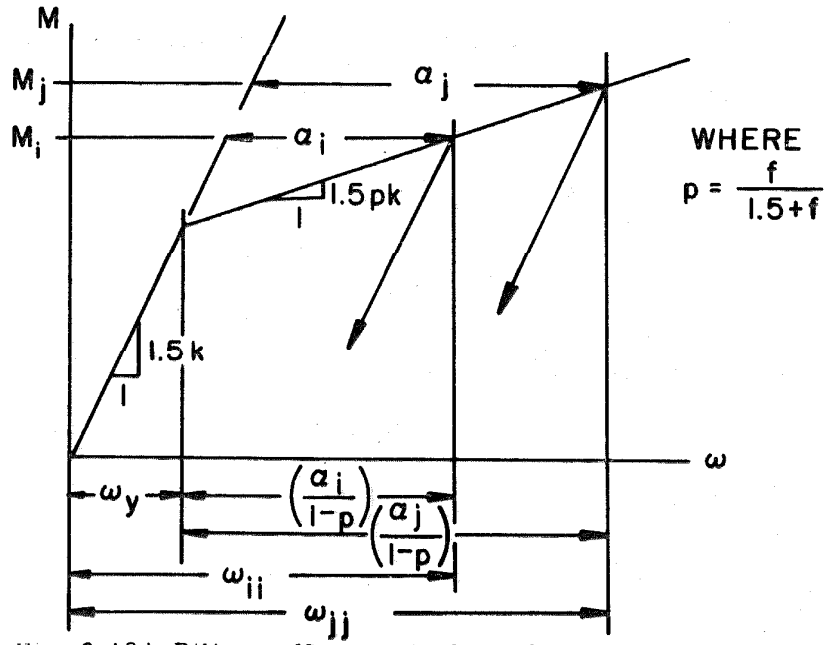

Fig. 2.13d Bilinear Hysteresis Loop for the Simple Yielding Beams for Ends (i) and (j) of the One-component Model of a Nonlinear Beam

the bending moment M_i can be directly related to ω'_{ii} (see Fig. 2.13b) which represents the curvature of the central beam at end (i) as shown below (similarly for end (j)).

The bending moment-end rotation equations for the simple yielding beams are:

$$\begin{aligned} &\text{for end (i)}, && M_i = 1.5\, k(\omega_{ii} - \alpha_i) \ ; \\ &\text{for end (j)}, && M_j = 1.5\, k(\omega_{jj} - \alpha_j) \ . \end{aligned} \tag{2.33}$$

Furthermore, in the nonlinear state of yield, the equations relating incremental bending moments to incremental plastic angles for the simple yielding beams are the same ones used at the ends of the one-component model:

$$\begin{aligned} &\text{at end (i)}, && \Delta M_i = f_i k \Delta\alpha_i \ ; \\ &\text{at end (j)}, && \Delta M_j = f_j k \Delta\alpha_j \ . \end{aligned} \tag{2.34}$$

By equating Eq. 2.33 to Eq. 2.28.1, the general moment rotation equations for M_i and M_j, the following equations for the simple yielding beams are obtained:

$$\begin{aligned} 1.5k(\omega_{ii} - \alpha_i) &= k[\,(\omega_i - \alpha_i) + \tfrac{1}{2}(\omega_j - \alpha_j)] \ ; \\ 1.5k(\omega_{jj} - \alpha_j) &= k[\,\tfrac{1}{2}(\omega_i - \alpha_i) + (\omega_j - \alpha_j)] \ . \end{aligned} \tag{2.35}$$

Hence, equations for ω_{ii} and ω_{jj} are found:

$$\omega_{ii} = \frac{2}{3}\left[\omega_i + \tfrac{1}{2}\alpha_i + \tfrac{1}{2}(\omega_j - \alpha_j)\right] ;$$

$$\omega_{jj} = \frac{2}{3}\left[\tfrac{1}{2}(\omega_i - \alpha_i) + \omega_j + \tfrac{1}{2}\alpha_j\right] . \qquad (2.36)$$

Since

$$\omega_i = \omega_{ii} + \omega_{ji}$$

and

$$\omega_j = \omega_{jj} + \omega_{ij} ,$$

an equation relating the cross-effect angles is obtained:

$$0 = \omega_{ij} + \omega_{ji} ,$$

or

$$\omega_{ij} = - \omega_{ji} . \qquad (2.37)$$

This means that the effect of the rotation at end (i) on end (j) is just the negative of the effect of the rotation at end (j) on end (i). Since a definite relationship exists between the bending moment M_i and the angle ω_{ii} and between M_j and ω_{jj} for all states of yield, it is possible to draw a meaningful bilinear hysteresis loop with the abscissa labeled ω_{ii} and ω_{jj} as is done in Fig. 2.13d for both ends of the one-component model of a nonlinear beam. In this case (see Section 2.4),

$$f_i = f_j = f \qquad \text{and} \qquad p = \frac{f}{1.5 + f} .$$

The above derivation of the equations for ω_{ii}, ω_{jj}, and ω_{ij} (ω_{ji}) are also valid for a curvilinear hysteresis loop such as the one

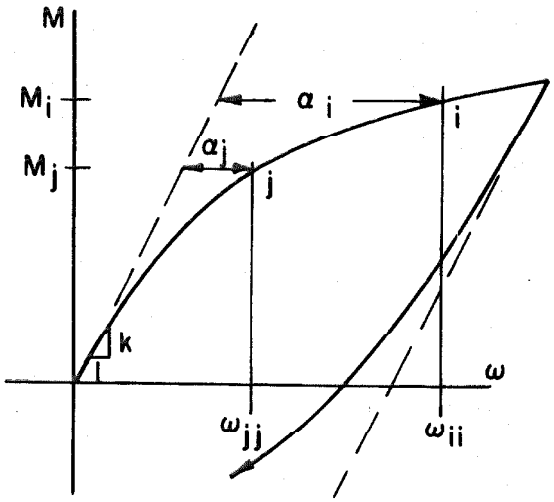

Fig. 2.14a Curvilinear Moment-Rotation Hysteresis Loop of the Simple Yielding Systems for Ends (i) and (j)

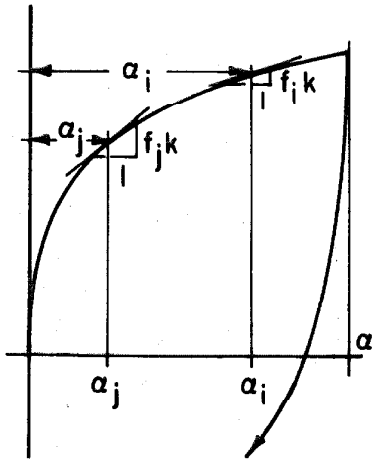

Fig. 2.14b Associated Moment-Plastic Angle Hysteresis Loop of the Simple Yielding Systems for Ends (i) and (j)

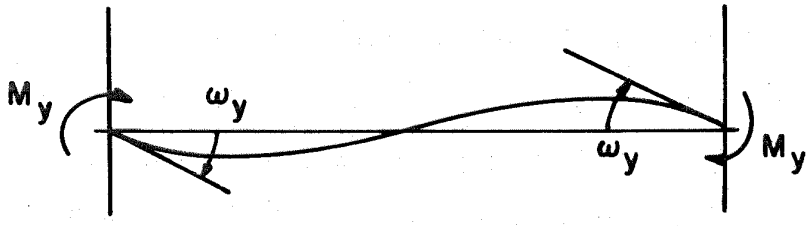

Fig. 2.15 Configuration of Beam at Point of Incipient Yielding

shown in Fig. 2.14a. Such freedom of choice of the hysteresis loop makes the one-component model quite versatile. The points (i) and (j) on the hysteresis loop in Fig. 2.14a represent the moments, rotations, and plastic angles of the ends of a one-component beam at a particular point in time. The values of f_i and f_j to be used in the next time increment are found from the slopes at the corresponding points (i) and (j) on the associated $M - \alpha$ hysteresis loop of Fig. 2.14b.

In the two-component model, the total bending moment at an end is dependent upon the curvatures of both components of the beam at that end. Since these two curvatures have different functional dependencies on the end rotations and plastic angles, when a beam encounters multiple states of yield (at different times), it becomes impossible to draw a two-dimensional hysteresis loop for the two-component model such as was done above for the one-component model.

2.7 Ductility Factors for Nonlinear Beams

In the previous section (Section 2.6) it was shown that by using the concept of simple yielding beams, bilinear and curvilinear hysteresis loops with precise meanings could be found for each end of a one-component model of a nonlinear beam. Consequently, the three ductility factors for simple yielding systems given in Section 2.3 can be directly applied to either end of a one-component beam. Even though an exact hysteresis loop cannot be found for the two-component

beam, two ductility factors can be defined for the two-component model which approximate the corresponding definitions for the one-component model.

It should be recalled that the definition of the plastic angle (α) for the two-component model is different from the definition of (α) for the one-component model. Since no definite hysteresis loop can be drawn for the two-component model which is valid for all states of yield, one can only obtain an approximate relationship between the incurred plastic angles, α, of the two systems. Since the beam usually yields approximately symmetrically, one can approximately relate the α's of the two systems by

$$\alpha_{\substack{\text{one-}\\\text{component}}} \doteq (1 - p)\alpha_{\substack{\text{two-}\\\text{component}}} . \qquad (2.38)$$

The incipient yield angle ω_y appears in the definitions which are used for bilinear hysteretic systems. Knowing the yield moment M_y, ω_y is found from either a simple yielding beam or from the entire beam rotating symmetrically as shown in Fig. 2.15. In either case, the equation relating M_y to ω_y is

$$M_y = \frac{6EI}{\ell} \omega_y$$

One-component Model

The first definition of ductility factor measures the maximum absolute value of the end rotation of the simple yielding beam with respect to the incipient yield angle (for end (i)):

$$\mu_{1i} = \frac{|\omega_{ii}|_{max}}{\omega_y} \quad , \qquad (2.39.1)$$

but since

$$\omega_{ii} = \omega_y + \left(\frac{\alpha_i}{1-p}\right) \quad , \qquad (2.40.1)$$

where

$$p = \frac{f}{1.5 + f} \quad ,$$

Eq. 2.39.1 becomes

$$\mu_{1i} = 1 + \frac{\left|\frac{\alpha_i}{(1-p)}\right|_{max}}{\omega_y} \quad . \qquad (2.41.1)$$

The second definition measures the maximum absolute value of the nonlinear angle, which is the plastic angle (α) for this model, with respect to the incipient yield

Two-component Model

The first definition measures the maximum absolute value of the end rotation of the elasto-plastic component (or of the total beam) with respect to the incipient yield angle of the total beam (for end (i)):

$$\mu_{1i} = \frac{|\omega_i|_{max}}{\omega_y} \quad , \qquad (2.39.2)$$

but since

$$\omega_i = \omega_y + \alpha_i \quad , \qquad (2.40.2)$$

Eq. 2.39.2 becomes*

$$\mu_{1i} = 1 + \frac{|\alpha_i|_{max}}{\omega_y} \quad . \qquad (2.41.2)$$

Using Eq. 2.38, it is seen that Eq. 2.41.1 and Eq. 2.41.2 are approximately equal.

The second definition measures the maximum absolute value of the (approximate) nonlinear angle, which is

$$(1 - p) \cdot \alpha \quad ,$$

*This is the definition of ductility factor (or ductility requirement) used in the FHA Study, reference 1.

One-component Model, cont.

angle (for end (i)):

$$\mu_{2i} = 1 + \frac{|\alpha_i|_{max}}{\omega_y} \quad . \quad (2.42.1)$$

The third definition measures the maximum absolute value of the total end rotation of the simple yielding beam with respect to the linear end rotation (for end (i)):

$$\mu_{3i} = \left| \frac{\omega_{ii}}{\omega_{ii} - \alpha_i} \right|_{max} \quad . \quad (2.43.1)$$

The form of Eq. 2.43.1 is useful for curvilinear hysteresis loops; however, for bilinear hysteresis loops, Eq. 2.40.1 can be used to eliminate ω_{ii}. The resulting equation is (for end (i)):

$$\mu_{3i} = 1 + \left| \frac{\alpha_i}{\omega_y + \frac{p\alpha_i}{(1-p)}} \right|_{max} \quad . \quad (2.44.1)$$

Eq. 2.44.1 is the one used in the

Two-component Model, cont.

where (α) is the plastic angle for this model, with respect to the incipient yield angle (for end (i)):

$$\mu_{2i} = 1 + \frac{(1-p)|\alpha_i|_{max}}{\omega_y} \quad . \quad (2.42.2)$$

Using Eq. 2.38, it is seen that Eq. 2.42.1 and Eq. 2.42.2 are approximately equal.

The third definition of ductility factor is not written for the two-component model since it is impossible to use this model for curvilinear hysteresis loops.

One-component Model, cont.	Two-component Model, cont.
computer studies for finding the ductility factors of one-component beams. The primary reason for using this definition is that it can be used for curvilinear hysteresis loops as well.	

In this chapter two models of a nonlinear beam were discussed and compared. It was seen that the one-component model was more realistic from the physical point of view. Furthermore, the one-component model was shown to be more versatile since it could treat curvilinear hysteresis loops whereas the two-component model was restricted to bilinear hysteresis loops. Consequently, the one-component model is considered to be the better model of a nonlinear beam and is therefore used in the computer studies of nonlinear tall structures for this report.

In the next chapter the equations of motion of a nonlinear tall structure are derived wherein either of these beam models can be used to represent the girders and columns of the structure.

CHAPTER III

EQUATIONS OF MOTION FOR NONLINEAR TALL STRUCTURES

3.1 Introduction

The purposes of this chapter are to describe the properties of a class of nonlinear tall structures, to present the damping mechanisms appropriate to such structures, to derive the equations of motion, to discuss the integration techniques, and to show how the equations of motion are solved with the aid of a digital computer.

3.2 A Class of Nonlinear Tall Structures

The class of nonlinear structures considered in this report is intended to be representative of modern high-rise buildings with glass and other lightweight walls such as are being built today (1967). This class of structures is characterized by the following assumed properties.

1. The foundation is infinitely rigid.
2. The structure is symmetric in plan view; hence, torsional deformation is neglected.
3. The girders provide all of the stiffness to the floors and they are flexible (not infinitely rigid).
4. The columns provide all of the stiffness to the walls.
5. There no shear walls; hence, the structures are of the moment-resisting type. This means that the structures resist deformation only by the moments developed at the

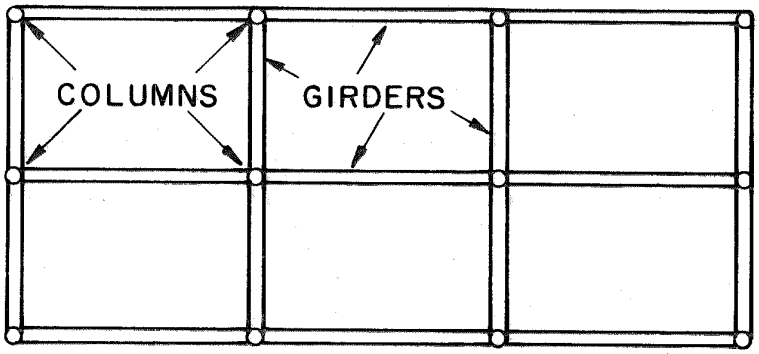

Fig. 3.1a Plan View of a
Moment-resisting Frame

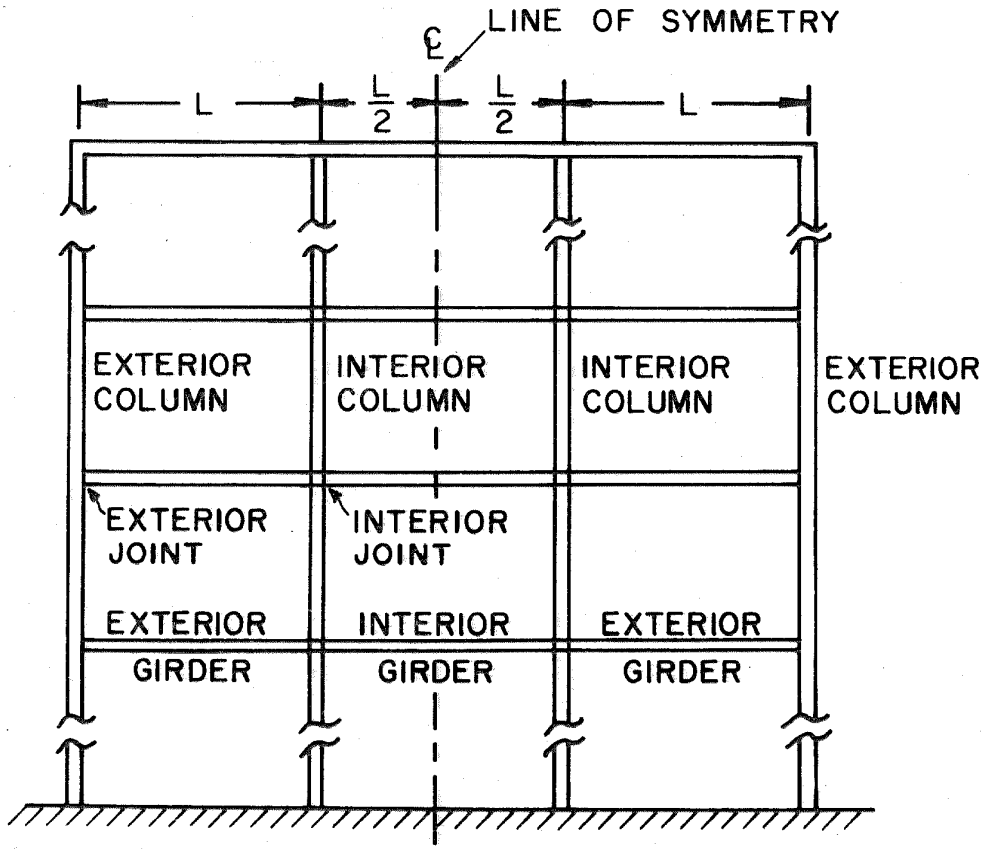

Fig. 3.1b Side View of a
Moment-resisting Frame

ends of the girders and columns.

6. Shear deflection in the girders and columns is neglected.
7. As modeled in Chapter II, the girders and columns can yield at each end according to a bilinear bending moment-end rotation hysteresis loop.
8. There is no extension (or contraction) between any joints within the same floor.
9. There is no extension (or contraction) between floors since it is assumed that the structures are infinitely rigid in the vertical direction.
10. All mass is concentrated at the floor levels.
11. The mass of each floor moves only horizontally. Vertical motion is not considered because of property (9). Although the joints rotate, the rotational inertia associated with each joint is neglected.
12. Gravitational effects are negligible for the base overturning moment.

The particular computer program developed in this study can analyze only one (two-dimensional) structural frame at a time. In addition to the properties listed above, the structural frame must also have the following:

A. It must have three bays (four columns) and be symmetric with respect to the centerline as shown in Fig. 3.1b.
B. The structural properties of the frame are representative of the three-dimensional structure being considered.

Such a three-dimensional structure is shown in Fig. 3.1a.

C. The columns are uniformly spaced L inches apart.

D. There are twenty floors above ground level. With minor alteration, however, the computer program can handle an arbitrary number of floors.

Several properties of the frame are optional:

E. The mass of each floor;

F. The distance between any two adjacent floors;

G. The stiffness properties of the girders and columns in the linear state of yield;

H. The ratio of the second slope to the first slope of the bilinear hysteresis loops (see Chapter II); and

I. The yield moments in the hysteresis loops.

Since the frame is subjected to an earthquake accelerogram in the computer program, it is of interest to note the properties of the accelerograms which can be used:

J. The time history must be digitized.

K. The time increments can be either uniform or nonuniform.

L. A multiplicative amplitude scale factor can be used.

One frame having the above structural properties is extensively studied in this report. This is the A/20/2.2/2/6 frame which was designed by R. W. Clough and K. L. Benuska and was used in their studies for the Federal Housing Administration.* An advantage of using this particular frame is that a basis for comparison exists for

*See Section 2.2 of the FHA Study, reference 1.

verifying that the computer program for analyzing nonlinear frames written for this report operates properly (see Section 4.5). Independent studies using this program with various modifications can then be made with confidence in the results.

This frame was designed by means of static analysis using a digital computer. The design moments and design forces in the frame were found in a conventional manner by taking into consideration static gravity loads plus lateral loads as specified by the Uniform Building Code (of 1965). The design of the girders and columns is such that after the static gravity loads (but not the lateral loads) are applied, the positive and the negative yield bending moment for each hysteresis loop have the same absolute value, M_y, as shown in Fig. 2.12.

The identifying numbers of this frame have the following significance:

A: structural properties are tapered;

20: number of floors above ground level;

2.2: (approximate) period of the fundamental mode;

2: ratio of the yield moments to the corresponding design moments in the girders; and

6: ratio of the yield moments to the corresponding design moments in the columns.

For completeness, the structural properties of this frame are given in Fig. 3.2. The stiffness and strength properties of structures are often unrelated and this is reflected in this frame.

A section of a frame having the properties listed above is

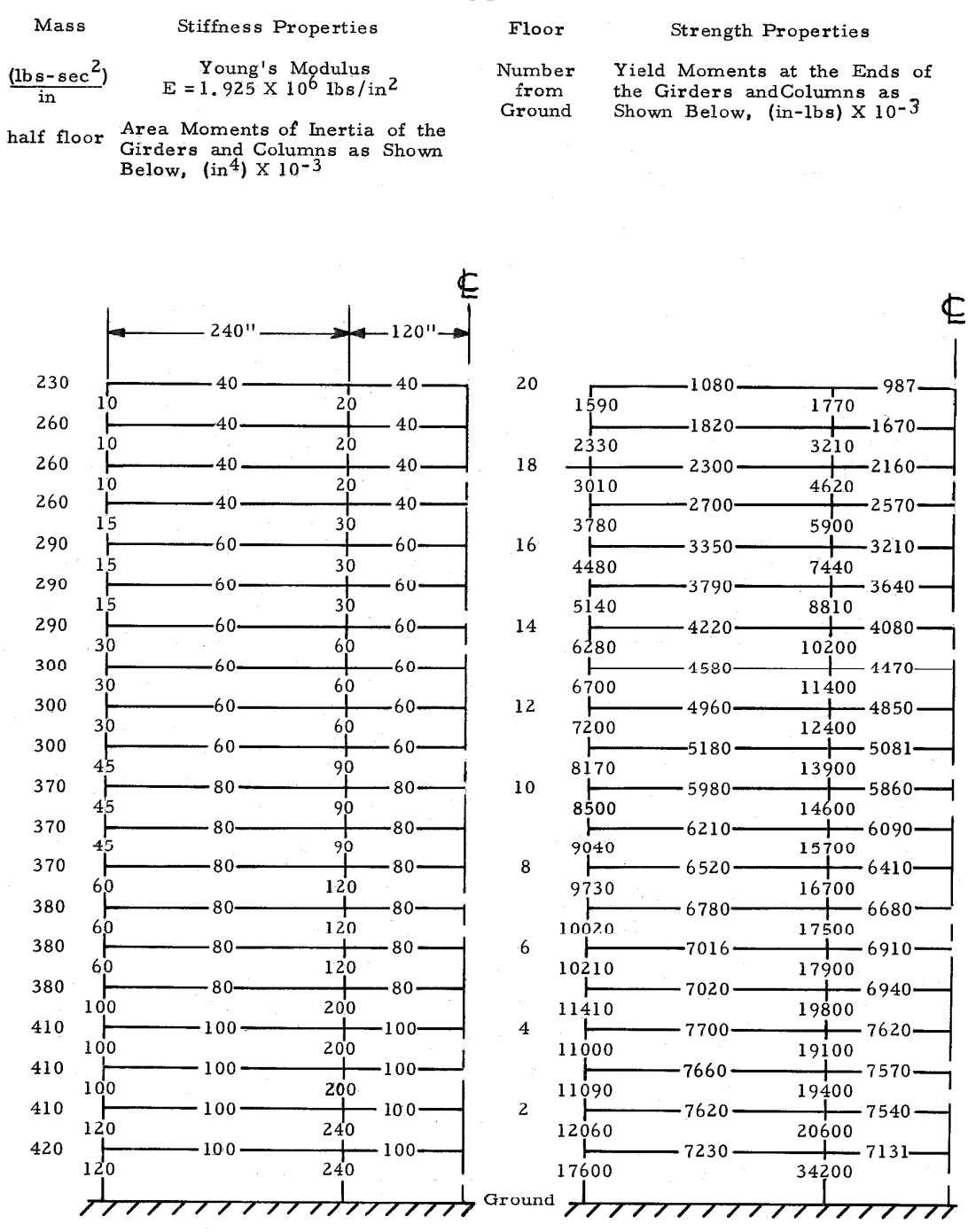

Fig. 3.2. Structural Properties of the A/20/2.2/2/6 Frame. Only Half of the Frame is Shown Since it is Symmetric With Respect to the Centerline (see Fig. 3.1b). The Height Between Floors is 144 Inches Except Between Ground and the First Floor Which is 180 Inches.

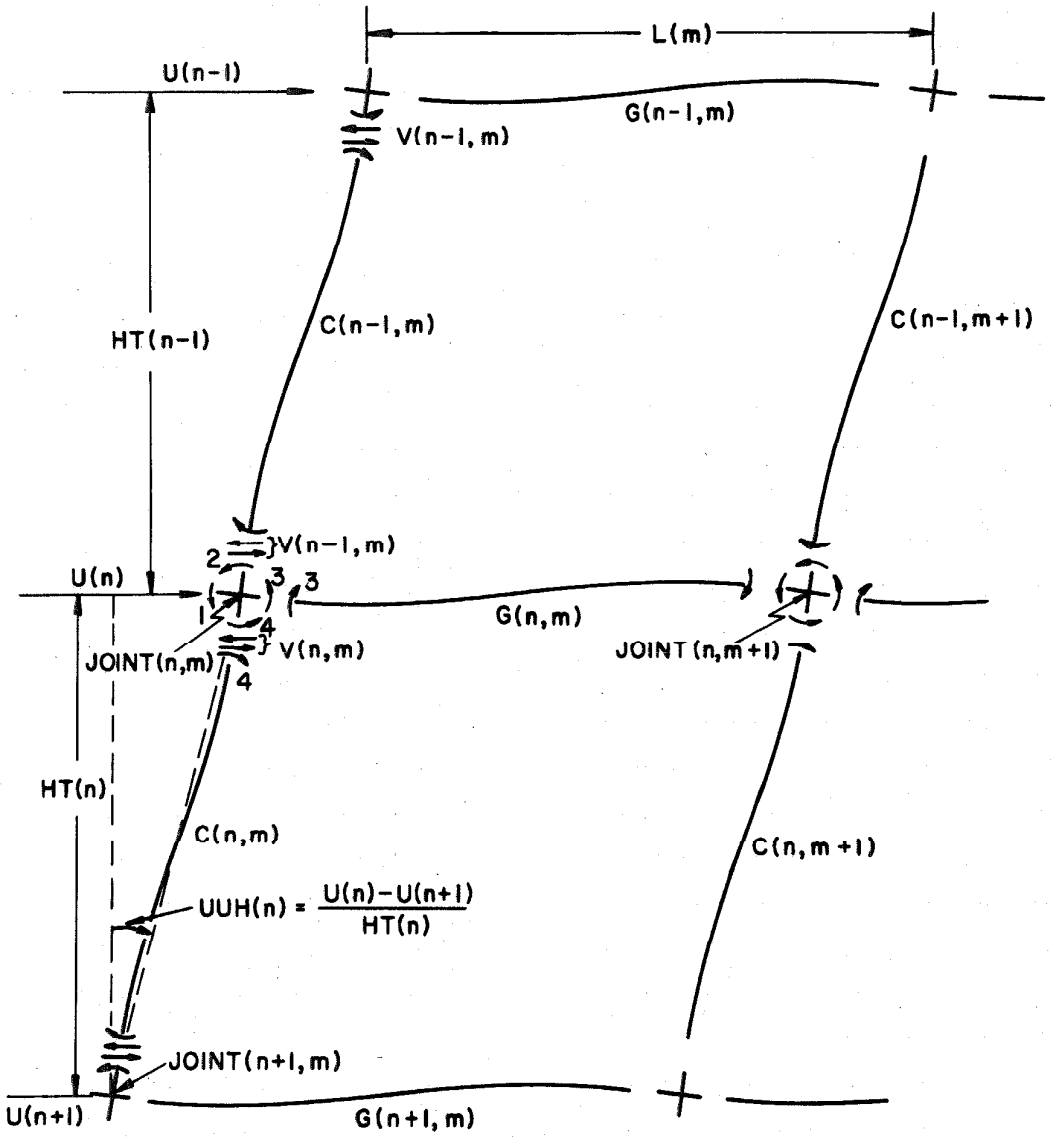

Fig. 3.3 Section of a Typical Moment-resisting Frame in a Deformed Position

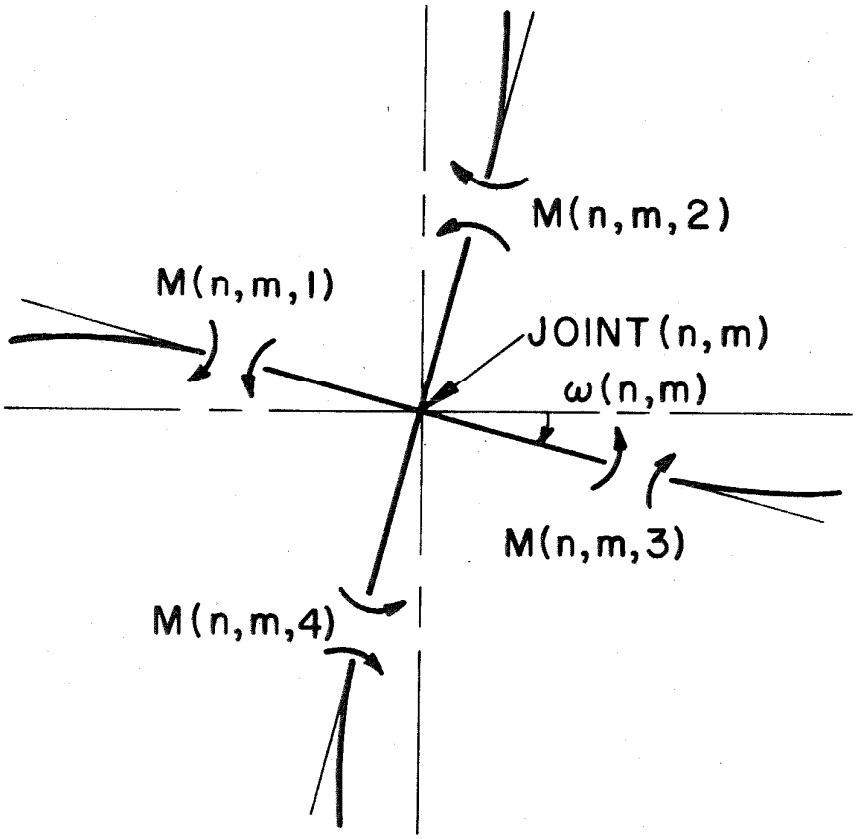

Fig. 3.4 Bending Moments M(n, m, j) at Joint (n, m)

shown in a deformed position in Fig. 3.3 and a typical joint is shown in a rotated position in Fig. 3.4. The coordinates, moments and forces used in this chapter are depicted in Figs. 3.3 and 3.4 in the positive direction unless labeled to the contrary. These coordinates, etc., are defined below.*

n	: index, the floor number starting from the roof;
m	: index, the column line starting from the left;
j (j = 1,2,3,4)	: index, the number of the intersecting beam (column or girder);
joint (n,m)	: the joint formed by the intersection of girder (n) and column (m);
$\omega(n,m)$	: the angular rotation of joint (n,m);
$\alpha(n,m,j)$	: the plastic angle of the j^{th} beam at joint (n,m) (see Section 2.5);
M(n,m,j)	: the (total) bending moment of the j^{th} beam at joint (n,m);
V(n,m)	: the shear force in column C(n,m);
HT(n,m)	: the height between floor number (n) and (n+1) (numbered from roof);
U(n)	: the horizontal displacement of floor (n); and
$UUH(n) = \frac{U(n) - U(n+1)}{HT(n)}$	: the interfloor shear angle between floors (n) and (n+1).

Usually floors are numbered from the ground up as shown in Fig. 3.2. However, this method of numbering is not convenient for the computer program since the natural sequence of most operations is from the roof down. Consequently, in the derivation of the

*Some of these coordinates, etc., are labeled in "Fortran notation," e.g., UUH(n): the interfloor shear angle, since this type of notation is used in the resulting computer program (see Appendix E).

equations of motion, the floors are numbered from the roof down, as shown in Fig. 3.3.

3.3 Damping

Because of the difficulty of determining the damping mechanisms in structures, conventional approximations are made. Two approximate damping mechanisms which can be adapted to the equations of motion are described below.

1. As a structure vibrates during an earthquake, energy is dissipated aerodynamically. This dissipation is approximately proportional to the product of the area of the face of the structure perpendicular to the motion and the absolute (or total) velocity of the face. However, for the studies performed here, a pseudo-aerodynamic energy dissipation is assumed which uses the velocity relative to ground. Assuming that the area of the face at a given floor level is proportional to the mass of the associated floor, "mass proportional" viscous damping is obtained. The expression for this type of damping, for an N-floor structure is:

$$\alpha M \dot{\underline{U}} \tag{3.1}$$

where

α: scalar constant,

M: mass matrix (N × N), and

$\dot{\underline{U}}$: vector (N × 1) of the velocities of the floors relative to the ground.

2. When a structure vibrates in an earthquake, the interfloor displacements are large enough to cause the walls and partitions to crack and to rub together, thus dissipating energy. This type of damping in a structure is proportional to the relative interfloor velocity, and a convenient approximation is the expression

$$\beta K \dot{\underline{U}} \qquad (3.2)$$

where

β: a scalar constant,

K: a stiffness matrix $(N \times N)$, and

$\dot{\underline{U}}$: vector $(N \times 1)$ of the velocities of the floors relative to ground.

The form of the stiffness matrix which can be used in Expression 3.2 depends to some degree upon the method of solving the equations of motion. Since the equations of motion are nonlinear, they are solved by means of an incremental integration approach (see Sections 3.4 and 3.5). In this case, if K is a tridiagonal stiffness matrix such as the one for a structure with rigid girders*, Expression 3.2 can be readily adapted to the equations of motion. This type of damping is often referred to as "stiffness proportional" viscous damping.

These two damping mechanisms together comprise "Rayleigh damping" for the structure with rigid girders.

*The $(N \times N)$ stiffness matrix for an N-floor structure with flexible girders is, in general, full.

Assume for the moment that the structure with flexible girders remains linear (does not yield) and that the damping matrix, C, is given by

$$C = \alpha M + \beta K' \tag{3.3}$$

where

α, β : scalar constants;

M : diagonal mass matrix ($N \times N$); and

K' : stiffness matrix ($N \times N$) for the structure with flexible girders.

The resulting linear homogeneous matrix equation of motion is

$$M\ddot{\underline{U}} + (\alpha M + \beta K')\dot{\underline{U}} + K'\underline{U} = 0 \tag{3.4}$$

where

$\underline{U}$ is the displacement vector ($N \times 1$).

In this case, classical normal modes can be found.[44] Hence, for mass proportional damping ($\beta = 0$) only, it can be shown that the fraction of critical damping in the n^{th} mode is

$$\xi_n^m = \frac{\alpha}{2\omega_n}, \quad n = 1, 2, \ldots, N,$$

where ω_n is the natural frequency (circular); therefore,

$$\xi_1^m > \xi_2^m > \xi_3^m > \ldots > \xi_N^m . \quad \text{(See footnote, page 61.)}$$

Furthermore, for stiffness proportional damping ($\alpha = 0$) only, it can be shown that

$$\xi_n^s = \frac{\beta \omega_n}{2} \quad , \quad n = 1, 2, \ldots, N \; ;$$

therefore,

$$\xi_1^s < \xi_2^s < \xi_3^s < \ldots < \xi_N^s \; . \quad *$$

When written in the form of the incremental equations of motion for structures with flexible girders, the Rayleigh damping described above in Expressions 3.1 and 3.2 appears as

$$\Delta \underline{D} = [\alpha M + \beta K] \, \Delta \dot{\underline{U}} \tag{3.5}$$

where

$\Delta \underline{D}$: incremental damping force vector $(N \times 1)$;

α, β : scalar constants;

M : diagonal mass matrix $(N \times N)$;

K : tridiagonal stiffness matrix $(N \times N)$ for the structure with rigid girders; and

$\Delta \dot{\underline{U}}$: vector $(N \times 1)$ of the velocities of the floors relative to ground.

---

*For the A/20/2.2/2/6 structure described in the previous section (Section 3.2),

$$\omega_n = 2.84, \; 6.91, \; 11.5, \; 16.0, \; 20.9, \; \ldots \; .$$

Hence, for mass proportional damping, with 0.10 fraction of critical damping in the fundamental mode,

$$\xi_n^m = 0.10, \; 0.041, \; 0.025, \; 0.018, \; 0.014, \; \ldots \; ,$$

and for stiffness proportional damping with 0.10 fraction of critical damping in the fundamental mode,

$$\xi_n^s = 0.10, \; 0.24, \; 0.40, \; 0.56, \; 0.74, \; \ldots \; .$$

In the next section it will be seen how Eq. 3.5 is incorporated into the equations of motion for this class of structures.

3.4 Equations of Motion

In this section the equations of motion are derived for the structural frame described in Section 3.2 having the damping mechanisms of Section 3.3. Two general equations are derived. One of these is for the angular rotation of the joints and the other governs the horizontal motion of the floors. The equation for the rotation of the joints is derived from a balance of bending moments at the ends of the girders and columns at a typical joint and the equation of motion for the floors is derived from a balance of forces on a typical floor. Since the frame is symmetric, as shown in Fig. 3.1b or in Fig. 3.5, these equations of motion are written for only one-half of the frame (the left half) taking symmetry into consideration.

Both of these equations use incremental bending moment-end rotation equations for the girders and columns in the frame. In Chapter II, the moment-rotation equations, Eq. 2.29.1 and Eq. 2.29.2, are developed for beams without making a distinction between columns and girders. However, for the present purpose, a distinction must be made since the interfloor shear angle appears in the moment-rotation equations for the columns but does not appear in the equations for the girders. Using the one-component beam model, the incremental moment-rotation equations for the girders and columns which intersect to form joint (n,m) (see Fig. 3.4) are given in Eq. 3.6. Similar equations can be written for the two-

component beam model.

$$\Delta M(n,m,1) = k_G(n,m-1)\left\{[\Delta\omega(n,m) - \Delta\alpha(n,m,1)] + \tfrac{1}{2}[\Delta\omega(n,m-1) - \Delta\alpha(n,m-1,3)]\right\} ;$$

$$\Delta M(n,m,2) = k_C(n-1,m)\left\{[\Delta\omega(n,m) - \Delta UUH(n-1) - \Delta\alpha(n,m,2)] + \tfrac{1}{2}[\Delta\omega(n-1,m) - \Delta UUH(n-1) - \Delta\alpha(n-1,m,4)]\right\} ; \quad (3.6)$$

$$\Delta M(n,m,3) = k_G(n,m)\left\{[\Delta\omega(n,m) - \Delta\alpha(n,m,3)] + \tfrac{1}{2}[\Delta\omega(n,m+1) - \Delta\alpha(n,m+1,1)]\right\} ;$$

$$\Delta M(n,m,4) = k_C(n,m)\left\{[\Delta\omega(n,m) - \Delta UUH(n) - \Delta\alpha(n,m,4)] + \tfrac{1}{2}[\Delta\omega(n+1,m) - \Delta UUH(n) - \Delta\alpha(n+1,m,2)]\right\} ;$$

where

$k_G(n,m)$: stiffness of girder $G(n,m)$* in the linear state of yield** ;

$k_C(n,m)$: stiffness of column $C(n,m)$ in the linear state of yield.

Just as the $\Delta\alpha$'s were eliminated from the incremental bending moment-end rotation equations in Chapter II, the $\Delta\alpha$'s can be eliminated from Eq. 3.6. Consequently, equations analogous to

---

*See Section 3.2 for nomenclature of G(n,m); C(n,m).

**See Section 2.5 for nomenclature of "linear state of yield."

Eq. 2.31.1 relating the incremental bending moments to end rotations via effective stiffness parameters can be written

for the girders:

$$\begin{Bmatrix} \Delta M(n,m,3) \\ \Delta M(n,m+1,1) \end{Bmatrix} = \begin{bmatrix} SGA(n,m) & SGB(n,m) \\ SGB(n,m) & SGC(n,m) \end{bmatrix} \begin{Bmatrix} \Delta\omega(n,m) \\ \Delta\omega(n,m+1) \end{Bmatrix} \quad ; \qquad (3.7)$$

for the columns:

$$\begin{Bmatrix} \Delta M(n,m,4) \\ \Delta M(n+1,m,2) \end{Bmatrix} = \begin{bmatrix} SCA(n,m) & SCB(n,m) \\ SCB(n,m) & SCC(n,m) \end{bmatrix} \begin{Bmatrix} \Delta\omega(n,m) - \Delta UUH(n) \\ \Delta\omega(n+1,m) - \Delta UUH(n) \end{Bmatrix} . \qquad (3.8)$$

In Eq. 3.7, SGA(n,m) is the Fortran name for the effective stiffness parameter which corresponds to S_A in Eq. 2.31.1 (or Eq. 2.31.2) for the girder G(n,m) having stiffness $k_G(n,m)$ in the linear state of yield. A similar correspondence applies for the other effective stiffnesses of the girders and columns.

One of the properties of the class of nonlinear structures considered is that the rotational inertia associated with each joint is neglected (see Section 3.2). Hence, the sum of the bending moments at the ends of the girders and columns at each joint is zero (see Fig. 3.4):

$$\sum_{j=1}^{4} \Delta M(n,m,j) = 0 \quad . \qquad (3.9)$$

By substituting Eq. 3.7 and Eq. 3.8 with appropriate indices, n, m, and j, into Eq. 3.9, the general equation for the incremental rotation

of joint (n,m) is found:

$$\Delta\omega(n,m) = - \frac{\begin{bmatrix} + SGB(n,m-1)\cdot\Delta\omega(n,m-1) + SGB(n,m)\cdot\Delta\omega(n,m+1) \\ + SCB(n-1,m)\cdot\Delta\omega(n-1,m) + SCB(n,m)\cdot\Delta\omega(n+1,m) \\ - [SCB(n-1,m) + SCC(n-1,m)]\cdot\Delta UUH(n-1) \\ - [SCA(n,m) + SCB(n,m)]\cdot\Delta UUH(n) \end{bmatrix}}{[SGC(n,m-1) + SGA(n,m) + SCC(n-1,m) + SCA(n,m)]} \quad (3.10)$$

For the left half of the symmetrical three bay frame of Fig. 3.5, the joints (n,2) and (n,3) are seen to be typical of the joints of column lines m = 2 and m = 3, respectively. By considering the characteristics of each of these two joints, different equations for the incremental joint rotations of each column line are derived.

Because the girders G(n,1), n = 1, ... , 22, do not exist in the structure (see Fig. 3.5), an equation for the incremental joint rotation $\Delta\omega(n,2)$ is found by removing the constants SGB(n,1) and SGC(n,1) from Eq. 3.10. Hence, for each joint of column line m = 2:

$$\Delta\omega(n,2) = - \frac{\begin{bmatrix} + SGB(n,2)\cdot\Delta\omega(n,3) \\ + SCB(n-1,2)\cdot\Delta\omega(n-1,2) + SCB(n,2)\cdot\Delta\omega(n+1,2) \\ -[SCB(n-1,2) + SCC(n-1,2)]\cdot\Delta UUH(n-1) \\ -[SCA(n,2) + SCB(n,2)]\cdot\Delta UUH(n) \end{bmatrix}}{[SGA(n,2) + SCC(n-1,2) + SCA(n,2)]} \quad (3.11)$$

For column line m = 3, the symmetry of the frame is considered in either of two ways. The first and simpler method is to look at the entire frame and to observe that

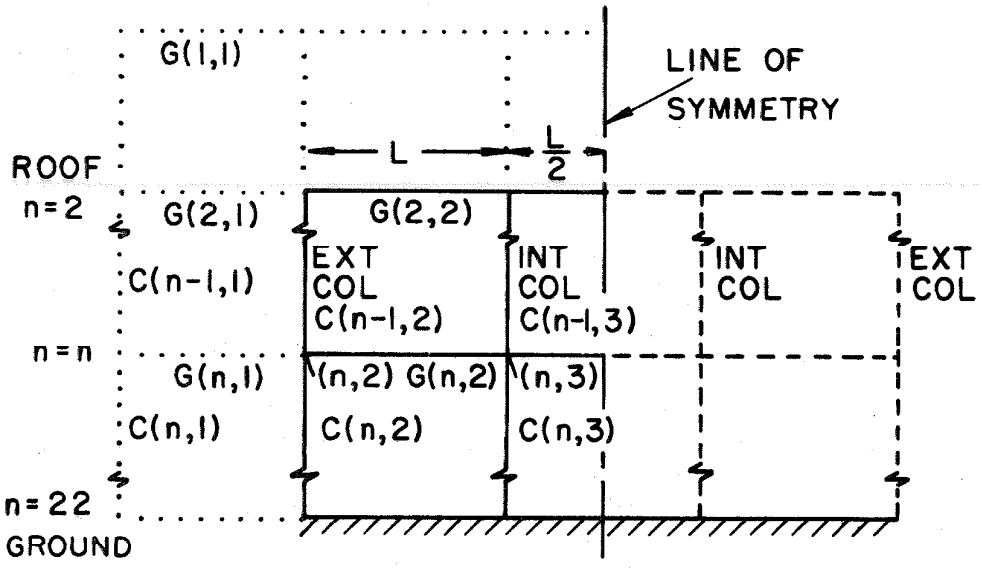

Fig. 3.5 (·····) Dummy Columns and Girders With Stiffnesses Identically Equally to Zero. (Computer Program Artifice)

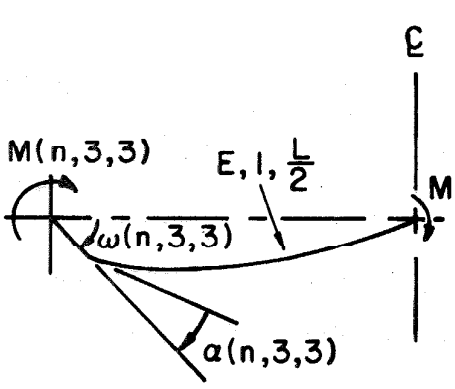

Fig. 3.6a Treatment of Symmetry for One-component Model of Girder G(n, 3). Moment at Centerline Equals Zero.

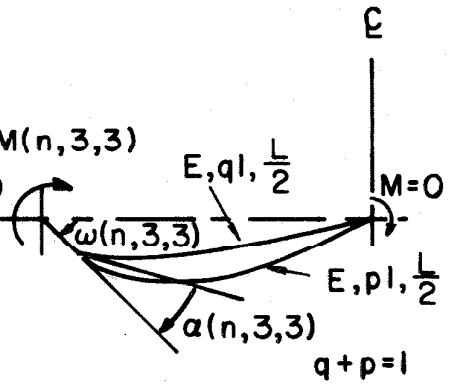

Fig. 3.6b Treatment of Symmetry for Two-component Model of Girder G(n, 3). Moment at Centerline of Each Component Equals Zero.

$$\Delta\omega(n,4) = \Delta\omega(n,3) \; . \qquad (3.12)$$

Consequently, for the interior girders, G(n,3), n = 2, ... , 21, a simplified bending moment equation is obtained for $\Delta M(n,3,3)$ by substituting Eq. 3.12 into Eq. 3.7):

$$\Delta M(n,3,3) = [\, SGA(n,3) + SGB(n,3)] \cdot \Delta\omega(n,3) . \qquad (3.13)$$

Because the joints at both ends of each interior girder rotate identically the same amount, only the two yield states (a) and (d) need to be considered for either beam model:

State (a) - linear at both ends:

$$\Delta M(n,3,3) = 1.5 \cdot k \cdot \Delta\omega(n,3) \; ; \qquad (3.14)$$

State (d) - nonlinear at both ends:

$$\Delta M(n,3,3) = 1.5 \cdot p \cdot k \cdot \Delta\omega(n,3) \; ; \qquad (3.15)$$

(see Eq. 2.32.1 and Eq. 2.32.2 at the end of Section 2.5). In the second method the girders are cut at the centerline and pinned so that the bending moment at the centerline remains identically zero. For the one-component model this is shown in Fig. 3.6a. For the two-component model, in order to properly treat symmetry, each component must be separately pinned at the centerline as shown in Fig. 3.6b. If, instead, the two components are first connected together and then pinned, certain problems arise, which are discussed in Appendix C. Assuming symmetry is considered correctly, the second method yields the same incremental moment-rotation equations as the first.

Since there are only two states of yield for the interior girders, Eq. 3.14 and Eq. 3.15 are of the form

$$\Delta M(n,3,3) = SGA(n,3) \cdot \Delta\omega(n,3) \tag{3.16}$$

which uses only one effective stiffness parameter: SGA(n,3). Hence, for the interior girders, SGB(n,3) is unnecessary and is merged into SGA(n,3). The new values of SGA(n,3) are given (for both beam models) by:

state of yield	SGA(n,3)
linear :	$1.5 \cdot k$
nonlinear :	$1.5 \cdot p \cdot k$.

Using Eq. 3.16 for $\Delta M(n,3,3)$ instead of Eq. 3.13, the following equation is obtained for each joint of column line m = 3:

$$\Delta\omega(n,3) = - \frac{\left[\begin{array}{l} + SGB(n,2)\cdot\Delta\omega(n,2) \\ + SCB(n-1,3)\cdot\Delta\omega(n-1,3) + SCB(n,3)\cdot\Delta\omega(n+1,3) \\ -[SCB(n-1,3) + SCC(n-1,3)]\cdot\Delta UUH(n-1) \\ -[SCA(n,3) + SCB(n,3)]\cdot\Delta UUH(n) \end{array}\right]}{[SGC(n,2) + SGA(n,3) + SCC(n-1,3) + SCA(n,3)]} \tag{3.17}$$

The differential equation of motion for a typical floor of the structural frame is derived from a balance of forces wherein the sum of the shear forces in the columns, V, damping forces, D, and initial force, I, acting on each floor must be zero. These forces, along with the displacement, U, are depicted in incremental form in Fig. 3.7 and are positive as shown.

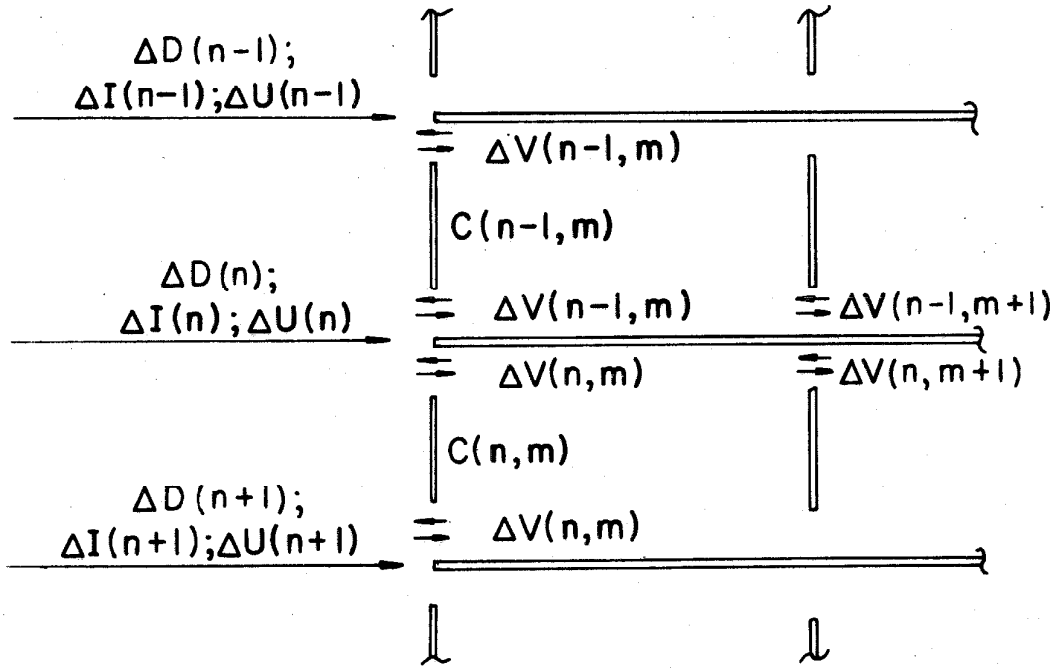
ΔD(n-1);
ΔI(n-1);ΔU(n-1)
ΔV(n-1,m)
C(n-1,m)
ΔD(n);
ΔI(n);ΔU(n)
ΔV(n-1,m)
ΔV(n-1,m+1)
ΔV(n,m)
ΔV(n,m+1)
C(n,m)
ΔD(n+1);
ΔI(n+1);ΔU(n+1)
ΔV(n,m)

Fig. 3.7

Several constants having Fortran names are used in the equations of motion. Besides those introduced previously, the following ones are used:

$$\begin{aligned}
\text{SCAB}(n,m) &= \text{SCA}(n,m) + \text{SCB}(n,m) \ ; \\
\text{SCBC}(n,m) &= \text{SCB}(n,m) + \text{SCC}(n,m) \ ; \\
\text{SCABH}(n,m) &= \frac{\text{SCAB}(n,m)}{\text{HT}(n)} \ ; \\
\text{SCBCH}(n,m) &= \frac{\text{SCBC}(n,m)}{\text{HT}(n)} \ ; \\
\text{SCD}(n) &= \sum_{m=2}^{3} [\text{SCAB}(n,m) + \text{SCBC}(n,m)] \ ; \text{ and} \\
\text{SCDHH}(n) &= \frac{\text{SCD}(n)}{[\text{HT}(n)]^2} \ .
\end{aligned} \tag{3.18}$$

The incremental shear force in column C(n,m) is

$$\Delta V(n,m) = - \left[\frac{\Delta M(n,m,4) + \Delta M(n+1,m,2)}{\text{HT}(n)} \right] . \tag{3.19}$$

Substituting from Eq. 3.8 and from the list of Fortran constants, Eq. 3.18, Eq. 3.19 becomes

$$\Delta V(n,m) = - \frac{1}{\text{HT}(n)} \left[\begin{array}{l} \text{SCAB}(n,m) \cdot [\Delta\omega(n,m) - \Delta\text{UUH}(n)] \\ + \text{SCBC}(n,m) \cdot [\Delta\omega(n+1,m) - \Delta\text{UUH}(n)] \end{array} \right] ; \tag{3.20}$$

where

$$\Delta\text{UUH}(n) = \frac{\Delta U(n) - \Delta U(n+1)}{\text{HT}(n)} . \tag{3.21}$$

The resultant incremental shear force acting on (the left half of) the

n^{th} floor of the structural frame in Fig. 3.7 is given by the expression

$$\sum_{m=2}^{3} [\Delta V(n-1,m) - \Delta V(n,m)] \quad . \tag{3.22}$$

Using Eq. 3.20 for ΔV and Eq. 3.21 for ΔUUH, and substituting from the list of Fortran constants, Eq. 3.18, an equation for the resultant incremental shear force acting on the n^{th} floor is found:

$$\begin{aligned}
&\sum_{m=2}^{3} [\Delta V(n-1,m) - \Delta V(n,m)] \\
&= - \sum_{m=2}^{3} SCABH(n-1,m)\cdot\Delta\omega(n-1,m) - \sum_{m=2}^{3} SCBCH(n-1,m)\cdot\Delta\omega(n,m) \\
&\quad + \sum_{m=2}^{3} SCABH(n,m)\cdot\Delta\omega(n,m) + \sum_{m=2}^{3} SCBCH(n,m)\cdot\Delta\omega(n+1,m) \\
&\quad + SCDHH(n)\cdot\Delta U(n+1) + SCDHH(n-1)\cdot\Delta U(n-1) \\
&\quad - [SCDHH(n) + SCDHH(n-1)]\cdot\Delta U(n) \quad .
\end{aligned} \tag{3.23}$$

The incremental damping force acting on the n^{th} floor is found from Eq. 3.5:

$$\Delta D(n) = \left\{ \begin{array}{l} - 2\xi_1^m \omega_1 M(n)\cdot\Delta\dot{U}(n) \\ + \left(\dfrac{2\xi_1^s}{\omega_1}\right)\cdot \left\{ \begin{array}{l} + SCDHH(n-1)\cdot\Delta\dot{U}(n-1) \\ -[SCDHH(n-1) + SCDHH(n)]\cdot\Delta\dot{U}(n) \\ + SCDHH(n)\cdot\Delta\dot{U}(n+1) \end{array} \right\} \end{array} \right\}; \tag{3.24}$$

where

ξ_1^m : the fraction of critical damping for the fundamental mode of the linear structure with only mass proportional damping;

ξ_1^s : the fraction of critical damping for the fundamental mode of the linear structure with only stiffness proportional damping;

ω_1 : circular frequency of the fundamental mode,

M(n) : mass of one half of the n^{th} floor, and

$\Delta\dot{U}(n)$: incremental velocity of the n^{th} floor relative to the foundation.

Although both mass proportional and stiffness proportional damping are included in the equations of motion, only mass proportional damping is used in the computer program for this report (see Subroutine CONV in Appendix E).

The incremental inertial force, ΔI, acting on the n^{th} floor is given by

$$\Delta I(n) = -M(n)\cdot(\Delta\ddot{U}(n) + \Delta\ddot{y}) \qquad (3.25)$$

where

M(n) : mass of one half of the n^{th} floor,

$\Delta\ddot{U}(n)$: incremental acceleration of the n^{th} floor relative to the foundation; and

$\Delta\ddot{y}$: incremental acceleration of the foundation.

Since the incremental force balance for the n^{th} floor is

$$\Delta I(n) + \Delta D(n) + \sum_{m=2}^{3} [\Delta V(n-1,m) - \Delta V(n,m)] = 0 \quad , \qquad (3.26)$$

by substituting from Eq. 3.23, Eq. 3.24, and Eq. 3.25, the differential equation of motion in incremental form is obtained:

$$M(n)\cdot\Delta\ddot{U}(n)$$

$$+ [\, 2\xi_1^m\cdot\omega_1\cdot M(n) + 2\left(\frac{\xi_1^s}{\omega_1}\right)\cdot(SCDHH(n) + SCDHH(n-1))]\cdot\Delta\dot{U}(n)$$

$$+ [\, SCDHH(n) + SCDHH(n-1)]\cdot\Delta U(n)$$

$$+ \sum_{m=2}^{3} [\, SCBCH(n-1,m) - SCABH(n,m)]\cdot\Delta\omega(n,m)$$

$$- \left(\frac{2\xi_1^s}{\omega_1}\right)\cdot SCDHH(n-1)\cdot\Delta\dot{U}(n-1) - SCDHH(n-1)\cdot\Delta U(n-1)$$

$$+ \sum_{m=2}^{3} SCABH(n-1,m)\cdot\Delta\omega(n-1,m)$$

$$- \left(\frac{2\xi_1^s}{\omega_1}\right)\cdot SCDHH(n)\cdot\Delta\dot{U}(n+1) - SCDHH(n)\cdot\Delta U(n+1)$$

$$- \sum_{m=2}^{3} SCBCH(n,m)\cdot\Delta\omega(n+1,m)$$

$$= - M(n)\cdot\Delta\ddot{y} \quad . \qquad (3.27)$$

In this section, three equations of motion have been derived for the left half of the n^{th} floor of the structural frame taking into consideration the symmetrical properties of the frame. In the next section, the methods used to solve the set of $3N$ equations of motion

for an N-floor nonlinear frame are discussed.

3.5 Solution of the Equations of Motion

As previously mentioned, because of yielding, the stiffness properties of the girders and columns change as a function of time during a strong earthquake. Hence, the equations of motion are nonlinear, which eliminates, for all practical purposes, solutions using modal analysis methods. Nevertheless, the nonlinear equations of motion can be solved by means of an integration technique which uses finite increments of time. In the integration technique used here, it is assumed that the stiffness properties of each girder and column in the structure remain constant throughout each time increment, but that they can change from one time increment to the next. Consequently, within each time increment, the equations of motion are linear.

One criteria for determining the magnitude "ΔT" of the time increment is

$$\Delta T = \frac{1}{10} \tau_{20}$$

where τ_{20} is the period of the 20^{th} mode of the linear structure. It should be mentioned that in the studies for this report the A/20/2.2/2/6 structure is analyzed with

$$\Delta T = 0.005 \text{ sec} \approx \frac{1}{6} \tau_{20} .$$

The integration technique used to solve the equations of

motion is one presented by E. L. Wilson and R. W. Clough[40] and is similar to the Newmark β-method[42,43] with $\beta = 1/6$. For this method, it is assumed that:

1. the acceleration is linear within each time interval; and
2. the acceleration, velocity and displacement at the beginning of the time interval are known.

With these assumptions, for each floor, a set of equations relating the incremental acceleration, velocity, and displacement that occur during the t^{th} time interval can be found. One form of this set of equations is the following:

$$\Delta U(n)_t = \Delta U(n)_t \ ;$$

$$\Delta \dot{U}(n)_t = \frac{3}{\Delta t} \Delta U(n)_t + B(n) \ ; \qquad (3.28)$$

$$\Delta \ddot{U}(n)_t = \frac{6}{(\Delta t)^2} \Delta U(n)_t + A(n) \ ;$$

where

$$A(n) = - \frac{6}{\Delta t} \ddot{U}(n)_{(t-\Delta t)} - 3\dot{U}(n)_{(t-\Delta t)} \ ;$$

$$B(n) = - 3\dot{U}(n)_{(t-\Delta t)} - \frac{\Delta t}{2} \ddot{U}(n)_{(t-\Delta t)} \ ;$$

$t-\Delta t$: time at the beginning of the increment; and

t : time at the end of the increment.

A(n) and B(n) appear as constants in Eq. 3.28 since they are determined at the beginning of the time increment. Using Eq. 3.28, the incremental acceleration and incremental velocity are eliminated from the equations of motion leaving only the incremental displacement; hence, it is said to be the "ΔU" form. Furthermore,

Eq. 3.28 can be manipulated into any one of six forms, ΔU, $\Delta \dot{U}$, $\Delta \ddot{U}$, U, $\dot{U}$, or $\ddot{U}$, where the name of the form corresponds to the variable that remains in the equations of motion.

Using Eq. 3.28, the equation of motion, Eq. 3.27, becomes

$$\left\{ \left(\frac{6}{(\Delta t)^2} + \frac{6\xi_1^m \omega_1}{\Delta t} \right) \cdot M(n) + \left(1 + \frac{6\xi_1^s}{\Delta t \cdot \omega_1} \right) \cdot (SCDHH(n) + SCDHH(n-1)) \right\} \cdot \Delta U(n)$$

$$+ \sum_{m=2}^{3} [SCBCH(n-1,m) - SCABH(n,m)] \cdot \Delta\omega(n,m)$$

$$- \left(1 + \frac{6\xi_1^s}{\Delta t \cdot \omega_1} \right) \cdot SCDHH(n-1) \cdot \Delta U(n-1)$$

$$+ \sum_{m=2}^{3} SCABH(n-1,m) \cdot \Delta\omega(n-1,m)$$

$$- \left(1 + \frac{6\xi_1^s}{\Delta t \cdot \omega_1} \right) \cdot SCDHH(n) \cdot \Delta U(n+1)$$

$$- \sum_{m=2}^{3} SCBCH(n+1,m) \cdot \Delta\omega(n+1,m)$$

$$= - [\Delta \ddot{y} + A(n) + 2 \cdot \xi_1^m \cdot \omega_1 \cdot B(n)] \cdot M(n) + \left(\frac{2\xi_1^s}{\omega_1} \right) \cdot \left[\begin{array}{l} -[SCDHH(n) + SCDHH(n-1)] \cdot B(n) \\ + SCDHH(n-1) \cdot B(n-1) + SCDHH(n) \cdot B(n+1) \end{array} \right] \tag{3.29}$$

where the incremental displacements and joint rotations are the only unknowns.

Two methods for solving the equations of motion, Eq. 3.11, Eq. 3.17, and Eq. 3.29, for a nonlinear tall structure are considered here. One is entitled relaxation-iteration and the other, matrix substitution. In the relaxation-iteration method, for each time increment, the three equations of motion are solved for each floor beginning with the top floor (roof) keeping all of the other floors fixed. Then, the equations for the adjacent floor are solved keeping all of the other floors, including the roof, fixed. This procedure is continued sequentially until the equations for all the floors have been "relaxed." By iteration, or repetition of this relaxation process until a convergence criteria is satisfied, the incremental displacements and rotations are found for this time increment. Then the succeeding time interval is considered and the relaxation-iteration process is applied again.

In the matrix substitution method,[37] which is described in more detail later in this section, a matrix equation is written involving a large tridiagonal matrix with 3×3 matrices as elements. For each time increment the equations of motion are solved according to a recursion relationship. No iteration is necessary.

In summary, any one of three forms, ΔU, $\Delta \dot{U}$, or $\Delta \ddot{U}$, of the integration technique can conceivably be used with either one of the two methods, iteration or matrix, for solving the incremental equations of motion.

Four of the combinations were studied: $\Delta \ddot{U}$-iteration, $\Delta \ddot{U}$-matrix, ΔU-iteration, and ΔU-matrix. Of these, only the

$\Delta\ddot{U}$-matrix does not work well. In this case, some of the 3×3 matrices are ill-conditioned since the relative magnitudes of the elements may be on the order of 10^6. The other three combinations yield approximately the same answers (resulting displacements are the same to about 4 decimal places) and take about the same amount of computer time, using single precision arithmetic (8 decimal digits). The combination used in all of the computer studies presented in this report is the ΔU-matrix method using double precision arithmetic* (16 decimal digits) for solving the incremental equations of motion and finding the displacements, joint rotations and interfloor displacements. However, the bending moments, shear forces, etc., are calculated using single precision.

Throughout this chapter the roof has been numbered $n = 2$ and the first floor above ground level $n = 21$ for reasons mentioned earlier. However, there is one exception: for the matrix operations described below, $n = 1$ corresponds to the roof and $n = 20$ corresponds to the first floor above the ground level. It should be mentioned that this notation, including the exception for the matrix operations, is consistent with the computer program.

The three equations of motion for the n^{th} floor of the structural frame, Eq. 3.11, Eq. 3.17 and Eq. 3.29, can be written together as a matrix equation, Eq. 3.30, which is of the form of Eq. 3.31. The equations of motion for a tall structure are obtained by writing Eq. 3.31 for each floor. Together, the equations for a

---

*By using double precision instead of single precision arithmetic, the computing time for the usual run, i.e., without Subroutine MODE, was increased approximately 75%.

$$
\left[\begin{array}{c|c|c|c|c|c|c|c|c}
-\left(1+\frac{6\xi_1^2}{\Delta t\cdot\omega_1}\right)\cdot & & & \left(1+\frac{6\xi_1^s}{\Delta t\omega_1}\right)\left(\begin{array}{c}\text{SCDHH(n-1)}\\ +\text{SCDHH(n)}\end{array}\right) & & & -\left(1+\frac{6\xi_1^s}{\Delta t\cdot\omega_1}\right)\cdot & & \\
\cdot\ \text{SCDHH(n-1)} & +\text{SCABH(n-1,2)} & +\text{SCABH(n-1,3)} & +\text{M(n)}\left(\frac{6}{\Delta t}+\frac{6\xi_1^m\omega_1}{\Delta t}\right) & +\text{SCBCH(n-1,2)} & +\text{SCBCH(n-1,3)} & \cdot\ \text{SCDHH(n)} & -\text{SCBCH(n,2)} & -\text{SCBCH(n,3)} \\
 & & & & -\text{SCABH(n,2)} & -\text{SCABH(n,3)} & & & \\
\hline
-\text{SCBCH(n-1,2)} & +\text{SCB(n-1,2)} & 0 & \begin{array}{l}+\text{SCBCH(n-1,2)}\\ -\text{SCABH(n,2)}\end{array} & \begin{array}{l}+\text{SCC(n-1,2)}\\ +\text{SGA(n,2)}\\ +\text{SCA(n,2)}\end{array} & +\text{SGB(n,2)} & +\text{SCABH(n,2)} & +\text{SCB(n,2)} & 0 \\
\hline
-\text{SCBCH(n-1,3)} & 0 & +\text{SCB(n-1,3)} & \begin{array}{l}+\text{SCBCH(n-1,3)}\\ -\text{SCABH(n,3)}\end{array} & +\text{SGB(n,2)} & \begin{array}{l}+\text{SGC(n,2)}\\ +\text{SCC(n-1,3)}\\ +\text{SGA(n,3)}\\ +\text{SCA(n,3)}\end{array} & +\text{SCABH(n,3)} & 0 & +\text{SCB(n,3)}
\end{array}\right]
\left\{\begin{array}{l}\Delta U(n-1)\\ \Delta\omega(n-1,2)\\ \Delta\omega(n-1,3)\\ \hline \Delta U(n)\\ \Delta\omega(n,2)\\ \Delta\omega(n,3)\\ \hline \Delta U(n+1)\\ \Delta\omega(n+1,2)\\ \Delta\omega(n+1,3)\end{array}\right\}
=
\left\{\begin{array}{l}-\text{M(n)}\cdot\left[\Delta\ddot{y}+\text{A(n)}+2\xi_1^m\cdot\omega_1\cdot\text{B(n)}\right]\\ \quad+\left(\frac{2\xi_1^s}{\omega_1}\right)\left[\begin{array}{l}+\text{SCDHH(n-1)}\cdot\text{B(n-1)}\\ -\left[\text{SCDHH(n)}+\text{SCDHH(n-1)}\right]\cdot\text{B(n)}\\ +\text{SCDHH(n)}\cdot\text{B(n+1)}\end{array}\right]\\ \hline 0\\ \hline 0\end{array}\right\}
\qquad (3.30)
$$

which is of the form

$$
\left[C_{n-1}^T \quad S \quad C_n\right]\left\{\begin{array}{l}\text{DR(n-1)}\\ \text{DR(n)}\\ \text{DR(n+1)}\end{array}\right\} = \text{DP(n)} \qquad (3.31)
$$

where

$$
\text{DR(n)} = \left\{\begin{array}{l}\Delta U(n)\\ \Delta\omega(n,2)\\ \Delta\omega(n,3)\end{array}\right\}
$$

and

$$
\text{DP(n)} = \left\{\begin{array}{l}-\text{M(n)}\cdot\left[\Delta\ddot{y}+\text{A(n)}+2\xi_1^m\cdot\omega_1\cdot\text{B(n)}\right]+\left(\frac{2\xi_1^s}{\omega_1}\right)\left[\begin{array}{l}+\text{SCDHH(n-1)}\cdot\text{B(n-1)}\\ -\left[\text{SCDHH(n)}+\text{SCDHH(n-1)}\right]\cdot\text{B(n)}\\ +\text{SCDHH(n)}\cdot\text{B(n+1)}\end{array}\right]\\ \hline 0\\ \hline 0\end{array}\right\}
$$

twenty floor structure form the tridiagonal matrix equation, Eq. 3.32:*

$$\begin{bmatrix} S_1 & C_1 & & & & & & & & \\ C_1^T & S_2 & C_2 & & & & & 0 & & \\ & C_2^T & S_3 & C_3 & & & & & & \\ & & & \cdot & & & & & & \\ & & & & \cdot & & & & & \\ & & & & & \cdot & & & & \\ & & & & & C_{17}^T & S_{18} & C_{18} & & \\ & 0 & & & & & C_{18}^T & S_{19} & C_{19} & \\ & & & & & & & C_{19}^T & S_{20} & \end{bmatrix} \cdot \begin{Bmatrix} DR_1 \\ DR_2 \\ DR_3 \\ \cdot \\ \cdot \\ \cdot \\ DR_{18} \\ DR_{19} \\ DR_{20} \end{Bmatrix} = \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \\ \cdot \\ \cdot \\ \cdot \\ DP_{18} \\ DP_{19} \\ DP_{20} \end{Bmatrix} \tag{3.32}$$

This set of equations is solved by successive substitution followed by successive back-substitution as shown below. The equation for $n = 1$ is

$$S_1 \cdot DR_1 + C_1 \cdot DR_2 = DP_1 \ .$$

Multiplying by S_1^{-1}, an equation for DR_1 is obtained:

$$DR_1 = -S_1^{-1} \cdot C_1 \cdot DR_2 + S_1^{-1} \cdot DP_1 \ . \tag{3.33}$$

DR_1, which is composed of the unknown displacement and joint rotations for floor $n = 1$, is eliminated from the equation for floor $n = 2$:

---

*This closely follows the matrix-substitution method presented by R. W. Clough, I. P. King, and E. L. Wilson in reference 37.

$$C_1(-S_1^{-1}\cdot C_1\cdot DR_2 + S_1^{-1}\cdot DP_1) + S_2\cdot DR_2 + C_2\cdot DR_3 = DP_2$$

or

$$\bar{S}_2\cdot DR_2 + C_2\cdot DR_3 = \overline{DP}_2 \qquad (3.34)$$

where

$$\bar{S}_2 = S_2 - C_1^T\cdot S_1^{-1}\cdot C_1$$

and

$$\overline{DP}_2 = DP_2 - C_1^T\cdot S_1^{-1}\cdot DP_1 \ .$$

From Eq. 3.34 a recursive relationship is seen for the n^{th} floor:

$$\bar{S}_n\cdot DR_n + C_n\cdot DR_{n+1} = \overline{DP}_n \qquad (3.35)$$

where

$$\bar{S}_n = S_n - C_{n-1}^T\cdot S_{n-1}^{-1}\cdot C_{n-1}$$

and

$$\overline{DP}_n = DP_n - C_{n-1}^T\cdot S_{n-1}^{-1}\cdot DP_{n-1} \ .$$

Noting that both C_{20} and DR_{21} are nonexistent for a 20 floor structure, the equation for floor $n = 20$ is

$$\bar{S}_{20}\cdot DR_{20} = \overline{DP}_{20} \qquad (3.36)$$

where $\bar{S}_{20}$ and $\overline{DP}_{20}$ are found by successive substitution. Since the only unknown in Eq. 3.36 is DR_{20}, it can be found from the equation

$$DR_{20} = \bar{S}_{20}^{-1}\cdot \overline{DP}_{20} \ . \qquad (3.37)$$

Having DR_{20}, by means of back-substitution using the recursive relationship

$$DR_n = \overline{S}_n^{-1}(\overline{DP}_n - C_n \cdot DR_{n+1}) \tag{3.38}$$

which is an inverse form of Eq. 3.35, solutions are found for DR_{19}, DR_{18} ... DR_1. In this manner, the incremental equations of motion are solved for each time increment.

In this chapter, the class of nonlinear tall structures treated in this report was described in detail, and the equations of motion for this class of structures were derived using the bending moment-end rotation equations for beams developed in Chapter II. Also, the method used to solve the equations of motion with the aid of a digital computer was outlined.

In order to verify that the resulting computer program operates properly, a number of tests were made. These are discussed in the next chapter.

CHAPTER IV

VERIFICATION OF THE COMPUTER PROGRAM

4.1 Introduction

The computer program developed in connection with this report is based upon the equations of motion derived in the previous chapter. As an overall check on the program, several tests covering various aspects of it were made. The purpose of this chapter is to briefly describe these tests.

4.2 Error Checks in the Program

In the computer solution of the equations of motion, one can anticipate certain round-off error. To evaluate these errors, the "error parameters" $\epsilon_1(n,m)$ and $\epsilon_2(n)$ were introduced into the moment balance and force balance equations, respectively.

For each time increment, the equations of motion were solved for the joint rotations and the displacements of the floors using double precision arithmetic (16 decimal digits). The individual bending moments and forces were found for the girders and columns of the frame using single precision arithmetic (8 decimal digits) since the nature of the round-off error in these computations is not so critical. Having determined the moments and forces , the error $\epsilon_1(n,m)$ in the bending moment balance at each joint in the frame is found according to Eq. 4.1, and the error $\epsilon_2(n)$ in the balance of forces acting on each floor is found according to Eq. 4.2:*

*See Chapter III for the terminology used in these equations.

$$\sum_{j=1}^{4} M(n,m,j) = \epsilon_1(n,m) \; ; \tag{4.1}$$

$$I(n) + D(n) + \sum_{m=2}^{3} [\, V(n-1,m) - V(n,m)] = \epsilon_2(n,m) \tag{4.2}$$

In the printed output at the end of the program, the maximum absolute values of $\epsilon_1(n,m)$ and $\epsilon_2(n)$ are printed along with the times at which they occur. For a typical computer run using an earthquake accelerogram, it is found that

$$|\epsilon_1(n,m)|_{max} \sim 10^{-6} |M(n,m,j)|_{max} \; ; \tag{4.3}$$

and

$$|\epsilon_2(n)|_{max} \sim 10^{-6} \left| \sum_{m=2}^{3} V(n,m) \right|_{max} . \tag{4.4}$$

It should be noted that for single precision arithmetic, the limit on accuracy is 10^{-8}.

If the values of the error parameters in the printed output of a computer run are consistent with the typical values given in Eq. 4.3 and Eq. 4.4, then there is reason to accept the results of the run. On the other hand, unusual behavior of the error parameters would lead one to suspect some type of program or computer malfunction. Such difficulties were not encountered in the computer runs for the present investigation.

4.3 The Wilson-Clough Integration Technique [40]

To investigate the properties of the Wilson-Clough integration technique, several different programs based on it were examined. At the same time, other programs were developed which used different integration techniques to solve the equations of motion for either the same or an equivalent system. These programs are described below, and the results are compared.

The equations of motion of the damped, three-mass, bilinear hysteretic system of Fig. 4.1, excited by a constant frequency, sinusoidal base acceleration $\ddot{y}(t)$, were solved using three different computer programs. The first program, explicitly written for this system, solved the equations of motion by means of a third order Runge-Kutta integration technique.* The second program, also explicitly developed for this system, used the Wilson-Clough integration technique. The third program was an adaptation of the regular program for analyzing nonlinear frames developed in connection with this report. In this adaptation the number of floors was reduced from twenty to three, and the girders were made rigid. The resulting model is shown in Fig. 4.2. The same approximately steady state initial conditions and the same sinusoidal excitation were used for all three programs. In each case the integration was carried out for fifteen cycles of the input acceleration. The resulting time history plots of the corresponding displacements were within

*This program, written by Dr. W. D. Iwan of California Institute of Technology, uses the standard integration subroutine of the California Institute of Technology Computing Center.

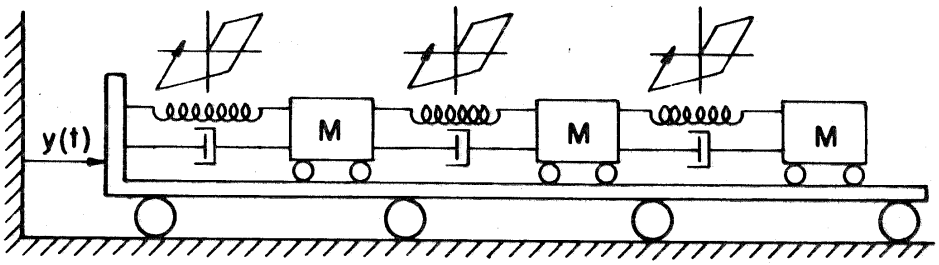

Fig 4.1 Three-mass, Damped, Base Excited System
With Bilinear Hysteretic Springs

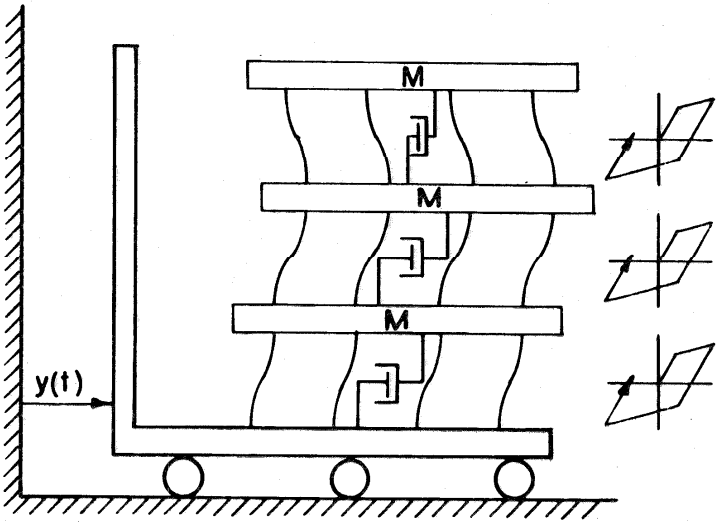

Fig. 4.2 Three-mass, Damped, Base Excited System
With Bilinear Hysteretic Springs

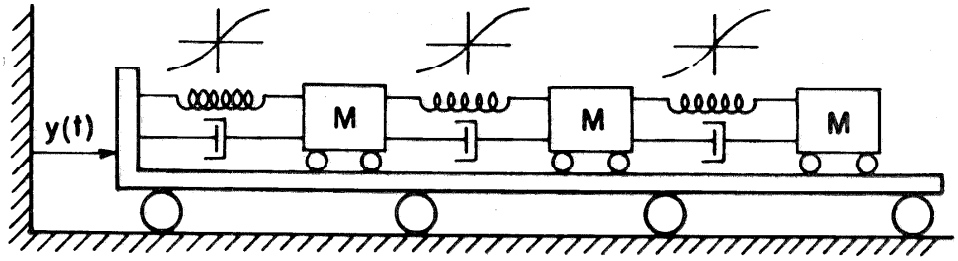

Fig. 4.3 Three-mass, Damped, Base Excited System
With Cubic Nonhysteretic Springs

2% of each other throughout the entire length of the computer runs.

A second system used for comparing the first two programs described above consisted of the damped, three-mass system with cubic-nonhysteretic springs shown in Fig. 4.3. Again, the resulting time history plots of the corresponding displacements were within 2% of each other for the fifteen cycles of the computer run. Even with zero initial conditions, this system achieves steady state motion after nine cycles.

Because of the consistency of the above results, it would appear that the Wilson-Clough integration technique is suitable for the present investigation.

4.4 Excitation of the Linear A/20/2.2/2/6 Frame Near Its Third Natural Frequency

For an additional test of the more complicated system later used in the nonlinear analysis, the computer program was modified so that the base excitation was a constant frequency, sinusoidal accelerogram. Included in this program was a means of checking that the response was approximately steady state.

The A/20/2.2/2/6 frame was used with the yield condition suppressed so that the system remained linear. The specified damping in the fundamental mode was 0.10 fraction of critical. Since mass proportional damping only was used, the effective damping in the higher modes is reduced, being 0.025 fraction of critical in the third mode. The frequencies of the half power points of the steady state resonance curve for a single degree of freedom system

having as a natural frequency the third modal frequency of the frame were analytically calculated (see Table 4-1).

Three computer runs were made in this test: one at each of the three frequencies given in Table 4.1. In each run the initial conditions were zero and the amplitude of the sinusoidal forcing function was the same. After the steady state criteria* were satisfied, the amplitude of the displacement of the 20^{th} floor and the phase angle between that floor and the forcing function were found. The three resulting amplitudes, normalized with respect to the amplitude for the third natural frequency, along with the corresponding phase angles are given in Table 4-1.

For a one degree of freedom system at steady state, the normalized amplitudes at the half-power points are 0.707 instead of 0.69 and 0.77, and the phase angles are 45^{o} and 135^{o} instead of 47^{o} and 140^{o}, respectively. The unnormalized displacement of the 20^{th} floor at the third natural frequency as calculated by the computer was 5% smaller than that found by analytical means using eigenvalue theory wherein it was assumed that the contributions to the excitation by all modes except the third were negligible.

---

*Within any one complete cycle of the input acceleration, if

$$|U\ MAX(n) + U\ MIN(n)| < 0.05 \cdot U\ MAX(n)$$

for each floor, where

U MAX(n) : the most positive displacement for the n^{th} floor during that cycle, and

U MIN(n) : the most negative displacement for the n^{th} floor during that cycle;

then an approximation to steady state is obtained.

TABLE 4-1

	Lower half-power point	Third natural frequency	Upper half-power point
Circular frequency	11.1905	11.4748	11.7591
Theoretical amplitude, normalized	0.707	1.00	0.707
Amplitude of the displacement of the 20th floor, normalized (computer output)	0.69	1.00	0.77
Theoretical phase angle	45°	90°	135°
Phase angle between displacement of the 20th floor and the forcing function* (computer output)	47°	90°	140°

These discrepancies in amplitudes and phase angles can be explained by the following factors:

1. Because the steady state criteria are not exact, the results of the computer program are only approximately steady state.
2. In the analytical calculations it is assumed that only one mode--the third--contributes to the excitation. However, in this test, base excitation is used which does not excite a pure mode in a multi-degree of freedom structure.

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*The forcing function used here is

$$f_n(t) = - M(n)\ddot{y}(t)$$

where

$M(n)$: mass of one half of the n^{th} floor, and
$\ddot{y}(t)$: base acceleration.

Consequently, even though the third mode is the dominant one in this frequency range, the others do contribute.

Since the discrepancies in the results are small and can be explained by these factors, this test provides further assurance that the computer program, including the mass proportional damping mechanism, is working properly.

Another point which reflects upon the accuracy of the computer program concerns the joint rotations of the linear A/20/2.2/2/6 frame. The stiffness properties of this frame (see Fig. 3.2) are designed* so that at each joint within the same floor the ratio of the sum of the column stiffnesses to the sum of the girder stiffnesses is the same. For example, for the first floor above the ground, this ratio for each joint is 2.4. Hence, as long as this frame remains linear, the rotations of the joints within the same floor should be equal. Even though the rotation of each joint is calculated independently, it is seen in the computer output that the rotations of joints within the same floor are the same to eight significant decimal digits, thus giving an additional check on the program. However, such a check cannot be made for the particular nonlinear frame studied in this report. For this frame, the strength properties are not directly proportional to the corresponding stiffness properties, and, therefore, the rotations within the same floor would not be expected to be equal. This difference in joint rotations within the same floor of the nonlinear frame can be seen in the time history plots of Chapter VI.

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*See Section 2.2 of the FHA Study, reference 1.

It is especially evident in the plots for the joints of the 20^{th} floor.

4.5 A Comparison With Results Presented in the FHA Study

In another comparison test, the A/20/2.2/2/6 frame was subjected to earthquake excitation and the resulting responses were compared with corresponding ones presented in the FHA Study. Since the yield criteria and the treatment of symmetry used in the regular nonlinear analysis program for this report differ from those used in the program for the FHA Study, the computer program used for this test includes modifications to remove these differences.

It is the author's understanding that the computer program used for the FHA Study is basically the same one presented in a report to the Office of Civil Defense by T. Y. Lin and Associates.[2] That program can treat an arbitrary number of girders and columns, but since the two-component beam model is used (see Section 2.5), the symmetrical properties of the three bay (four column) frame cannot be treated exactly by cutting the interior girders at the centerline as is done there. Nevertheless, this is an expedient approximation for that program. The problems that are encountered by using such an approximation are discussed in Appendix C. In order to account for the treatment of symmetry used in the FHA Study, the two-component beam model is used in the program for this test with appropriate modifications to the incremental moment-rotation-plastic angle equations.

As mentioned above, the yield criteria used in the regular

nonlinear analysis program for this report differ from those used in the program for the FHA Study. In order that the criteria be the same in both computations, the criteria used in the FHA Study are adopted for purposes of the comparison test. It is noted that the yield criteria used in the FHA program can, in special cases, lead to incorrect responses. The plastic angles and resulting ductility factors are particularly sensitive to this difficulty. The special situations under which these problems may become troublesome are discussed in Appendix B.

The same input data consisting of the structural properties and the accelerogram of the El Centro (N-S) earthquake of 18 May 1940 were used in both computations. This accelerogram is the same as the one used at the Earthquake Engineering Laboratory of the California Institute of Technology, but rounded off to four decimal places.

The responses obtained from the modified program using this input data are the same as those (within the tolerance of the plots) presented in the FHA Study.

In this Chapter several different tests of the computer program were outlined and the results of each test indicate that the program performs as expected. Consequently, it is believed that the computer program for determining the response of nonlinear tall structures subjected to earthquake excitation operates properly and provides the desired accuracy.

CHAPTER V

MODAL ANALYSIS OF CONTINUOUS CANTILEVER BEAMS AND OF TALL STRUCTURES

5.1 Introduction

In the past a number of authors, M. A. Biot,[6] G. N. Bycroft,[34] R. W. Clough,[36] R. L. Jennings,[19] K. Kanai,[32] K. Muto,[29] H. M. Westergaard[30] and others have mentioned that "higher modes" are important and that "whiplash" is observed in tall structures. This chapter is intended to determine the conditions under which higher modes are important as well as to suggest a definition of whiplash for tall structures.

By studying continuous beams and discretized models of tall structures, it is possible to gain a fuller understanding of why the displacements, strains (interfloor displacements), shear forces, and total accelerations have differing dependencies upon the various modes as well as upon the position in the structure. Two linear elastic continuous cantilever beams which represent limiting cases of tall structures are studied. In one, the stiffness and mass distributions are uniform, while in the other, these distributions taper to a point at the top. Furthermore, a discretized model of a typical structural frame (A/20/2.2/2/6)* is also studied by modal analysis. In Section 5.2 the continuous beams, which are subjected to an ensemble of earthquakes, are analyzed. In particular, the average

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*See Section 3.2 for a description of the structural properties.

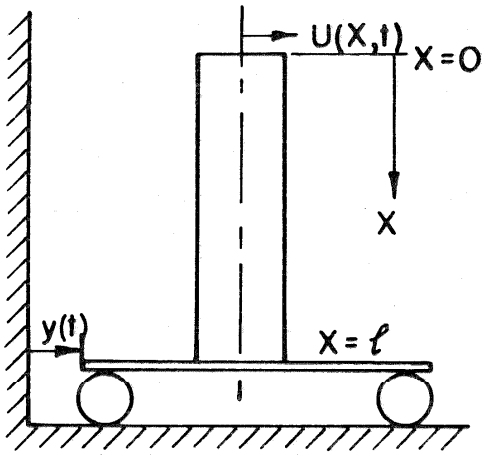

Fig. 5.1 Uniform Cantilever Beam

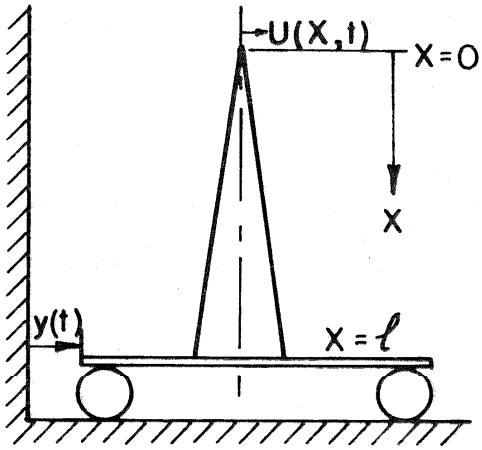

Fig. 5.2 Tapered Cantilever Beam

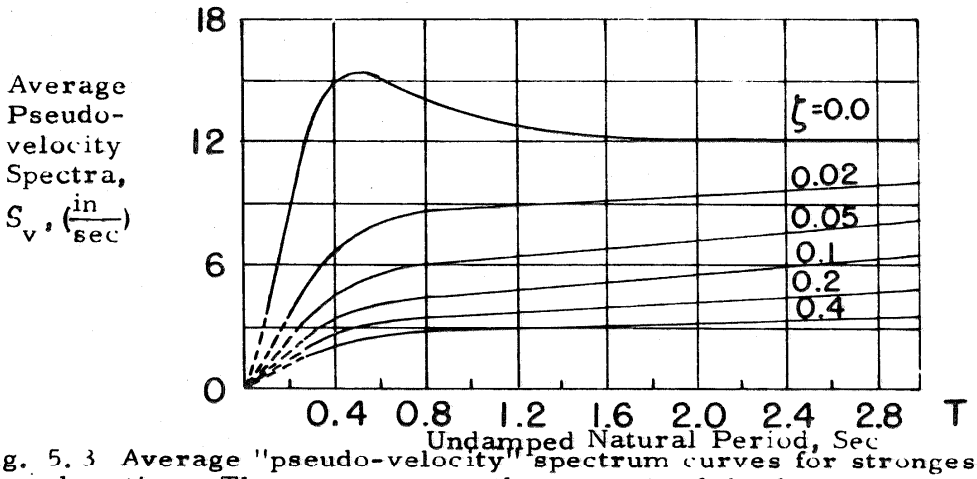

Fig. 5.3 Average "pseudo-velocity" spectrum curves for strongest ground motion. These curves are the average of the four strongest ground motions so far recorded: El Centro, California, May 18, 1940; El Centro, California, Dec. 30, 1934; Olympia, Washington, Apr. 13, 1949; Taft, California, July 21, 1952. Before averaging, each spectrum was normalized so that there was a unit area under the zero-damped curve. To correspond to the respective ground motions, the ordinates must be multiplied by factors as follows: El Centro, 1940 (2.7); El Centro, 1934 (1.9); Olympia, 1949 (1.9); Taft, 1952 (1.6). (Reference 9).

maximum absolute modal contributions to the displacements, strains, shear forces, and total accelerations are related to the average pseudo-velocity spectra S_v. In this chapter, the phrase "average maximum absolute" means average over an ensemble of earthquakes of the maximum absolute value over time of each member of the ensemble. In order to show how these average maximum absolute modal contributions depend upon the taper of the beam and upon the position $(\frac{x}{\ell})$ in the beam (see Fig. 5.1) some of them are later presented in tabular form.

In Section 5.3, the method used to obtain "equivalent" modal participation factors and the "equivalent" modal contributions to displacements and to interfloor displacements for yielding structures is treated. The results obtained by using this method for a typical yielding structural frame are then discussed.

5.2 Modal Analysis of Continuous Beams

One possible model for approximating a tall structure of uniform properties subjected to horizontal earthquake excitation is the shear-type continuous cantilever beam shown in Fig. 5.1. The general equation for this type of cantilever beam excited by the base acceleration $\frac{d^2y}{dt^2}(t)$ is

$$\frac{\partial}{\partial x}\left(K(x)\,\frac{\partial u}{\partial x}(x,t)\right) = \gamma(x)\left(\frac{\partial^2 u}{\partial t^2}(x,t) + \ddot{y}\right) \qquad (5.1)$$

with the boundary conditions:

$$\text{at } x = 0 , \qquad \frac{\partial u}{\partial x} = 0 \ ; \qquad \text{and}$$

$$\text{at } x = \ell , \qquad u = 0 \ ;$$

where

$K(x)$ is the shear stiffness in units of lbs ;

$\gamma(x)$ is the mass density in units of $\frac{\text{mass}}{\text{unit length}}$

$u(x,t)$ is the relative displacement; and

$\ddot{y} = \frac{d^2y}{dt^2}(t)$.

For the uniform beam, the properties of stiffness and mass density are constants:

$$K(x) = K_o , \text{ and}$$

$$\gamma(x) = \gamma_o .$$

The velocity of wave propagation, c, is defined by

$$\frac{1}{c^2} = \frac{\gamma_o}{K_o} .$$

By substituting these values into Eq. 5.1, the general equation for a uniform beam is obtained:

$$\frac{\partial^2 u}{\partial x^2}(x,t) - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}(x,t) = \frac{1}{c^2}\ddot{y}(t) . \qquad (5.2)$$

The boundary conditions remain the same:

$$x = 0 \qquad \frac{\partial u}{\partial x} = 0 , \qquad \text{and}$$

$$x = \ell \qquad u = 0 .$$

The solution of this eigenvalue problem with zero initial conditions is

$$u(x,t) = \frac{2c}{\ell} \sum_{n=1}^{\infty} \frac{(-1)^n}{\omega_n^2} \cos\left(\frac{\omega_n x}{c}\right) \int_0^t \ddot{y}(\tau) \cdot \sin\left(\omega_n (t-\tau)\right) d\tau \quad (5.3)$$

and the eigenfrequencies are given by

$$\frac{\omega_n \ell}{c} = \frac{(2n-1)\pi}{2} \quad , \quad n = 1,2,3,\ldots \quad . \quad (5.4)$$

In this section the primary interest in using modal analysis is in making comparisons between the average maximum absolute values of the individual modal contributions of the responses, and not in finding the sums of the modal contributions. Furthermore, there is some question as to the absolute convergence of the total acceleration series. This is discussed briefly later. Consequently, only the n^{th} terms of the various responses are considered.

From Eq. 5.4, the contribution of the n^{th} mode to the displacement is

$$u_n(x,t) = \frac{2c}{\ell} \frac{(-1)^n}{\omega_n^2} \cos\left(\frac{\omega_n x}{c}\right) \int_0^t \ddot{y}(\tau) \sin\left(\omega_n (t-\tau)\right) d\tau. \quad (5.5)$$

Differentiation of Eq. 5.5 with respect to x gives the contribution of the n^{th} mode to the strain:

$$\frac{\partial u_n}{\partial x}(x,t) = \frac{2}{\ell} \frac{(-1)^{n+1}}{\omega_n} \sin\left(\frac{\omega_n x}{c}\right) \int_0^t \ddot{y}(\tau) \sin\left(\omega_n (t-\tau)\right) d\tau \quad . \quad (5.6)$$

It is noted that strain for a continuous model is analogous to the interfloor displacement of a model of a structure.

The contribution of the n^{th} mode to the shear force is obtained by multiplying Eq. 5.6 by K_o:

$$K_o \frac{\partial u_n}{\partial x}(x,t) = \frac{2K_o}{\ell} \frac{(-1)^{n+1}}{\omega_n} \sin\left(\frac{\omega_n x}{c}\right) \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \ . \quad (5.7)$$

From Eq. 5.2 it is seen that the total acceleration is given by

$$\frac{\partial^2 u}{\partial t^2}(x,t) + \ddot{y}(t) = c^2 \frac{\partial^2 u}{\partial x^2}(x,t) \quad (5.8)$$

and the n^{th} modal contribution to the total acceleration is*

$$c^2 \frac{\partial^2 u_n}{\partial x^2}(x,t) = \frac{2c}{\ell}(-1)^{n+1} \cos\left(\frac{\omega_n x}{c}\right) \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \ . \quad (5.9)$$

By taking the maximum value over time of the absolute value of Eqs. 5.5, 5.6, 5.7 and 5.9, one finds the maximum absolute value of the n^{th} modal contribution to each response for a given position $(\frac{x}{\ell})$ in the beam. Note that a term common to all of these equations is the maximum over time of the absolute value of the integral

$$S_v(\omega_n) = \max_{0<t} \int_o^t \ddot{y}(\tau) \sin\left(\omega_n(t-\tau)\right) d\tau \quad (5.10)$$

which is sometimes called the "spectral velocity." Physically,

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*Absolute convergence of this series is discussed below.

$S_v(\omega_n)$ is the absolute value of the maximum pseudo-velocity for an undamped single degree of freedom oscillator with a natural frequency ω_n (or period T_n)* subjected to an earthquake accelerogram $\ddot{y}(t)$. Curves of $S_v(T)$ vs. period T have been made for an ensemble of earthquakes with several damping factors.(9) By averaging $S_v(T)$ over the ensemble, average spectral pseudo-velocity curves are obtained which are shown in Fig. 4.3. From these curves one can find the average over an ensemble of earthquakes of the maximum absolute value of the pseudo-velocity of a single degree of freedom oscillator with a given period and fraction of critical damping.

Hence, the average maximum absolute values of the n^{th} modal contributions to the various responses are as follows:

for displacement:

$$\overline{\left|u_n(x)\right|_{\substack{\max \\ 0<t}}} = \frac{2\ell}{c}\frac{4}{\pi^2(2n-1)^2}\cos\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \qquad (5.11)$$

for strain:

$$\overline{\left|\frac{\partial u_n}{\partial x}(x)\right|_{\substack{\max \\ 0<t}}} = \frac{2}{c}\frac{2}{\pi(2n-1)}\sin\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \qquad (5.12)$$

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*For linear systems "pseudo-velocity" equals the displacement times the natural frequency. Studies comparing pseudo-velocity spectra to velocity spectra for a one degree of freedom linear system excited by an ensemble of earthquakes have been performed by C. V. Chelapati, reference 15.

for shear force:

$$\overline{\left| K_o \frac{\partial u_n}{\partial x}(x) \right|_{\substack{\max \\ 0<t}}} = \frac{2K_o}{c} \frac{2}{\pi(2n-1)} \sin\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \qquad (5.13)$$

and for total acceleration:

$$\overline{\left| c^2 \frac{\partial^2 u_n}{\partial x^2}(x) \right|_{\substack{\max \\ 0<t}}} = \frac{2c}{l} \cos\left(\frac{\omega_n x}{c}\right) S_v(\omega_n) \quad . \qquad (5.14)$$

In Fig. 5.3 it is seen that for periods greater than 0.3 seconds (frequency less than $\frac{2\pi}{0.3}$ radians per second) and for very lightly damped systems the average spectral velocity $S_v(\omega)$ is approximately a constant, to be denoted by $\bar{S}_v$. Since the periods of only the first five modes of the A/20/2.2/2/6 structure are larger than 0.3 sec, only the first five modes of the continuous beams are considered below.

Looking again at Fig. 5.3, one sees that for periods less than 0.3 sec, as the periods approach zero ($\omega \to \infty$), the average spectral velocity $S_v(\omega)$ decreases approximately in proportion to T. From Eq. 5.4 then,

$$S_v(\omega_n) \sim \frac{1}{n} \quad .$$

This rate of decrease of $S_v(\omega_n)$ is sufficient to ensure absolute convergence of summations of Eqs. 5.5, 5.6, and 5.7 for displacement, strain, and shear force. However, there may be some

question of the absolute convergence of a summation of Eq. 5.9 in order to obtain the total acceleration, even though the average maximum absolute contribution of the n^{th} mode does decrease as (n) increases beyond some value. By assuming that the base excitation $\ddot{y}(t)$ has a bounded frequency spectrum, absolute convergence can be assured.

In order to show more explicitly the dependence of the average maximum absolute values of these responses upon position in the uniform beam, Eqs. 5.11, 5.12, 5.13, and 5.14 are evaluated at seven different positions in the beam with $S_v(\omega_n)$ replaced by $\overline{S}_v$. The results are tabulated in Tables 5-2, 5-4, 5-6, and 5-8. From these tables several general trends are seen regarding the dependencies of the various responses upon the modes and upon the position $(\frac{x}{\ell})$ in the beam.

Table 5-2 shows that the displacements can be expected to be dominated by the first two or three modes, particularly in the upper portion of the beam. In Tables 5-4 and 5-6 it is seen that the strains and shear forces in the lower portion of a uniform beam are highly dependent on the first few modes, while in the upper portion many modes can be expected to contribute significantly.

From Table 5-8 for the total acceleration it is seen that throughout the structure all modes can be expected to contribute. Note, however, that near the base of the beam the expected contribution of the first mode is the smallest of the first five modes.

In order to show the effect of taper on the response of a continuous cantilever beam, subjected to earthquake excitation, the

beam of Fig. 5.2 is examined.

In this case, the stiffness $K(x)$ and the density per unit length $\gamma(x)$ are functions of x:

$$K(x) = K_o \frac{x}{\ell}$$

$$\gamma(x) = \gamma_o \frac{x}{\ell} \quad .$$

However, the velocity of wave propagation, c, is constant since

$$\frac{1}{c^2} = \frac{\gamma(x)}{K(x)} = \frac{\gamma_o}{K_o} \quad .$$

By substituting these values into the general shear beam equation, Eq. 5.1, the equation for the tapered beam is obtained:

$$\frac{1}{x} \frac{\partial}{\partial x} \left(x \frac{\partial u}{\partial x} (x,t) \right) - \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2} (x,t) = \frac{1}{c^2} \ddot{y} \qquad (5.15)$$

with the same boundary conditions:

at $x = 0$, $\quad \frac{\partial u}{\partial x} = 0$; and

at $x = \ell$, $\quad u = 0$.

The solution of this eigenvalue problem with zero initial conditions is

$$u(x,t) = -\frac{2c}{\ell} \sum_{n=1}^{\infty} \frac{1}{\omega_n^2} \frac{J_o\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n \ell}{c}\right)} \int_o^t \ddot{y}(\tau) \sin\left(\omega_n (t-\tau)\right) d\tau \qquad (5.16)$$

and the frequency equation is

$$J_o\left(\frac{\omega_n \ell}{c}\right) = 0 \quad . \tag{5.17}$$

The roots of the Bessel function J_o are given in Table 5-1.

TABLE 5-1

Natural Frequencies of Uniform and Tapered Beams

	Uniform Beam		Tapered Beam	
	$\cos\left(\frac{\omega_n \ell}{c}\right) = 0$	Frequency Ratios	$J_o\left(\frac{\omega_n \ell}{c}\right) = 0$	Frequency Ratios
n	$\left(\frac{\omega_n \ell}{c}\right)$		$\left(\frac{\omega_n \ell}{c}\right)$	
1	1.57	1	2.4	1.00
2	4.71	3	5.52	2.30
3	7.85	5	8.65	3.61
4	10.99	7	11.8	4.92
5	14.13	9	14.9	6.21
6	17.27	11	18.1	7.54

Again, because the primary interest is in making comparisons between the average maximum absolute values of the individual modal contributions, the n^{th} terms of the various responses are discussed.

From Eq. 5.16, the n^{th} term of the displacement for the tapered beam is

$$u_n(x,t) = -\frac{2c}{\ell}\frac{1}{\omega_n^2}\frac{J_o\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n \ell}{c}\right)}\int_o^t \ddot{y}(\tau)\sin\left(\omega_n(t-\tau)\right)\,d\tau \quad . \qquad .(5.18)$$

By differentiating Eq. 5.18 with respect to x, the n^{th} modal contribution to the strain is obtained:

$$\frac{\partial u_n}{\partial x}(x,t) = \frac{2}{\ell}\frac{1}{\omega_n}\frac{J_1\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n \ell}{c}\right)}\int_o^t \ddot{y}(\tau)\sin\left(\omega_n(t-\tau)\right)d\tau \quad . \qquad (5.19)$$

The contribution of the n^{th} mode to the shear force in a tapered beam is found by multiplying Eq. 5.19 by $K_o\frac{x}{\ell}$:

$$K_o\frac{x}{\ell}\frac{\partial u_n}{\partial x}(x,t) = \frac{x}{\ell}\frac{2K_o}{\ell}\frac{1}{\omega_n}\frac{J_1\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n \ell}{c}\right)}\int_o^t \ddot{y}(\tau)\sin\left(\omega_n(t-\tau)\right)d\tau. \quad (5.20)$$

From Eq. 5.15 it is seen that the total acceleration for the tapered beam is given by

$$\frac{\partial^2 u}{\partial t^2}(x,t) + \ddot{y}(t) = \frac{c^2}{x}\frac{\partial}{\partial x}\left(x\frac{\partial u(x,t)}{\partial x}\right). \qquad (5.21)$$

Hence, the n^{th} modal contribution to the total acceleration is*

$$\frac{c^2}{x}\frac{\partial}{\partial x}\left(x\frac{\partial u_n}{\partial x}(x,t)\right) = \frac{2c}{\ell}\frac{J_o\left(\frac{\omega_n x}{c}\right)}{J_1\left(\frac{\omega_n \ell}{c}\right)}\int_o^t \ddot{y}(\tau)\sin\left(\omega_n(t-\tau)\right)d\tau \quad . \qquad (5.22)$$

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*See p.100 for discussion of absolute convergence of this series.

By taking the maximum over time of the absolute value of Eqs. 5.18, 5.19, 5.20 and 5.22; using Eq. 5.10; and letting $z_n = \frac{\omega_n \ell}{c}$; the average maximum absolute values of the contributions of the n^{th} mode to the various responses are obtained:

for displacement:

$$\overline{\left|u_n(x)\right|_{\substack{\max \\ 0<t}}} = \frac{2\ell}{c} \frac{1}{z_n^2} \frac{\left|J_1\left(z_n \frac{x}{\ell}\right)\right|}{\left|J_1(z_n)\right|} S_v(\omega_n) \quad , \tag{5.23}$$

for strain:

$$\overline{\left|\frac{\partial u_n(x)}{\partial x}\right|_{\substack{\max \\ 0<t}}} = \frac{2}{c} \frac{1}{z_n} \frac{\left|J_1\left(z_n \frac{x}{\ell}\right)\right|}{\left|J_1(z_n)\right|} S_v(\omega_n) \quad , \tag{5.24}$$

for the shear force:

$$\overline{\left|K_o \frac{x}{\ell} \frac{\partial u_n}{\partial x}(x)\right|_{\substack{\max \\ 0<t}}} = \frac{x}{\ell} \frac{2K_o}{c} \frac{1}{z_n} \frac{\left|J_1\left(z_n \frac{x}{\ell}\right)\right|}{\left|J_1(z_n)\right|} S_v(\omega_n) \, , \tag{5.25}$$

and for total acceleration:

$$\overline{\left|\frac{c^2}{x} \frac{\partial}{\partial x}\left(x \frac{\partial u_n}{\partial x}(x)\right)\right|_{\substack{\max \\ 0<t}}} = \frac{2c}{\ell} \frac{\left|J_o\left(z_n \frac{x}{\ell}\right)\right|}{\left|J_1(z_n)\right|} S_v(\omega_n) \, . \tag{5.26}$$

In order to show the dependencies of displacements, strains, shear forces, and total accelerations upon the first five modes and upon the position $(\frac{x}{\ell})$ in the tapered beam, Eq. 5.23, 5.24, 5.25 and 5.26 with $S_v(\omega_n)$ replaced by $\overline{S_v}$ have been tabulated in Tables 5-3, 5-5, 5-7 and 5-9. The general trends regarding the dependencies of the responses upon the modes of and upon the position $(\frac{x}{\ell})$ in the tapered beam are approximately the same as for the uniform beam with two exceptions. These exceptions are the increased emphasis on the higher modes in all of the responses in the tapered beam and the fact that the shear force in the tapered beam is proportional to

$$\frac{x}{\ell}\left(\frac{\partial u}{\partial x}\right)$$

whereas the shear force in the uniform beam is directly proportional to the strain.

The total acceleration coefficients of the uniform and tapered beams in Tables 5-8 and 5-9 are interesting since they indicate that each of the first five modes of the uniform beam can be expected to contribute about the same amount, except near the base where the contribution of the first mode becomes small. At the top of the tapered beam, however, the higher modes can be expected to dominate the total acceleration.

It appears, then, that a mass particle at the top of a severely tapered structure can be expected to feel very high accelerations. Furthermore, since it is more closely related to strain than to

TABLE 5-2

Displacement Coefficients of $\frac{2\ell}{c}\bar{S}_v$

for Uniform Beam from Eq. 5.11

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.0	0.405	0.045	0.016	0.008	0.005
0.167	0.391	0.032	0.004	0.002	0.004
0.333	0.351	0.000	0.014	0.007	0.000
0.500	0.287	0.032	0.011	0.006	0.004
0.667	0.203	0.045	0.008	0.004	0.005
0.833	0.105	0.032	0.016	0.008	0.004
1.000	0.000	0.000	0.000	0.000	0.000

Displacement Coefficients of $\frac{2\ell}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.0	1.000	0.111	0.040	0.020	0.012
0.167	1.000	0.081	0.011	0.005	0.009
0.333	1.000	0.000	0.040	0.020	0.000
0.500	1.000	0.111	0.040	0.020	0.012
0.667	1.000	0.222	0.040	0.020	0.025
0.833	1.000	0.304	0.149	0.076	0.034
1.000	0.000	0.000	0.000	0.000	0.000

TABLE 5-3

Displacement Coefficients of $\frac{2\ell}{c}\bar{S}_v$

for Tapered Beam from Eq. 5.23

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.0	0.333	0.096	0.049	0.031	0.022
0.167	0.320	0.077	0.027	0.008	0.001
0.333	0.282	0.031	0.011	0.012	0.004
0.500	0.223	0.016	0.018	0.004	0.006
0.667	0.151	0.038	0.004	0.006	0.005
0.833	0.074	0.029	0.014	0.007	0.003
1.000	0.000	0.000	0.000	0.000	0.000

Displacement Coefficients of $\frac{2\ell}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.0	1.000	0.290	0.148	0.093	0.065
0.167	1.000	0.241	0.084	0.024	0.003
0.333	1.000	0.108	0.038	0.044	0.014
0.500	1.000	0.073	0.079	0.017	0.026
0.667	1.000	0.254	0.027	0.042	0.035
0.833	1.000	0.387	0.196	0.098	0.040
1.000	0.000	0.000	0.000	0.000	0.000

TABLE 5-4

Strain Coefficients of $\frac{2}{c}\bar{S}_v$

for Uniform Beam from Eq. 5.12

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.0	0.000	0.000	0.000	0.000	0.000
0.167	0.165	0.150	0.123	0.088	0.050
0.333	0.318	0.212	0.064	0.045	0.071
0.500	0.450	0.150	0.090	0.064	0.050
0.667	0.551	0.000	0.110	0.079	0.000
0.833	0.615	0.150	0.033	0.024	0.050
1.000	0.637	0.212	0.127	0.091	0.071

Strain Coefficients of $\frac{2}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.0	0.000	0.000	0.000	0.000	0.000
0.167	1.000	0.911	0.746	0.533	0.304
0.333	1.000	0.667	0.200	0.143	0.222
0.500	1.000	0.333	0.200	0.143	0.111
0.667	1.000	0.000	0.200	0.143	0.000
0.833	1.000	0.244	0.054	0.038	0.081
1.000	1.000	0.333	0.200	0.143	0.111

TABLE 5-5

Strain Coefficients of $\frac{2}{c}\bar{S}_v$

for Tapered Beams from Eq. 5.24

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	0.157	0.220	0.234	0.211	0.162
0.333	0.296	0.310	0.162	0.014	0.105
0.500	0.400	0.225	0.077	0.108	0.041
0.667	0.457	0.033	0.134	0.078	0.018
0.833	0.462	0.137	0.025	0.032	0.057
1.000	0.416	0.181	0.116	0.085	0.067

Strain Coefficients of $\frac{2}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	1.000	1.398	1.486	1.342	1.030
0.333	1.000	1.047	0.548	0.048	0.356
0.500	1.000	0.563	0.192	0.270	0.103
0.667	1.000	0.072	0.294	0.170	0.039
0.833	1.000	0.296	0.053	0.068	0.123
1.000	1.000	0.436	0.278	0.204	0.161

TABLE 5-6

Shear Force Coefficients of $\frac{2K_o}{c}\bar{S}_v$

for Uniform Beam from Eq. 5.13

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	0.165	0.150	0.123	0.088	0.050
0.333	0.318	0.212	0.064	0.045	0.071
0.500	0.450	0.150	0.090	0.064	0.050
0.667	0.551	0.000	0.110	0.079	0.000
0.833	0.615	0.150	0.033	0.024	0.050
1.000	0.637	0.212	0.127	0.091	0.071

Shear Force Coefficients of $\frac{2K_o}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	1.000	0.911	0.746	0.533	0.304
0.333	1.000	0.667	0.200	0.143	0.222
0.500	1.000	0.333	0.200	0.143	0.111
0.667	1.000	0.000	0.200	0.143	0.000
0.833	1.000	0.244	0.054	0.038	0.081
1.000	1.000	0.333	0.200	0.143	0.111

TABLE 5-7

Shear Force Coefficients of $\frac{2K_o}{c}\bar{S}_v$

for Tapered Beam from Eq. 5.25

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	0.026	0.037	0.039	0.035	0.027
0.333	0.099	0.103	0.054	0.005	0.035
0.500	0.200	0.113	0.038	0.054	0.021
0.667	0.304	0.022	0.090	0.052	0.012
0.833	0.385	0.114	0.020	0.026	0.047
1.000	0.416	0.181	0.116	0.085	0.067

Shear Force Coefficients of $\frac{2K_o}{c}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	0.000	0.000	0.000	0.000	0.000
0.167	1.000	1.398	1.486	1.342	1.030
0.333	1.000	1.047	0.548	0.048	0.356
0.500	1.000	0.563	0.192	0.270	0.103
0.667	1.000	0.072	0.294	0.170	0.039
0.833	1.000	0.296	0.053	0.068	0.123
1.000	1.000	0.436	0.278	0.204	0.161

TABLE 5-8

Total Acceleration Coefficients of $\frac{2c}{\ell}\bar{S}_v$

for Uniform Beam from Eq. 5.14

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	1.000	1.000	1.000	1.000	1.000
0.167	0.966	0.707	0.259	0.259	0.707
0.333	0.866	0.000	0.866	0.866	0.000
0.500	0.707	0.707	0.707	0.707	0.707
0.667	0.500	1.000	0.500	0.500	1.000
0.833	0.259	0.707	0.966	0.966	0.707
1.000	---------- base acceleration ----------				

Total Acceleration Coefficients of $\frac{2c}{\ell}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	1.000	1.000	1.000	1.000	1.000
0.167	1.000	0.732	0.268	0.268	0.732
0.333	1.000	0.000	1.000	1.000	0.000
0.500	1.000	1.000	1.000	1.000	1.000
0.667	1.000	2.000	1.000	1.000	2.000
0.833	1.000	2.732	3.732	3.732	2.732
1.000	---------- base acceleration ----------				

TABLE 5-9

Total Acceleration Coefficients of $\frac{2c}{\ell}\bar{S}_v$

for Tapered Beam from Eq. 5.26

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	1.926	2.939	3.684	4.302	4.842
0.167	1.850	2.349	2.003	1.049	0.206
0.333	1.629	0.931	0.805	1.724	0.896
0.500	1.290	0.495	1.312	0.520	1.311
0.667	0.874	1.170	0.302	0.872	1.180
0.833	0.427	0.870	1.085	1.009	0.664
1.000	----------base acceleration ----------				

Total Acceleration Coefficients of $\frac{2c}{\ell}\bar{S}_v$

Normalized to Coefficients of Mode 1

$\frac{x}{\ell}$	Mode 1	2	3	4	5
0.000	1.000	1.526	1.912	2.233	2.513
0.167	1.000	1.270	1.083	0.567	0.112
0.333	1.000	0.571	0.494	1.058	0.550
0.500	1.000	0.384	1.017	0.403	1.016
0.667	1.000	1.339	0.346	0.998	1.350
0.833	1.000	2.039	2.541	2.365	1.555
1.000	----------base acceleration ----------				

displacement, structural damage appears to be more dependent upon the higher modes than upon the lower ones, particularly in the upper portion of a severely tapered structure.

From these tables of response coefficients it can be concluded that in order to minimize the strains and total accelerations in a structure, particularly in the upper portion, it is best to have the stiffness and mass distributions as uniform as possible.

It is also clear from these tables that the extent to which "higher modes" are important depends upon the response of interest, the taper of the structure, and the position within the structure.

Often an analogy is made between either excessive displacements (relative to ground), structural damage, or total accelerations near the top of a building and an energy pulse traveling toward and being reflected from the tip of a whip. These phenomena are said to result from "whiplash" of the structure. Since structural damage and total acceleration are directly related to excessive interfloor displacement and do not necessarily result from large displacements, a suggested definition of "whiplash" in structures is the magnitude of the interfloor displacements.

5.3 Modal Analysis of Discretized Models of Tall Structures

The application of eigenvalue theory to the discretized model of a structure is covered in this section. Using the equations developed here, a computer program subroutine was written which determines the "equivalent" modal participation factors, "equivalent"

modal contributions to the displacements and "equivalent" modal contributions to the interfloor displacements for arbitrarily selected floors in the structure. Using such results a comparison can be made with the results for the continuous beams.

Usually, modal analysis is restricted to structures that remain linear. However, modal analysis can also be useful for structures which yield. Since the eigenvectors of the linear structure form a complete set of base vectors, it is possible to expand any displacement vector in terms of these eigenvectors in order to find what will be called the "equivalent" modal participation factors. These factors can then be used to find "equivalent" modal contributions to the displacements and to the interfloor displacements. In this sense, the concept of the modal participation factor has the same significance for a yielding structure as for a linear one.

It should be noted that, for the excitation used, all of the columns in the A/20/2.2/2/6 structure remain linear except for those between the upper two or three floors. Furthermore, all of the girders yield--but not necessarily all at the same time. Hence, from an overall point of view, the structure is reasonably linear when compared to one in which all of the girders and columns yield at the same time.

The eigenvectors of a linear structure with N floors are found as follows: The homogeneous equation of motion is

$$M\ddot{\underline{u}} + K\underline{u} = 0 \tag{5.27}$$

where

$\underline{u}$: displacement vector $(N \times 1)$,

M : diagonal mass matrix $(N \times N)$, and

K : symmetric and positive definite stiffness matrix $(N \times N)$.

Letting

$$\underline{u} = M^{-\frac{1}{2}}\underline{\xi} \tag{5.28}$$

and premultiplying by $M^{-\frac{1}{2}}$, Eq. 5.27 becomes

$$I\ddot{\underline{\xi}} + \overline{K}\underline{\xi} = 0 \tag{5.29}$$

where

$$\overline{K} = M^{-\frac{1}{2}}KM^{-\frac{1}{2}} .$$

Since $\overline{K}$ is symmetric and positive definite, a theorem of matrix algebra guarantees the existence of a unitary transformation matrix ϕ such that

$$\phi^T\phi = I \quad \text{and} \quad \phi^T\overline{K}\phi = \begin{bmatrix} \omega_n^2 \end{bmatrix} . \tag{5.30}$$

Therefore, letting

$$\underline{\xi} = \phi\underline{\eta} \tag{5.31}$$

and premultiplying by ϕ^T, Eq. 5.29 becomes

$$\ddot{\underline{\eta}} + \begin{bmatrix} \omega_n^2 \end{bmatrix}\underline{\eta} = 0 \tag{5.32}$$

which is a diagonalized, or separated equation. That is, the solution for the n^{th} modal participation factor η_n is independent of every other mode.

By substituting Eq. 5.31 into Eq. 5.28, an equation relating the displacement vector, $\underline{u}$, to the modal participation factors, $\underline{\eta}$ is obtained:

$$\underline{u} = M^{-\frac{1}{2}} \phi \underline{\eta} \quad . \tag{5.33}$$

The inverse of Eq. 5.33 is

$$\underline{\eta} = \phi^T M^{+\frac{1}{2}} \underline{u} \quad . \tag{5.34}$$

Equations 5.33 and 5.34 are the ones used in the modal analysis of a tall structure.

As discussed in Section 3.5, a direct integration method is used in the computer program to solve the equations of motion for the yielding structure. Then, knowing the displacement vector $\underline{u}$, Eq. 5.34 is used to find the vector, $\underline{\eta}$, of "equivalent" modal participation factors. For η_n, Eq. 5.34 becomes

$$\eta_n = \sum_{k=1}^{20} \phi_{kn} M_k^{+\frac{1}{2}} u_k \tag{5.35}$$

for a structure with 20 floors. Similarly, for the displacement, u_j, of the j^{th} floor, Eq. 5.33 becomes

$$u_j = \sum_{n=1}^{20} M_j^{-\frac{1}{2}} \phi_{jn} \eta_n \quad . \tag{5.36}$$

Consequently, after obtaining η_n, the "equivalent" contribution of the n^{th} mode to the displacement of the j^{th} floor, u_{jn}, is found

from

$$u_{jn} = M_j^{-\frac{1}{2}} \phi_{jn} \eta_n \quad . \tag{5.37}$$

Using Eq. 5.37, the "equivalent" contribution of the n^{th} mode to the interfloor displacement, UU_{jn},* between floors (j) and (j+1) is found by subtraction:

$$UU_{jn} = u_{jn} - u_{(j+1)n} \tag{5.38}$$

As mentioned earlier, the interfloor displacement of the discretized structure corresponds to the strain in a continuous beam.

Only one modal analysis study of a yielding structure was made on the computer because of the extensive time involved.** In this study the A/20/2.2/2/6 structure with 0.0035 fraction of critical damping*** in the first mode was subjected to the four seconds of the El Centro Earthquake (N-S) of 18 May 1940. Furthermore, the two-component beam model described in Appendix C and the yield criteria with the "test bending moment" described in Appendix B were used. Although the beam model and yield criteria were later

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*UU is the Fortran name for interfloor displacement. Also, j = 2 at the top floor (roof) and j = 22 at the ground level. See Section 3.2 for more details of notation.

**The modal analysis is performed by subroutine MODE, which is optional. A listing of this subroutine is provided in Appendix E--Listing of the Computer Program.

***This fraction of critical damping is believed to be the same as that used by Clough and Benuska for their studies with essentially no damping in the FHA Study (reference 1).

TABLE 5-10

Modal Contributions to the Largest Displacements for the A/20/2.2/2/6 Structure

Floor	$\frac{x}{\ell}$	Mode 1	2	3	4	5	6	7	Displacement (inches)	Time of Occurrence
19	.05	-12.54	+.841	-.640	-.120	+.065	+.013	-.035	-12.3	3.155 sec
14	.30	- 9.52	-.121	+.316	+.076	+.143	-.042	-.254	- 9.12	3.230 sec
9	.55	- 5.93	-.597	-.133	-.136	-.154	+.014	+.044	- 6.86	3.155 sec
4	.80	- 2.058	-.331	-.713	+.063	+.135	+.035	+.007	- 2.92	3.010 sec

Modal Contributions Normalized to the Contribution of the First Mode

Floor	$\frac{x}{\ell}$	Mode 1	2	3	4	5	6	7
19	.05	1.00	.067	.051	.010	.005	.001	.003
14	.30	1.00	.013	.033	.008	.015	.004	.027
9	.55	1.00	.101	.022	.023	.026	.002	.007
4	.80	1.00	.161	.346	.031	.066	.017	.003

TABLE 5-11

Modal Contributions to the Largest Inter-floor Displacements for the A/20/2.2/2/6 Structure

Between Floors	$\frac{x}{\ell}$	Mode 1	2	3	4	5	6	7	Interfloor Displacement (inches)	Time of Occurrence
18-19	.08	-.038	-.132	-.373	-.360	-.199	+.120	+.020	-1.11	2.525 sec
13-14	.33	-.662	+.122	+.035	+.106	-.041	-.053	+.032	- .519	3.210 sec
8- 9	.58	-.685	-.001	-.018	-.103	+.012	+.019	+.062	- .756	3.335 sec
3- 4	.83	-.617	-.085	-.080	+.011	-.020	-.039	-.026	- .957	3.160 sec

Modal Contributions Normalized to the Contribution of the First Mode

Between Floors	$\frac{x}{\ell}$	Mode 1	2	3	4	5	6	7
18-19	.08	1.00	3.478	9.847	9.488	5.256	3.161	.536
13-14	.33	1.00	.184	.053	.160	.062	.080	.048
8- 9	.58	1.00	.001	.026	.151	.017	.027	.090
3- 4	.83	1.00	.137	.130	.018	.032	.064	.042

refined (see Chapter II and Appendix C, respectively), it is felt that the general trends of the results presented below are, nevertheless, correct.

The entries in Table 5-10 are the "equivalent" modal contributions to the displacements of the arbitrarily selected floors at the time when the entire displacement (not each modal contribution) reaches its maximum absolute value. Similarly, the entries in Table 5-11 are the "equivalent" modal contributions to the selected interfloor displacements at the time when the entire interfloor displacement (not each modal contribution) reaches its maximum absolute value. Hence, Tables 5-10 and 5-11 are not directly comparable to the corresponding tables for the continuous beams.

At various times during this study, modes as high as the eighth were found to contribute as much as 2% to the displacements. And what is more important, modes as high as the 13^{th} were found to contribute similar amounts to the interfloor displacements. If one either increased the level of the mass proportional damping, or preferably, introduced stiffness proportional damping, the number of modes which contribute significantly would probably decrease.

In Table 5-10 it is seen that, for the instances presented, the fundamental mode dominates the displacements throughout the structure. It is also seen that the higher modes contribute very little to the displacements in the upper portion but somewhat more in the lower portion.

On the other hand, for the cases presented in Table 5-11, the

third and fourth modes contribute the largest amounts to the interfloor displacement between the 18^{th}-19^{th} floor (which is the largest one occurring anywhere in the structure) with the fundamental mode contributing a relatively small amount. In the lower portion, however, the fundamental is the most important contributor.

Even though the results of this computer study are not directly comparable to the studies of continuous beams, the general trends of the modal contributions are the same. That is, (1) the fundamental mode dominates the displacements, particularly in the upper floors; and (2), for the interfloor displacements, the higher modes usually contribute more than the fundamental in the upper floors, but in the lower floors, the fundamental mode is more important.

In order to learn more about these trends, the time history plots of numerous responses are analyzed in the next chapter.

CHAPTER VI

RESPONSE RESULTS

6.1 Introduction

The purpose of this chapter is to present response results from computer studies of the A/20/2.2/2/6 structural frame (Sec. 3.2) subjected to earthquake excitation. These studies are carried out by the computer program described earlier in the report (Sec. 3.5) using the one-component beam model (Sec. 2.5) for the girders and columns of the frame. The plots given here are time histories of bending moments, interfloor shear forces, joint rotations, total accelerations, displacements and interfloor displacements for a number of stations in the structure.

The first section of this chapter shows the effect of yielding on the response of the structural frame. The excitation used in this investigation is the first four seconds of the accelerogram of the El Centro (N-S) earthquake of 18 May 1940. This is the strongest portion of the earthquake. In a later study in which the nonlinear frame was subjected to the entire length of the earthquake, it was observed that the maximum values of most of the responses occurred during the first four seconds, although several occurred afterwards. It is believed, however, that results based on the first four seconds are indicative of the effect of yielding.

The second section treats the response results for the nonlinear frame subjected to the entire length (30 sec) of the Jennings pseudo-earthquake number 6. In addition to time history plots, the

TABLE 6-1

Nomenclature Used in the Time History Plots in Chapter VI and in Appendix A

TOTAL ACCEL 17F	: The total horizontal acceleration of the 17^{th} floor
HORIZ DISP 4F	: The horizontal displacement of the 4^{th} floor relative to ground
OVRTURNG MOMT-BASE	: The overturning moment at the base of the three bay structure
SHEAR FORCE 18-19F	: One-half of the interfloor shear force between the 18^{th} and 19^{th} floors (from only the two columns used in the analysis)
INT-FL DISP GD-1F	: The interfloor displacement between the ground and the first floor
JNT ROT'N 17F-XC	: The rotation of the joint formed by the intersection of the exterior girder of the 17^{th} floor and the exterior column line
BND MOMT 17F-XG-XC	: The bending moment at station (e)
PL ANGLE 4F-XG-XC	: The plastic angle at station (f)
PL INDEX 4F-XG-XC	: The plastic index at station (f). This index indicates when yielding occurs and the direction of incremental yielding
BND MOMT 20F-IC-BT	: The bending moment at station (d)
MOD JNT ROT 20F-IC	: The rotation of the joint formed by the intersection of the 20^{th} floor and the interior column line modified to include the effect of the interfloor shear angle (see Eq. 3.8)
PL ANGLE 2F-XG-IC	: The plastic angle at station (g)
BND MOMT 2F-IG-IC	: The bending moment at station (h)

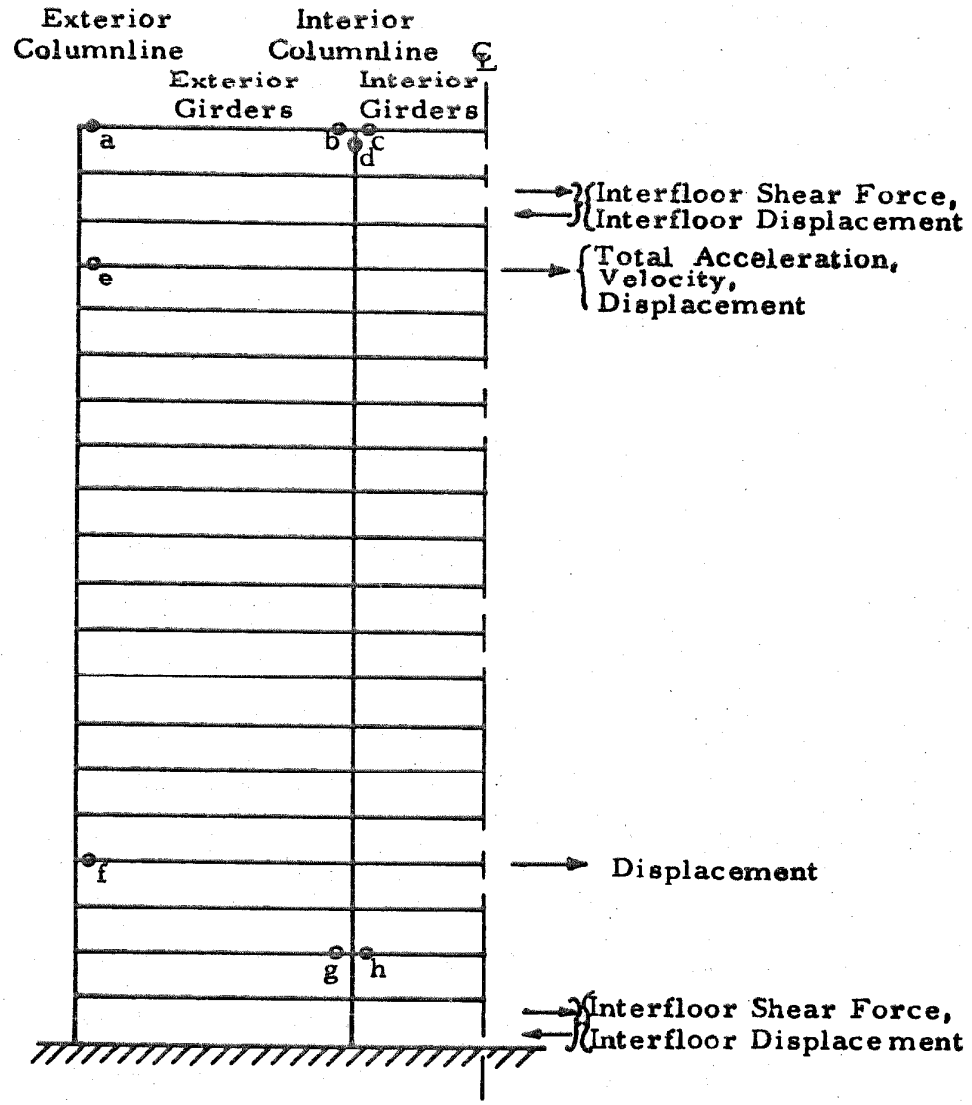


Fig. 6.1 o : Stations for Which the Bending Moment, Joint Rotation (or Modified Joint Rotation), Plastic Angle, and Plastic Index are Plotted.

displacement envelope and the ductility factors are shown. Response plots for similar studies of this frame using the entire length (29.38 sec) of the El Centro (N-S) earthquake of 18 May 1940 and several other pseudo-earthquakes are given in Appendix A.

The nomenclature used in these plots is described in Table 6-1. The unit system used is "in-lbs-sec" with the joint rotations given in radians. The stations for which the time histories of the responses are plotted are shown in Fig. 6.1.

6.2 Comparison of the Linear and Nonlinear Responses of a Structural Frame

In this section a comparison is made between the responses of the linear A/20/2.2/2/6 structural frame and the corresponding nonlinear frame. Each system had 0.10 fraction of critical damping in the fundamental mode and was subjected to approximately the first four seconds of the El Centro (N-S) earthquake of 18 May 1940.

The time histories of the total acceleration of the 17^{th} floor, the horizontal displacements of the 17^{th} floor and the 4^{th} floor as well as the overturning moment at the base of each frame are shown in Fig. 6.2a and Fig. 6.2b. For both the linear and the nonlinear cases, the displacement of the 17^{th} floor and the overturning moment at the base are primarily characterized by the frequency of the fundamental mode. In the time history of the displacement of the 4^{th} floor, higher frequencies are observed in addition to the fundamental for both the linear and nonlinear cases. Many frequencies are noted in the time history of the total acceleration of the 17^{th} floor. These

observations are in agreement with those for the continuous cantilever beam of Chapter V.

Comparing the total acceleration of the 17^{th} floor of the linear structure with that of the nonlinear structure, two features are noted which call for some discussion. The first is that an unusual peak occurs at 2.7 sec in the nonlinear plot in Fig. 6.2b which does not occur in the corresponding linear response, Fig. 6.2a. The second concerns the relatively larger high frequency components of the nonlinear plot.

For the above calculations mass proportional damping was used but no stiffness proportional damping was introduced. As a result, the higher modes are not so effectively damped as the lower. Consequently, there is a limitation on the model for the representation of certain responses which exhibit significant high frequencies.

The first feature to be discussed concerns the unusual peak in the total acceleration record of the 17^{th} floor in the nonlinear frame. This peak has an absolute value of 0.81g and is the maximum absolute value of the total acceleration for any floor during this run for the nonlinear case. However, for the linear frame, the maximum absolute value of the total acceleration at the 17^{th} floor is 0.58g while at the 20^{th} floor it is 1.0g. Since the maximum absolute value of the acceleration of the ground is 0.32g, it appears possible to have amplifications of the total acceleration on the order of 2 or more in this particular nonlinear structure. As a result of yielding, the amplitude and phase of the shear force waves in the nonlinear case

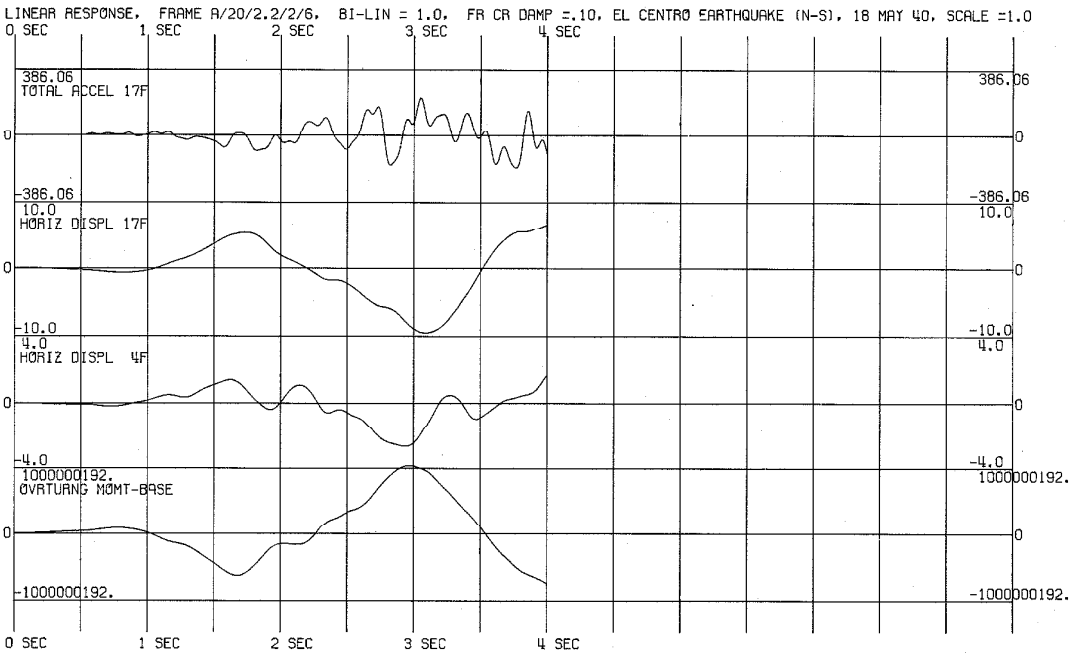


Fig. 6.2a

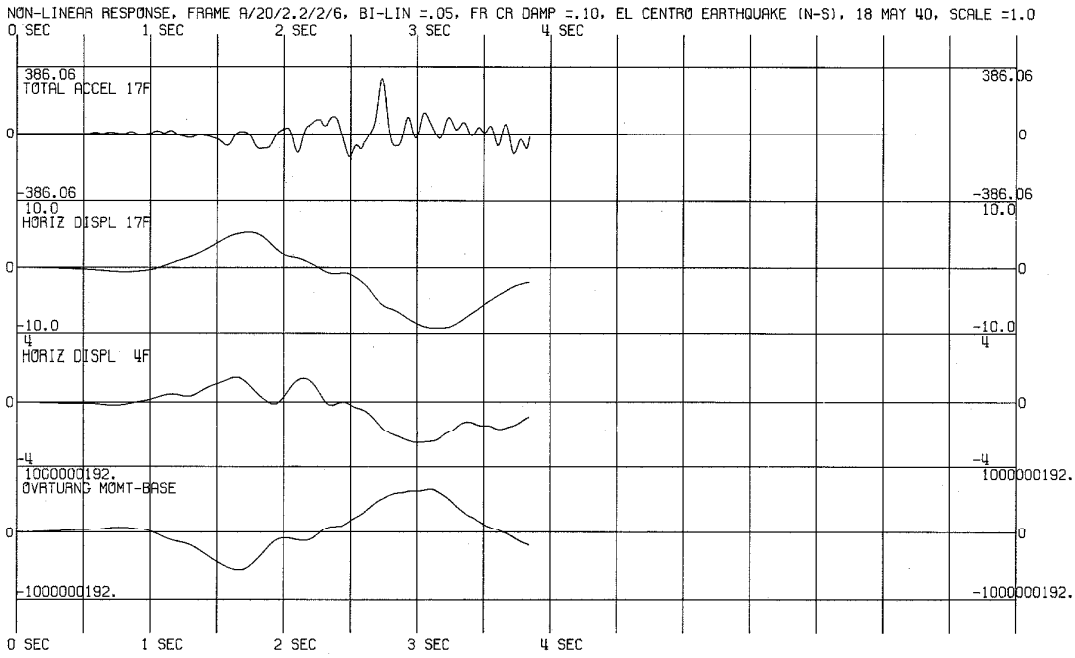


Fig. 6.2b

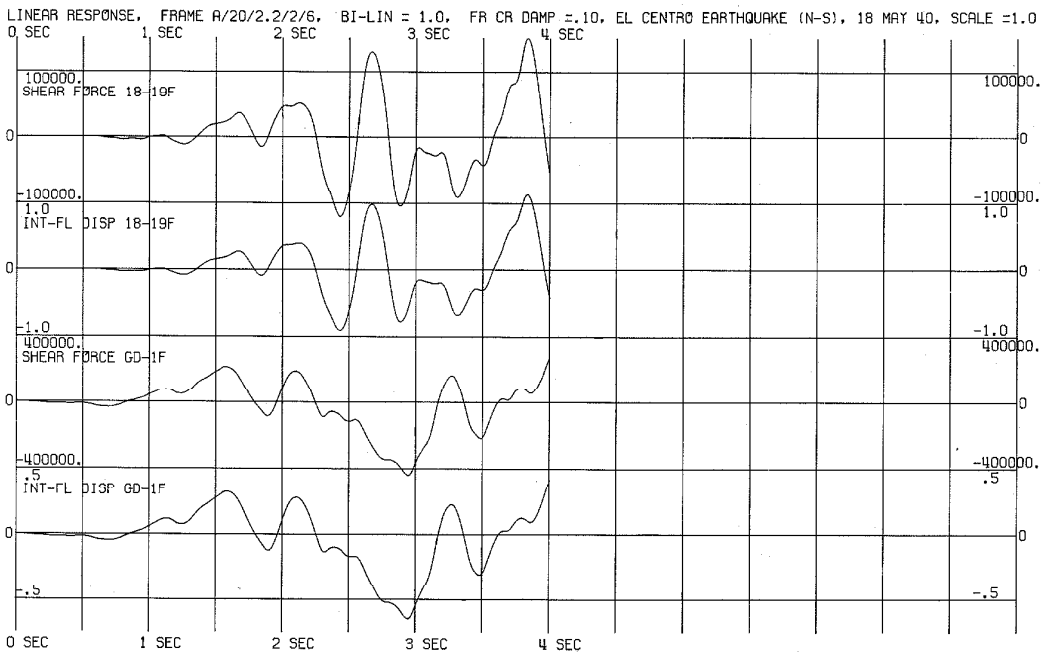


Fig. 6.3a

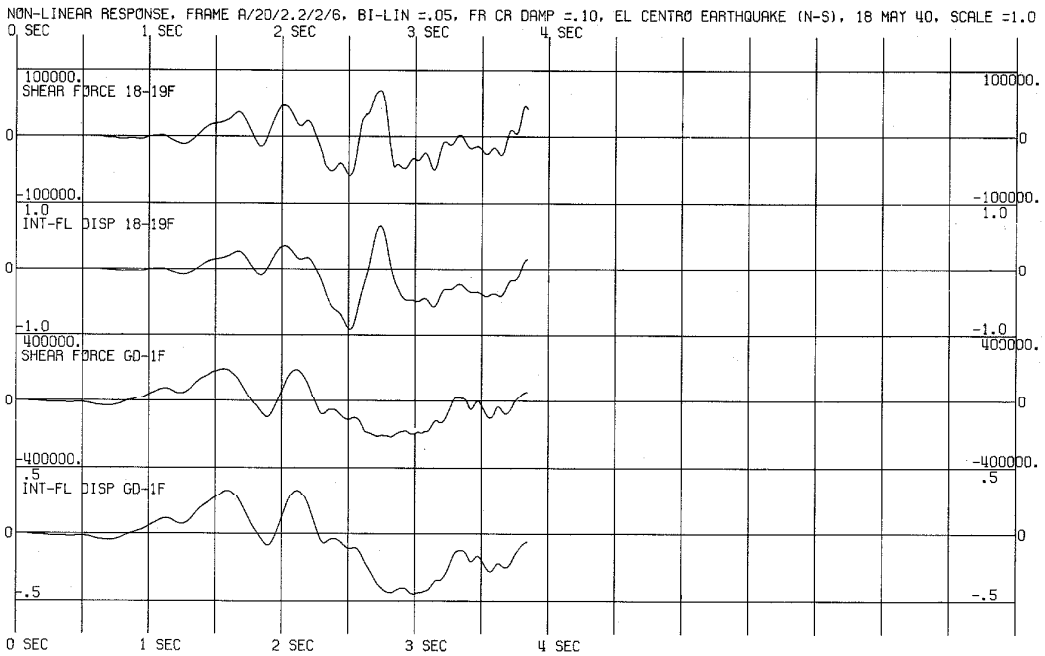


Fig. 6.3b

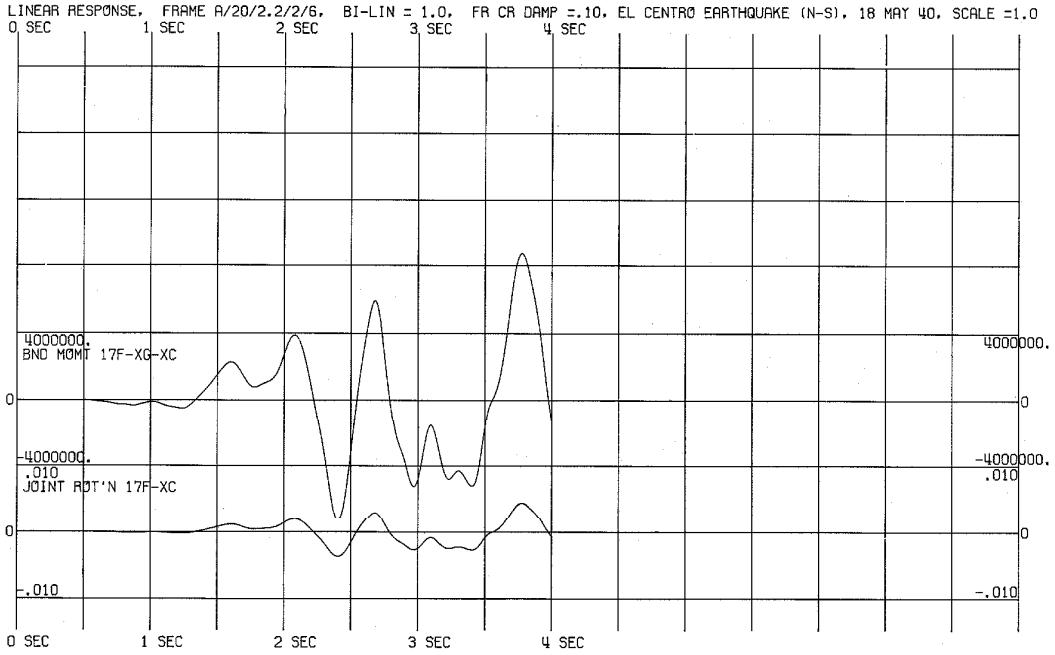


Fig. 6.4a

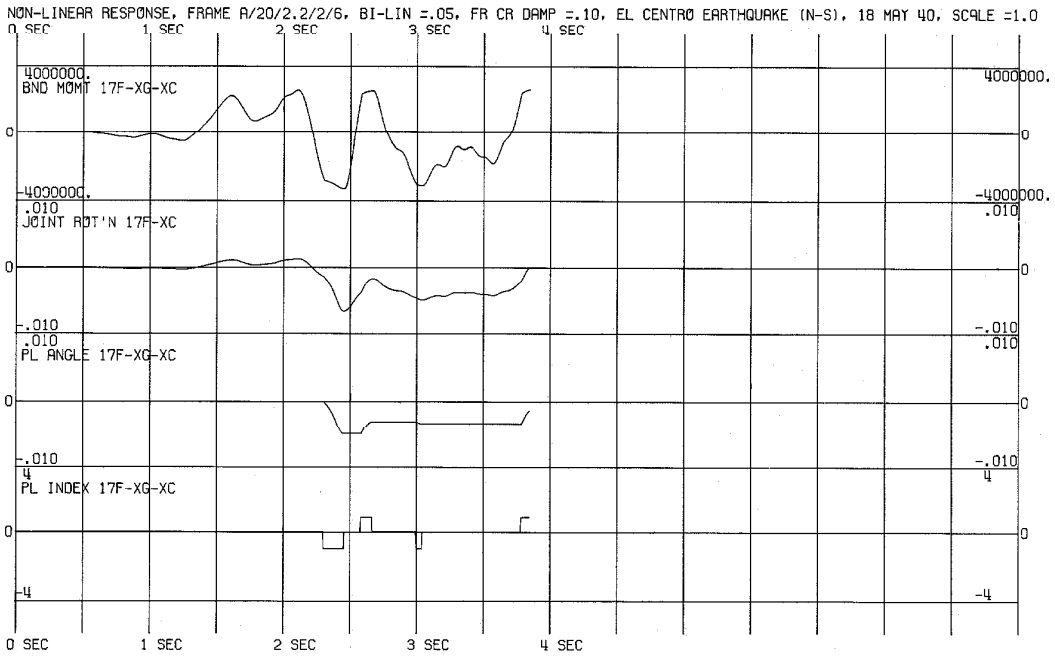


Fig. 6.4b Station (e)

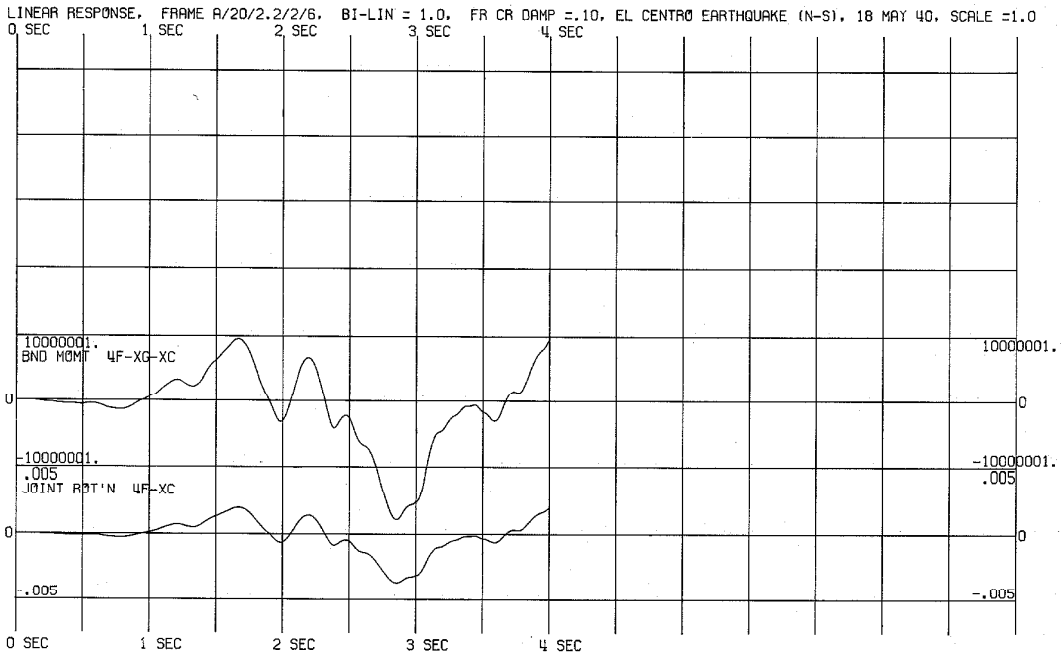


Fig. 6.5a

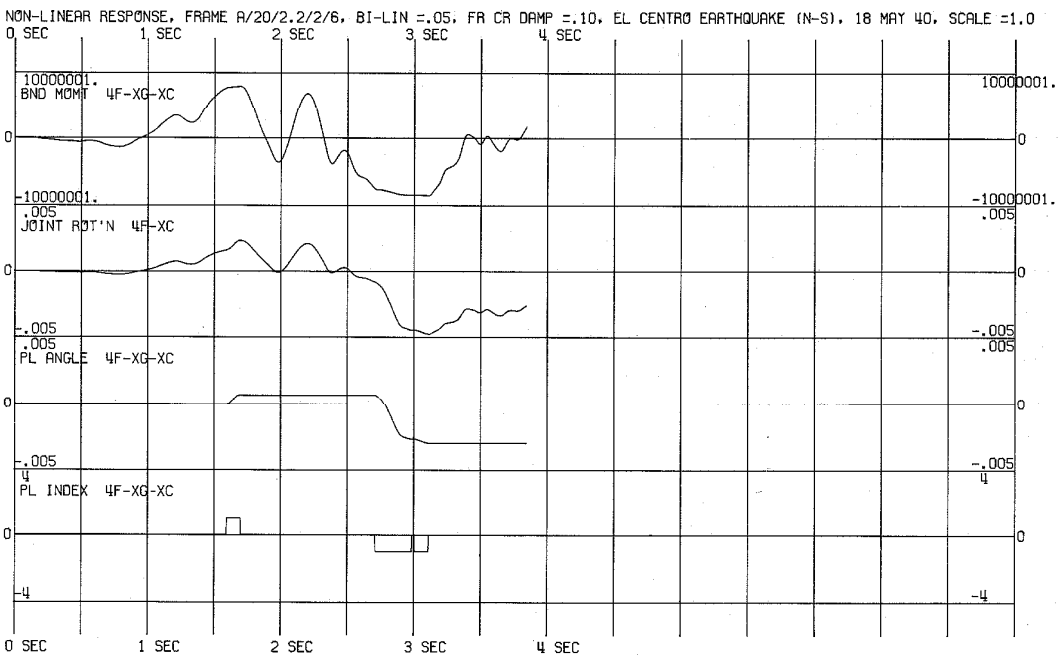


Fig. 6.5b Station (f)

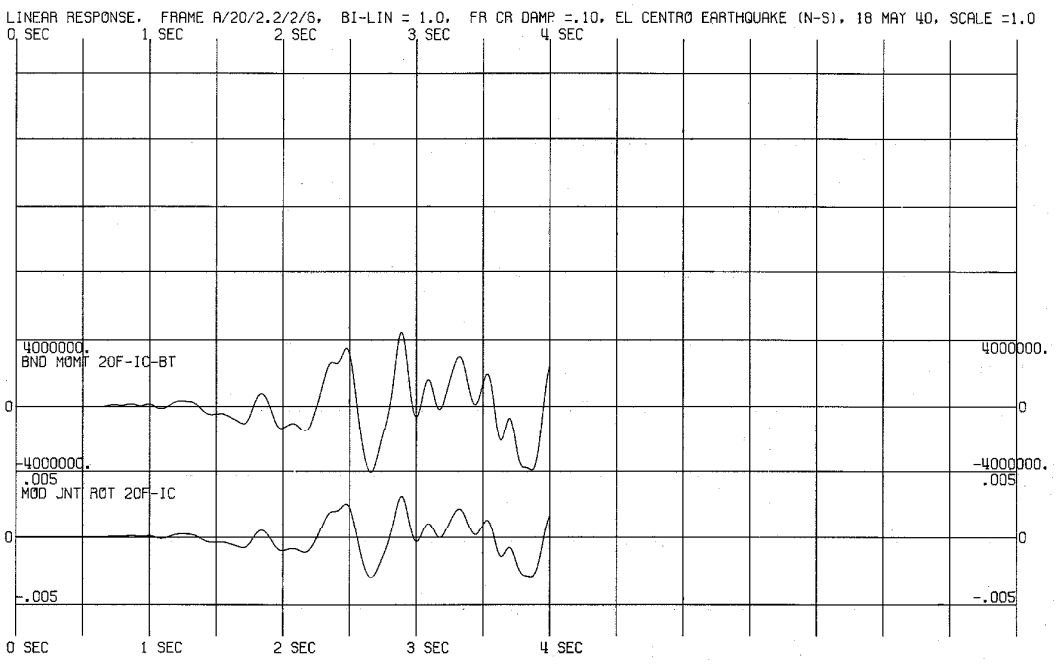


Fig. 6.6a

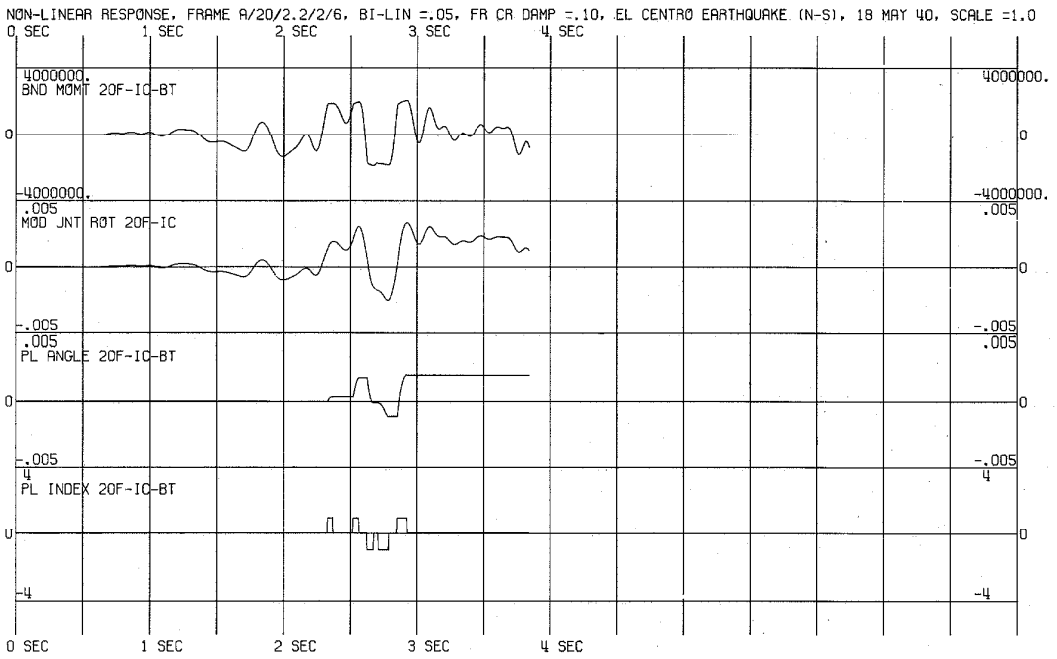


Fig. 6.6b Station (d)

TABLE 6-2

Floor	Sum of Shear* Forces Above and Below Floor (at Yield Level) lbs $\times 10^{-4}$	Weight* of Floor lbs $\times 10^{-4}$	Estimate of Maximum Absolute Value of Total Acceleration fraction g	Maximum Absolute Value of Total Acceleration from a Non-linear Study Using El Centro (N-S), 18 May 1940 fraction g
20	4.31**	8.88	0.49	0.54
	12.00	10.04	1.20	0.58
18	18.29	10.04	1.82	0.45
	24.04	10.04	2.39	0.81
16	29.99	11.20	2.67	0.65
	35.93	11.20	3.21	0.61
14	42.27	11.20	3.77	0.66
	48.03	11.58	4.14	0.45
12	52.36	11.58	4.52	0.53
	57.87	11.58	4.99	0.50
10	62.73	14.28	4.39	0.48
	67.44	14.28	4.72	0.43
8	71.06	14.28	4.97	0.39
	74.92	14.67	5.10	0.45
6	77.26	14.67	5.26	0.39
	82.39	14.67	5.61	0.45
4	85.15	15.83	5.37	0.50
	84.15	15.83	5.31	0.37
2	87.71	15.83	5.54	0.41
	102.92	16.21	6.34	0.37
Ground	-	-	-	0.32

*Calculations based on half of the A/20/2.2/2/6 frame (see Fig. 3.2).

**This one uses the yield moment in the 20th floor girder at the exterior column since it is lower than the one in the exterior column.

are changed from those in the linear case. From a review of the plots for this study, it appears that in the nonlinear case at 2.7 sec, shear force waves fortuitously meet at the 17^{th} floor in a manner sufficient to cause a high total acceleration.

To explore the way in which interfloor shear forces and accelerations may add up at various points in the structure, Table 6-2 has been compiled. The estimate of the maximum absolute value of the total acceleration of a floor is found by dividing the sum of the yield levels of the shear forces in the columns directly above and below that floor by the mass of the floor. These estimates of the maximum total acceleration that the structure could sustain are very high: from 0.49g at the roof to 6.3g near the base. This only means that with respect to total acceleration the columns near the base of the structure are much stronger than would be required for actual earthquake excitation. As shown in Table 6-2 for the El Centro (N-S) earthquake of 18 May 1940, the total acceleration of the 20^{th} floor was calculated to be 0.54g, and near the base it was calculated to be 0.37g. The 0.54g value is in excess of the estimated 0.49g because interfloor shear forces occurred between the 19^{th} and 20^{th} floors in excess of those sufficient to cause yielding in the columns.

The second point to be discussed concerns the larger high frequency components observed in the total acceleration for the nonlinear case in comparison with that for the linear case. This is likely to be a consequence of the sharp corner at the yield level in the bilinear hysteresis loop. A similar phenomenon was observed in the

previous investigation of the three-mass system with bilinear hysteretic springs shown in Fig. 4.1. In that case it was noted that higher frequency components corresponding to the higher natural modes appeared whenever a spring passed into the yield range. This type of response corresponds to that which would result from a step in the forcing function, i.e., the response would contain all of the natural frequencies of the system. The total acceleration of a real structure may or may not exhibit this high frequency characteristic depending upon the properties of the structure at the yield level. An estimate of the effect of such a sharp corner on the response of a tall structure might be obtained in a future study using curvilinear moment-rotation hysteresis loops such as shown in Fig. 2.14.

In order to observe the effect of yielding on other responses of the structure, Figs. 6.2a and 6.2b are again considered. Comparing the horizontal displacement of the 17^{th} floor for the linear case with that for the nonlinear case, one notes that there is little difference in the maximum displacements or in the time at which they occur. However, it is seen in Fig. A.1.3 of Appendix A that, as a result of yielding, a permanent displacement of about four inches is incurred in the displacement of the 17^{th} floor. Similarly, the maximum absolute displacement of the 4^{th} floor in the linear and the nonlinear cases are approximately equal and occur at about the same time. In the nonlinear case, a permanent displacement of about one inch is incurred.

The maximum absolute value of the overturning moment at the base, Fig. 6.2, is reduced about 30% by yielding. Since the

girders yield, the moments at the ends of the girders and the axial forces in the columns are limited. Consequently, the base overturning moment is also limited. Note that gravitational effects are not considered in the calculation of the base overturning moment of this model.

In the plots of the interfloor shear forces and interfloor displacements, Fig. 6.3, changes in the shape of the shear waves as a result of yielding can be seen. In particular, a shear wave pulse with a period of approximately 0.5 sec, about the period of the third mode, can be traced and changes in its shape observed. This shear force pulse appears between the ground and the 1^{st} floor from 1.6 sec to 2.1 sec in both the linear and nonlinear cases. In the linear case, this pulse appears approximately 0.6 sec later between the 18^{th} and 19^{th} floors. However, in the nonlinear case, the shear force pulse is observed between the 18^{th} and 19^{th} floors with different characteristics than those observed in the linear case. In comparison with the linear case, the amplitude is reduced 50%, the shape is different, and the peaks are delayed by approximately 0.1 sec. This delay is such that the peak in the shear force between the 18^{th}-19^{th} floors at 2.7 sec coincides with the 0.81g peak in the total acceleration plot in Fig. 6.26. The effect of yielding on the wave shape is even more noticeable after 2.8 sec in these time history plots.

Similar effects are observed for the bending moments at different stations in the structure as shown in Figs. 6.4, 6.5 and 6.6. Furthermore, it is seen that the time histories of the bending moments,

joint rotations, and plastic angles differ from one station to another in both the linear and in the nonlinear cases.

To summarize the effects of yielding on the response of the A/20/2.2/2/6 frame subjected to the strongest portion of the El Centro earthquake, it was observed that yielding limits the maximum absolute values of the shear forces to about 60% of the corresponding values in the linear case. However, yielding was not so effective in limiting displacements, interfloor displacements, and joint rotations. For these, the maximum absolute values were reduced to only about 80% of the linear values. Although the maximum absolute values of the total acceleration may be reduced by yielding, it was shown that for the structural properties used, this frame could withstand total horizontal accelerations in excess of 6g near the base and in excess of 1g near the top without the columns yielding (the girders do yield). It was also seen that during the time of severe yielding, the phase of the response was significantly altered.

In the next section, the response of this nonlinear structure to a pseudo-earthquake is presented. By a visual analysis of the time history plots of various responses, it is possible to obtain an understanding of the frequency components expected in each response.

6.3 Response of the Nonlinear A/20/2.2/2/6 Frame Subjected to Earthquake Excitation

Several response studies of the bilinear hysteretic A/20/2.2/2/6 frame subjected to different earthquakes are performed using the computer program developed in connection with this report.

In addition to the El Centro (N-S) earthquake of 18 May 1940, six of the pseudo-earthquakes generated from a random process by P. C. Jennings are used. Since the corresponding responses of the structure are similar in amplitude and frequency content for the pseudo-earthquakes used, the plots for one earthquake only are analyzed in this section. The plots for the other earthquakes are given in Appendix A.

The pseudo-earthquake accelerograms have spectral properties similar to those for the recorded strong motion accelerograms of real earthquakes. However, the pseudo-earthquake accelerograms are more nearly uniform in amplitude over time than are the real ones. In an attempt to obtain approximately the same amount of yielding in the structure for the pseudo-earthquakes as occurred in the structure for the El Centro earthquake, they were scaled so that the rms value of each accelerogram is approximately 1.2 times that of El Centro. The actual rms values used are given in Table 6-3. In all cases the bilinearity (the ratio of the second slope to the first in the hysteresis loop) was 0.05. The fraction of critical damping in the fundamental mode was 0.10 in all runs except one which used 0.05 fraction of critical damping in order to observe the effect of damping on the response. A summary of the combinations of parameters used in the studies is shown in Table 6-3.

The response plots for the nonlinear A/20/2.2/2/6 frame using pseudo-earthquake number 6 with 0.10 fraction of critical damping in the fundamental mode are presented in this section. In addition, the displacement responses (in inches) of seven one-degree of freedom

TABLE 6-3

Earthquake	Fraction Critical Damping in Frame	rms Scale Factor (with respect to El Centro)	Max. Abs. Acceleration of Earthquake (frac. grav.)	Spectrum Intensity $SI_{0.0}$	Max. Abs. Displacement (inches)	Max. Abs. Interfloor Displacement (inches)	Maximum Ductility Factors		
							All Girders	Interior Column	Exterior Column
El Centro (N-S) 18 May 1940	0.10	1.0	0.316	8.94	11.52	1.29	4.44	2.99	0.83
Pseudo-Earthquake No. 1	0.10	1.15	0.246	11.45	8.98	0.98	3.58	1.74	0.71
Pseudo-Earthquake No. 2	0.10	1.18	0.267	13.23	14.43	1.31	5.00	3.11	0.81
Pseudo-Earthquake No. 4	0.10	1.26	0.267	12.21	10.96	0.93	3.96	2.02	0.80
Pseudo-Earthquake No. 5	0.10	1.19	0.271	12.25	9.69	0.91	3.63	2.23	0.76
Pseudo-Earthquake No. 6	0.10	1.22	0.313	10.96	8.91	0.99	4.01	2.58	0.78
Pseudo-Earthquake No. 6	0.05	1.22	0.313	10.96	10.28	1.08	4.32	2.91	0.80
Pseudo-Earthquake No. 7	0.10	1.20	0.328	12.06	8.52	0.97	3.75	2.53	0.79

Notes:

These pseudo-earthquakes were chosen by flipping coins.

In all cases the bilinearity is 0.05.

Max. Abs. = Maximum Absolute.

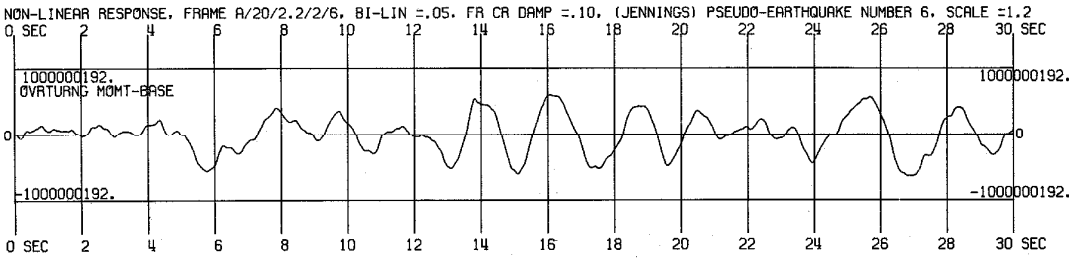

NON-LINEAR RESPONSE, FRAME A/20/2.2/2/6, BI-LIN =.05. FR CR DAMP =.10. (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6. SCALE =1.2
0 SEC 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 SEC
1000000192.
OVATURNG MOMT-BASE
0
-1000000192.
1000000192.
0
-1000000192.
0 SEC 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 SEC

Fig. 6.7

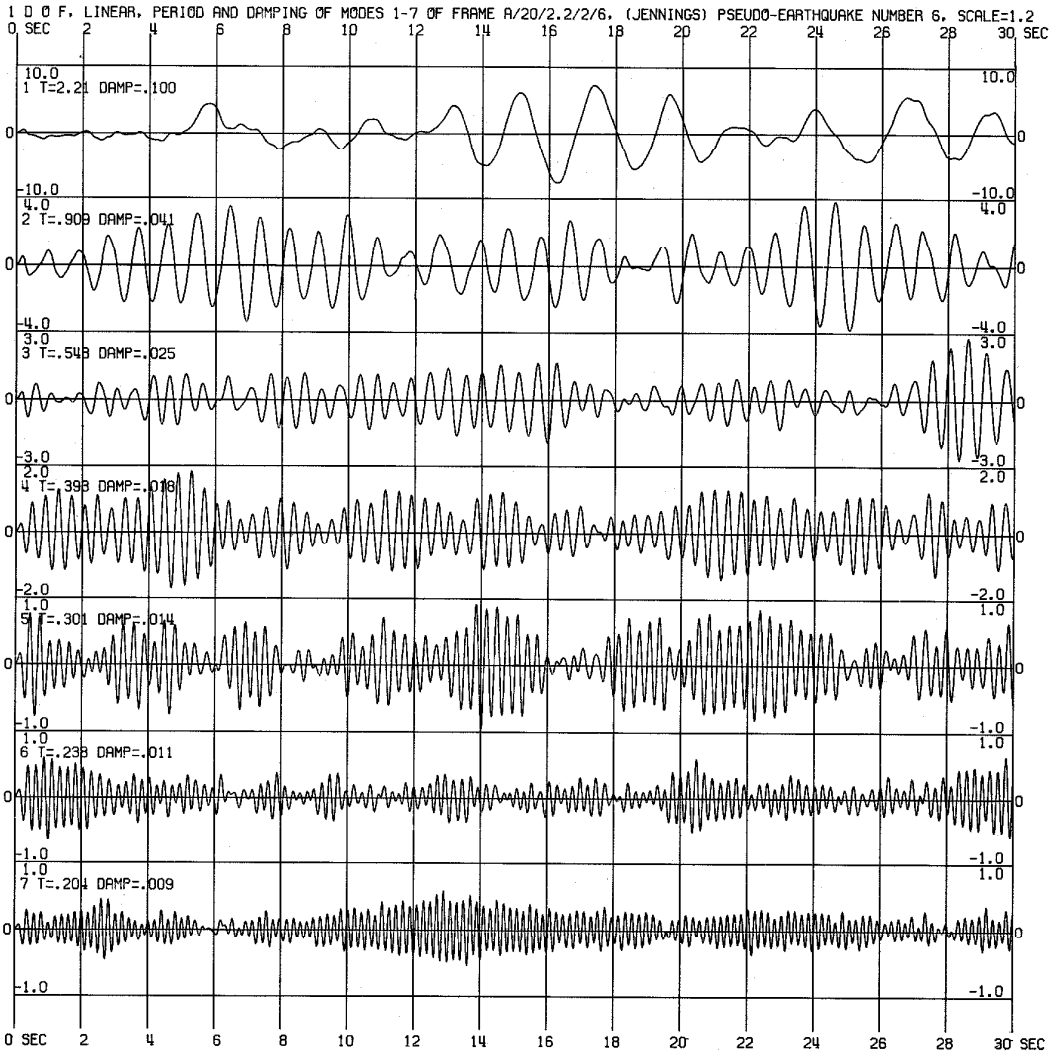

1 D O F, LINEAR, PERIOD AND DAMPING OF MODES 1-7 OF FRAME A/20/2.2/2/6, (JENNINGS) PSEUDO-EARTHQUAKE NUMBER 6, SCALE=1.2
0 SEC 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 SEC
10.0
1 T=2.21 DAMP=.100
-10.0
4.0
2 T=.909 DAMP=.041
-4.0
3.0
3 T=.543 DAMP=.025
-3.0
2.0
4 T=.393 DAMP=.018
-2.0
1.0
5 T=.301 DAMP=.014
-1.0
1.0
6 T=.238 DAMP=.011
-1.0
1.0
7 T=.204 DAMP=.009
-1.0
0 SEC 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 SEC

Fig. 6.8

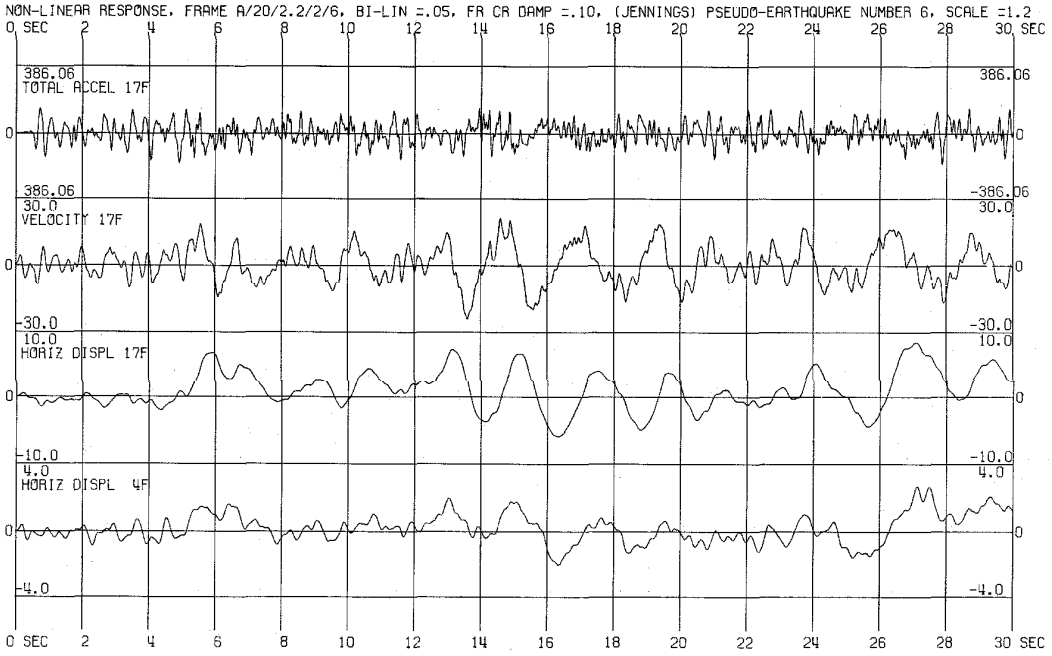


Fig. 6.9

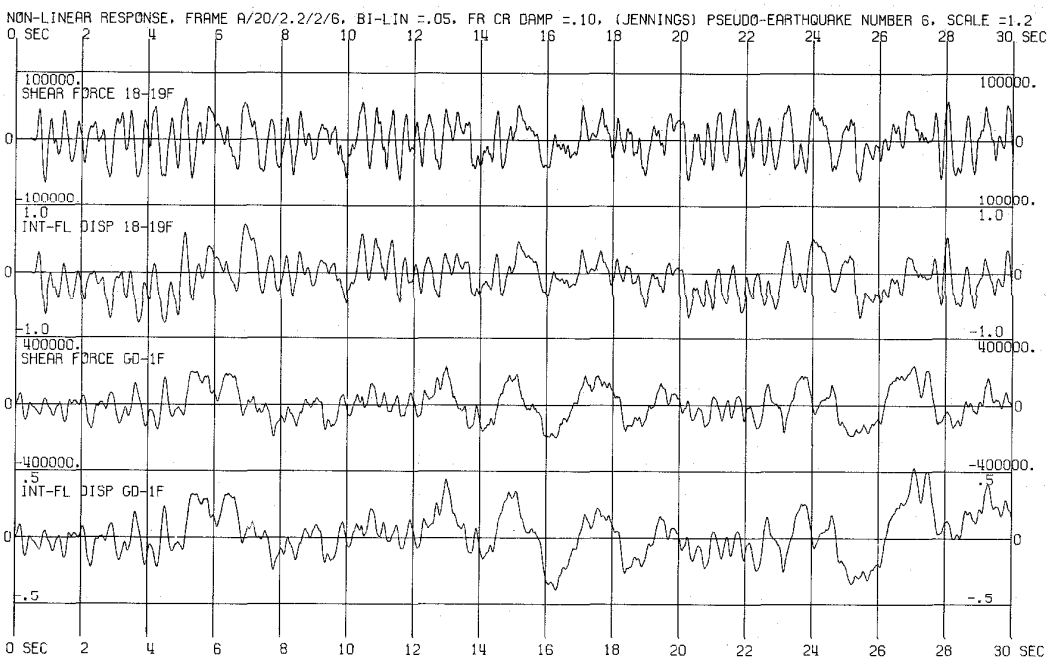


Fig. 6.10

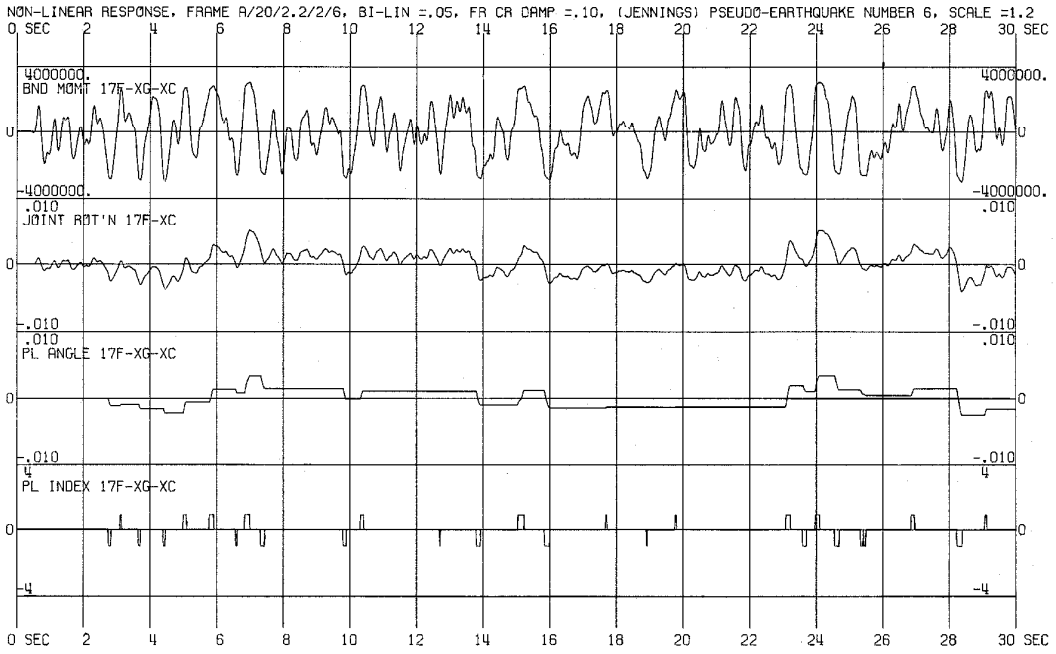


Fig. 6.11 Station (e)

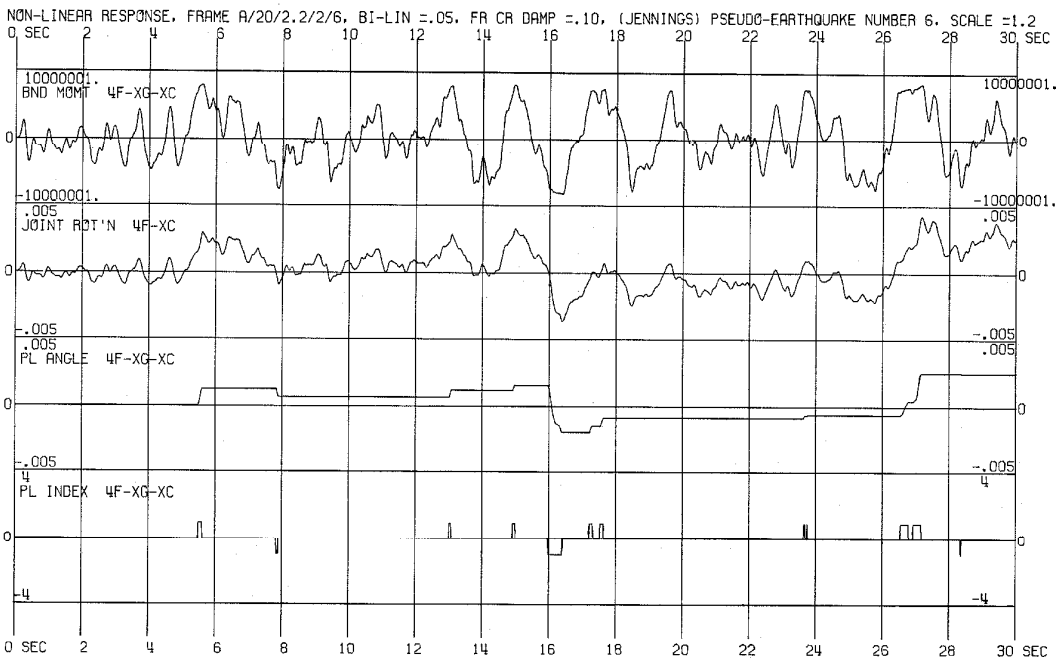


Fig. 6.12 Station (f)

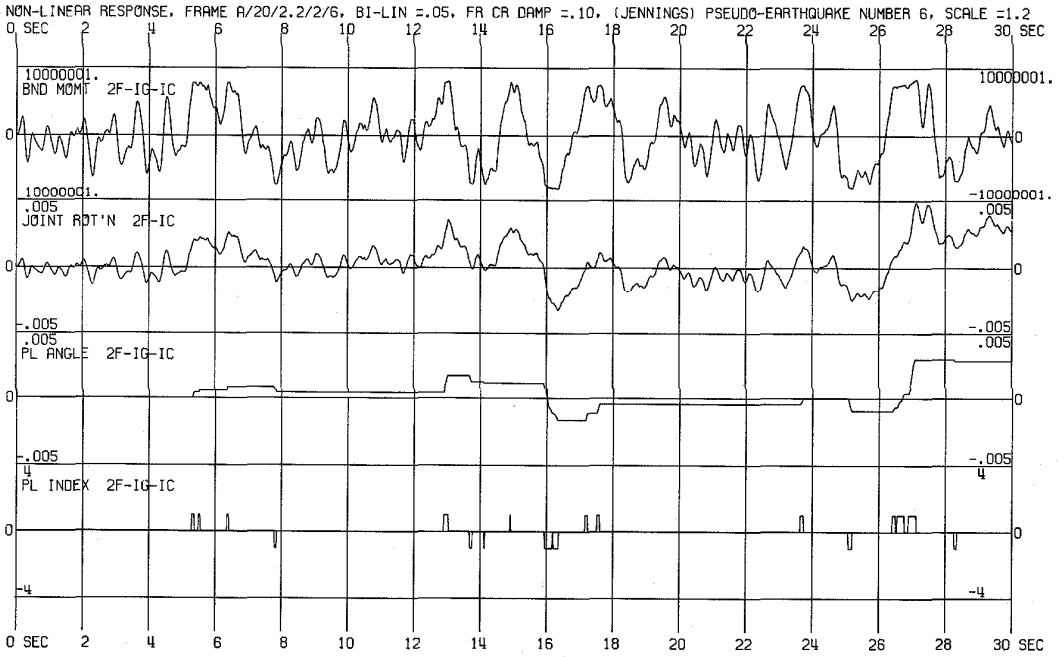


Fig. 6.13 Station (h)

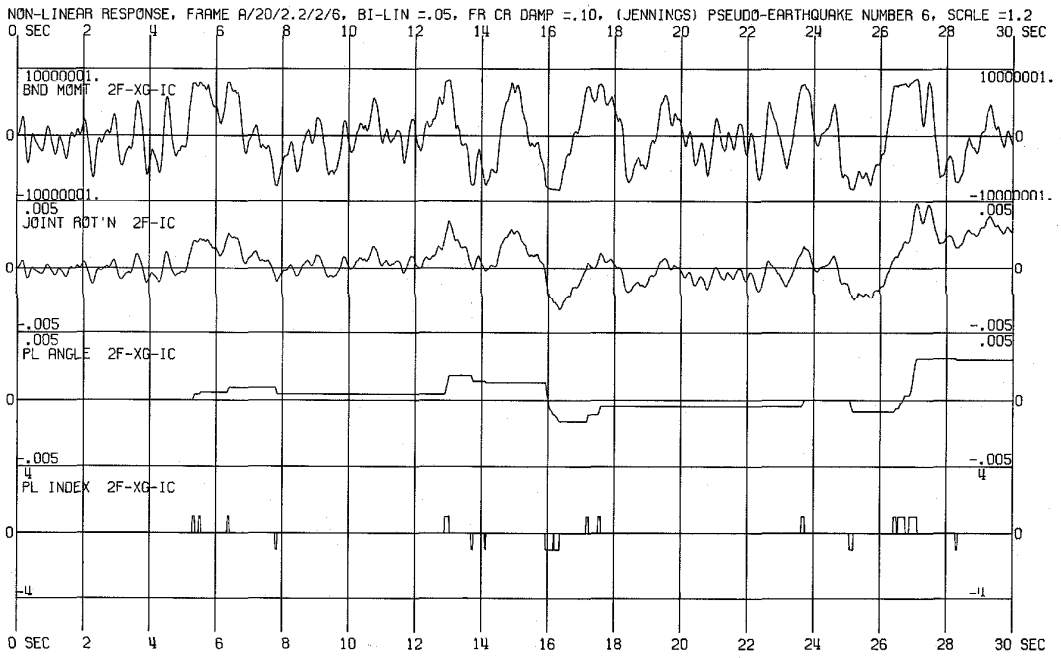


Fig. 6.14 Station (g)

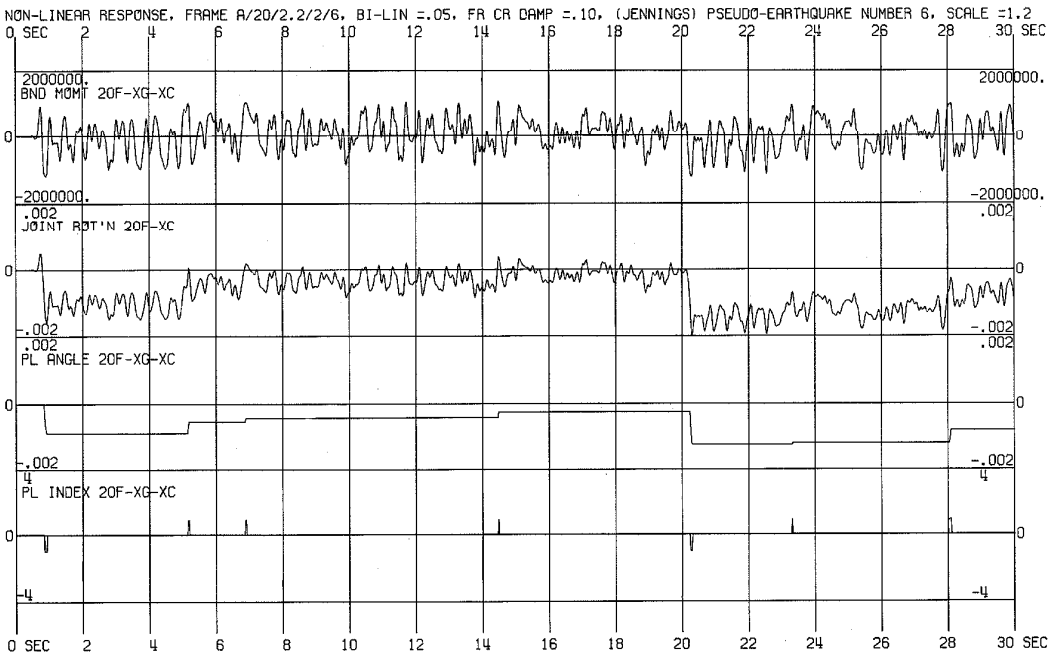


Fig. 6.15 Station (a)

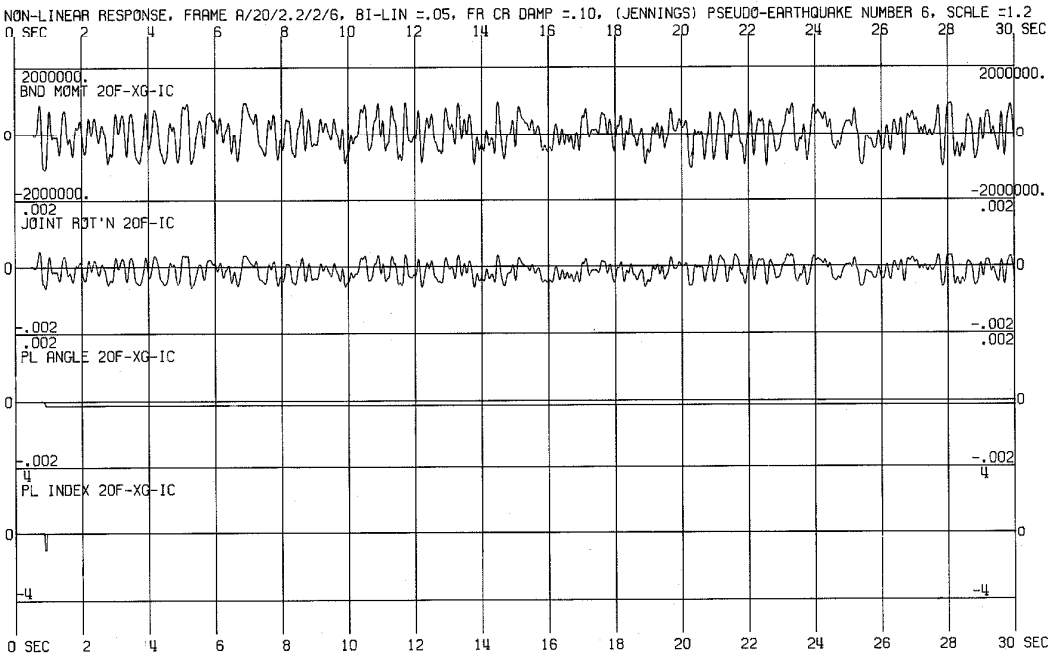


Fig. 6.16 Station (b)

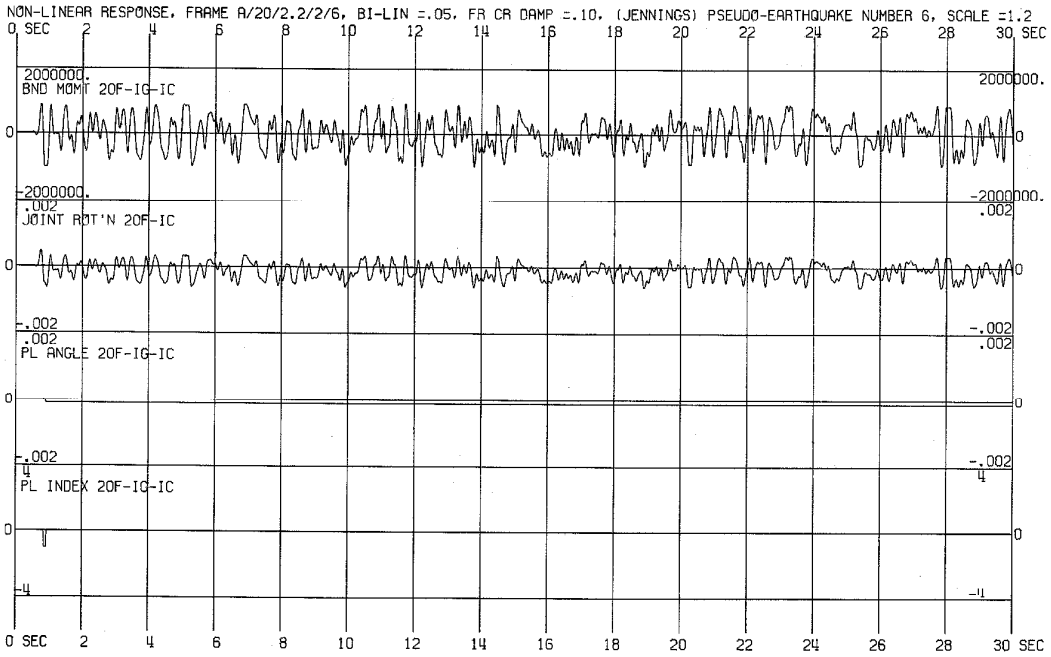


Fig. 6.17 Station (c)

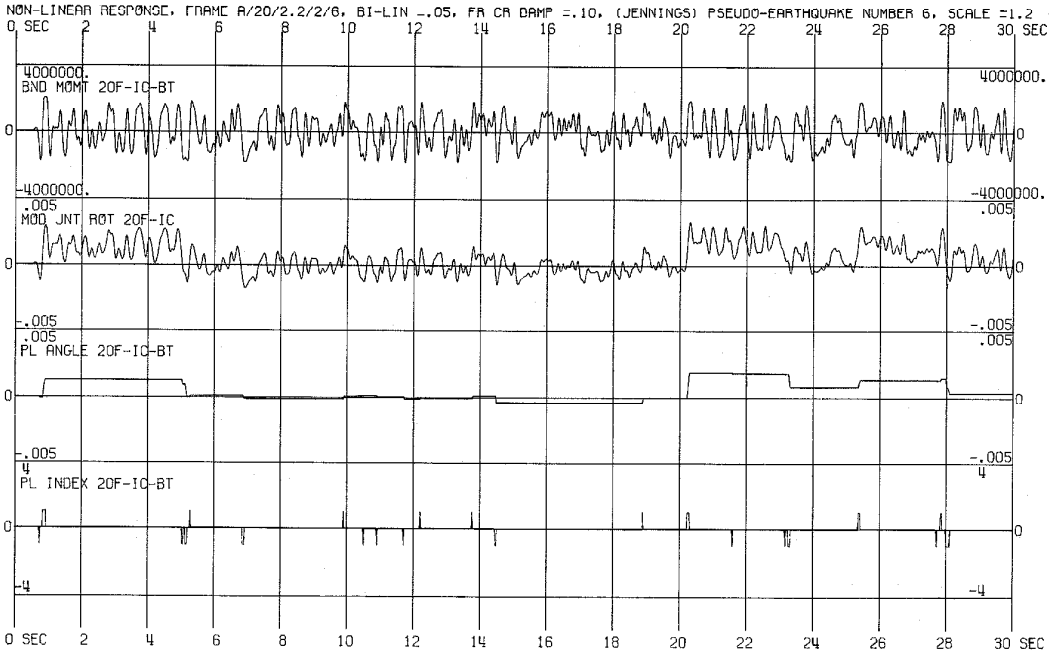


Fig. 6.18 Station (d)

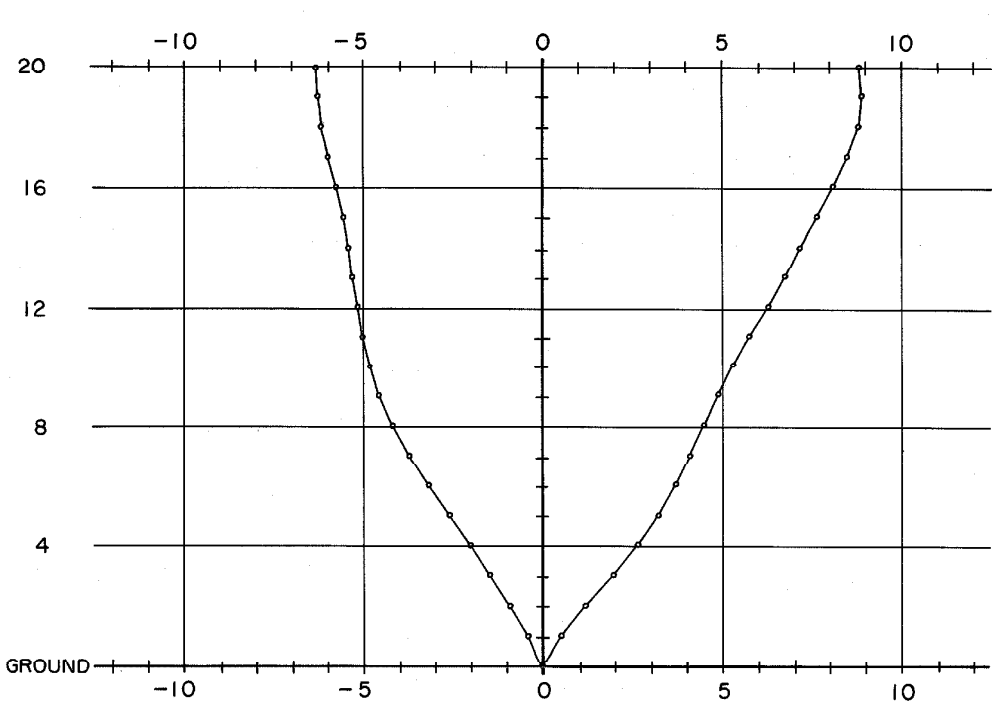


Fig. 6.19

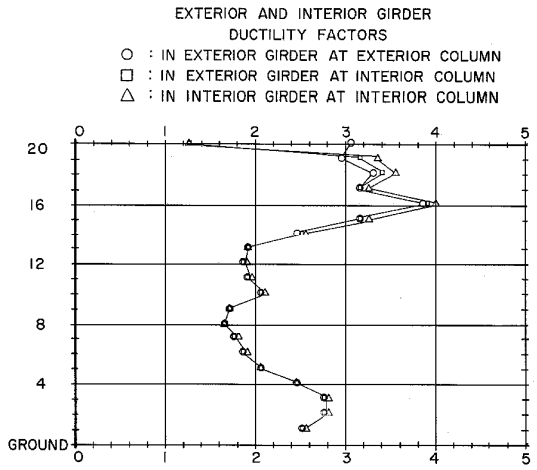


Fig. 6.20a

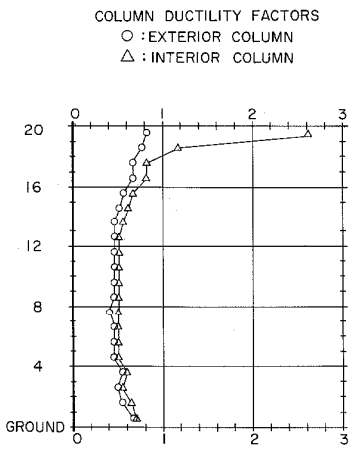


Fig. 6.20b

linear oscillators subjected to the same excitation are presented in Fig. 6.8. There is a one-to-one correspondence between these oscillators and the first seven modes of the linear A/20/2.2/2/6 frame with respect to period and damping. For example, the first oscillator has the period and the fraction of critical damping of mode 1. In Fig. 6.8 the response of this oscillator is labeled

$$1 \quad T = 2.21 \quad DAMP = .100 \quad .$$

The time history of the displacement of the 17^{th} floor is shown in Fig. 6.9. By comparison with the response of the linear oscillator corresponding to the first mode of the linear frame, henceforth called the first modal oscillator, it is seen that the two are closely correlated. This is in accordance with the analytical results for continuous cantilever beams in Chapter V which indicate that the horizontal displacements in the upper porition of the structure are expected to be dominated by the first few modes, primarily the fundamental.

By design, the distribution of yield bending moments in the A/20/2.2/2/6 frame is such that throughout most of the structure the girders yield at a much lower level than the neighboring columns. The purpose for such a design is "to maintain vertical load integrity of the structure."* For all of the earthquake excitations used, the interior columns of the nonlinear structure yield in only the upper 2 or 3 floors and the girders spend at most 10% of the time in yield. Consequently, the time histories of the gross responses of this particular structure,

*Clough and Benuska, FHA Study, p.1-7.

such as the displacements, should be highly correlated with those of the linear structure. If the yield moment distribution were changed so that more of the columns yielded, a lower correlation with the response of the linear system would probably occur.

The overturning moment at the base, Fig. 6.7, is observed to be closely correlated with the displacement at the 17^{th} floor and with the response of the first modal oscillator (with opposite sign). This correlation should exist for the following reason: When a floor is displaced, shear forces develop in the girders which appear as forces in the axial direction in the columns. Only when many of the floors are displaced in the same direction, such as in the fundamental mode, do the axial column forces add together to cause a significant resultant moment at the base.

A comparison of the 4^{th} floor displacement with the 17^{th} floor displacement, Fig. 6.9, indicates that higher frequency response (corresponding to the higher modes in a linear structure) of the structure influences the displacement (not interfloor displacement) of the 4^{th} floor more than it influences the displacement of the 17^{th} floor. This is in agreement with the studies of continuous beams.

In addition to a fundamental modal response component, higher frequency response is observed in the plot of the velocity (relative to ground) of the 17^{th} floor. This is expected since the velocity is a first derivative of the displacement and differentiations characteristically tend to amplify higher frequencies.

As seen in Fig. 6.9, the total acceleration, which is the sum

of the acceleration relative to ground and the acceleration of the ground, has many frequency components. Again, this is in agreement with the results of the continuous beam studies.

In Fig. 6.10, the time histories of the shear force and of the interfloor displacements between the 18^{th} and 19^{th} floors as well as between the ground and 1^{st} floors are shown. By comparison, it is seen that the time history of the shear force between the 18^{th} and 19^{th} floors and the time history of the corresponding interfloor displacement are very similar in appearance. The small differences between them occur because the shear force is a function of the joint rotations in addition to the interfloor displacement and because permanent displacement occurs as a result of yielding.

In the propagation of the first shear force pulse from the base to the 19^{th} floor in the linear A/20/2.2/2/6 frame a time lag of about 0.6 seconds is observed. This time lag is seen throughout most of the earthquake, but it is usually not so clear. For the shear-type continuous cantilever beams described in Chapter V, the time necessary for a shear force pulse to travel from the base to the top is:

<u>for the uniform beam</u>:

$$\frac{T_1}{4} = 0.55 \text{ sec} \quad \text{and}$$

<u>for the tapered (to a point) beam</u>:

$$\frac{T_1}{2.6} = 0.85 \text{ sec}$$

where T_1 is the period of the fundamental mode in each case. Since

this frame has mass and stiffness properties which are partially tapered (Fig. 3.2), the observed time lag of 0.6 sec is in good agreement with those predicted for the continuous beams. In these plots it is seen that the shear force and interfloor displacement between the ground and first floor have a strong dependence upon the fundamental mode. However, many modes contribute to the shear force and interfloor displacement near the top of the frame.

Time history plots of the bending moment, joint rotation, plastic angle, and plastic index are shown for several stations in the frame.* Each of the figures 6.11 through 6.18 corresponds to one of these stations. If the station is in a girder, the joint rotation is given; if the station is in a column, the modified joint rotation (Eq. 3.8) taking into account the interfloor displacement is plotted. During yielding the plastic index is (+1) or (-1) depending upon the direction in which the associated plastic angle moves. For example, at station (e) in the exterior girder in the 17^{th} floor, Fig. 6.11, the initial yielding occurs with negative bending moment; hence, the incurred plastic angle is negative and the plastic index is (-1). By making visual comparisons of the time history plot of the bending moment at station (e), Fig. 6.11, with the response of each of the linear modal oscillators, Fig. 6.8, correlations can be observed which indicate the importance of the modes to the response at that station. Between 3 sec and 10 sec a

*All of these time history plots were made on the California Computer Products "Model 763 Plotter" programmed with the IBM 7094-7040 Computer at the California Institute of Technology Computing Center. For the 30 seconds of the earthquake there were 6000 time increments. After every 4^{th} time increment, a point was plotted; hence, 50 points per earthquake second were plotted.

strong correlation with the 2nd modal response is observed; hence, it is concluded that the 2nd mode is responsible for most of the yielding during this time at this station. Between 13 sec and 19 sec, correlation with the first modal response occurs. Similarly, it is concluded that during this interval the first mode is responsible for most of the yielding at this station. At times combinations of modes are responsible for the yielding since the contribution of each mode by itself does not appear to be sufficient to cause yielding.

For an accurate determination of the relative importance of each mode at a particular instant, it is usually necessary to make a modal decomposition of the motion of the structure at that instant. Only at the times when one or two modes dominate a response does it appear possible to visually determine the importance of these modes. Such a modal analysis is made for this nonlinear structure in each time increment throughout the first four seconds of the El Centro earthquake according to a technique described in Section 5.3. The modal contributions to the largest displacements and interfloor displacements that occurred in that analysis are given in Table 5-10 and Table 5-11. The results presented in those tables indicate that often more modes than can be observed by the eye are important for the interfloor displacements. However, because extensive computer time is used in making modal decompositions, an analysis of this type was not performed for the present study. Consequently, in this chapter the discussion of modal contributions (or frequency components) is based upon visual analysis.

At station (f) in the 4^{th} floor, the fundamental mode appears to be the largest contributor to the bending moment and, hence, is the most important for yielding at this station. At times, however, the second and higher modes are observed to add to the fundamental mode, thereby contributing to the yielding.

As the frame is deformed, the ends of the girders within the same floor may or may not yield simultaneously depending upon the distribution of strength properties, i.e., yield moments, in that portion of the frame. In the lower portion, the yield moments in the interior and exterior girders within the same floor are approximately equal and the yield moments in the girders are lower than those in the neighboring columns. As a result, the columns remain linear and the girders yield. In the time history plots, Fig. 6.13 and Fig. 6.14, for stations (h) and (g), it is observed that the girders in the second floor yield almost simultaneously.

Near the top of the frame the yield moments within the same floor differ by as much as 10%. In the top two floors the yield moments in the girders approximate the yield moments in the neighboring columns. Consequently, in this portion of the frame, the girders within the same floor usually do not yield at the same time or by the same amount. This can be seen by comparing the time history plots, Fig. 6.15 and Fig. 6.16, of the plastic angles incurred at the opposite ends of the exterior girder in the 20^{th} floor. (Note that different scale factors are used in the various plots for the 20^{th} floor.)

For stations (h) and (g) in the second floor, there are some

TABLE 6-4

Times at which the most positive and most negative displacements and interfloor displacements occur for each floor of the A/20/2.2/2/6 frame with 0.10 fraction of critical damping (mass proportional) in the fundamental mode. Pseudo-earthquake number 1 is used. See Fig. A.2.3 in Appendix A for plots of corresponding displacements at the 4th and 17th floors and Fig. A.2.4 for the interfloor displacements between ground and the 1st floor and between the 18th and 19th floors.

Time (sec) of Occurrence of the Most Negative:			Time (sec) of Occurrence of the Most Positive:	
Displacement	Interfloor Displacement	Floor	Displacement	Interfloor Displacement
14.6 sec		20	18.0 sec	
	29.5 sec			24.5 sec
14.6		19	18.0	
	10.1			24.5
14.6		18	18.0	
	29.6			24.5
4.0		17	18.0	
	29.0			3.2
4.0		16	25.3	
	4.1			3.2
3.9		15	25.3	
	4.1			18.0
3.8		14	25.3	
	12.0			18.0
3.8		13	25.3	
	12.0			25.3
3.8		12	15.6	
	12.0			25.4
3.8		11	13.1	
	16.5			25.4
3.8		10	13.1	
	16.5			25.4
3.8		9	13.1	
	3.8			15.3
21.5		8	13.1	
	3.8			15.6
21.5		7	13.1	
	9.7			13.2
21.5		6	13.1	
	9.7			13.1
21.5		5	13.1	
	16.3			13.1
21.5		4	13.0	
	21.5			13.1
21.4		3	13.0	
	21.5			13.0
21.4		2	13.0	
	21.4			13.0
21.4		1	13.0	
	21.4			13.0
		Ground		

high frequency components in the bending moments, but it appears that most of the yielding occurs in connection with the response of the fundamental mode. On the other hand, in the time history plots of the bending moments in the 20^{th} floor, many frequencies are observed. It is not obvious that any one frequency dominates the bending moments or the yielding in the 20^{th} floor. Furthermore, the times at which yielding occurs in the 20^{th} floor are different from those at which yielding occurs in either the 2^{nd}, 4^{th}, or the 17^{th} floor.

In Fig. 6.19a the displacement envelope is given. Although the points are connected together, this does not necessarily imply that either all of the most positive displacements or all of the most negative displacements occur at the same time. Typically three or four times are represented for either side of the envelope as shown in Table 6-4 for the study using pseudo-earthquake number 1. However, in the study being analyzed in this section, which uses pseudo-earthquake number 6, all floors do reach their most positive displacements at the same time and the most negative displacements at the same time.

The 20^{th} floor (roof) usually, but not always, is the floor which incurs the maximum absolute displacement of any floor in the frame during an earthquake. As indicated in Table 6-3, for the various studies using 0.10 fraction of critical damping in the frame, the maximum absolute displacements range between 8.5 and 14.4 inches. The ductility factors for the individual girders and columns are shown in

Fig. 6.20a and Fig. 6.20b, respectively. The ductility factors greater than 1.0 indicate that yielding occurred and are calculated according to Eq. 2.44.1. However, when yielding does not occur, the ductility factors are less than 1.0 and are given by the maximum absolute values of the ratios of the incurred bending moments to the corresponding yield bending moments. For the girders, the ductility factor for each end is shown, but for the columns, only the maximum value that occurs at either end is plotted. The maximum values of the ductility factors for each study are presented in Table 6-3. For the pseudo-earthquakes, with 0.10 fraction of critical damping in the frame, the maximum values of the ductility factors for the girders range from 3.6 to 5.0. For the exterior columns, which do not yield, the maximum values range from 0.71 to 0.81. For the interior columns, which yield in the upper two or three floors only, the maximum values are between 1.74 and 3.11.

This range in the maximum values of the structural responses appears to result from statistical variation. Such a range might be expected because the exact time histories of the earthquake accelerograms differ and because there is some variation in the statistics of the properties of the accelerograms.

In order to observe the effect of viscous damping on the response of this frame, a computer run using pseudo-earthquake number 6 with 0.05 fraction of critical damping was made. The plots for this run are shown in Fig. A.6.1 through Fig. A.6.8 in Appendix A. The effect of viscous damping is seen by comparing the various responses in those plots with the corresponding ones

presented in this section. The summary of these response results tabulated in Table 6-3 indicates that by decreasing the fraction of critical damping from 0.10 to 0.05, the maximum absolute values of the various responses increase: the displacement by 15%; the interfloor displacement by 9%; the ductility factors in the girders by 8%, in the interior columns by 12%, and in the exterior columns (moment ratio) by 3%.

6.4 Relationships Between Structural Responses and the Strength of an Earthquake

Three common measurements of the strength of an earthquake are the following:

1. The maximum absolute value of the ground acceleration during the earthquake;
2. The rms value of the acceleration of the earthquake:

$$\left[\frac{1}{30}\int_0^{30} [\ddot{y}(t)]^2\, dt\right]^{\frac{1}{2}} ;$$

3. The spectrum intensity of the earthquake: The spectrum intensity SI_ξ is defined as the area under the velocity spectrum curve* from period $T = 0.1$ to $T = 2.5$ for a

*If $\dot{z}$ is the relative velocity of a linear, base excited oscillator having period T and fraction of critical damping ξ, then the velocity spectrum value for this system is

$$S_{vel}(\xi, \frac{2\pi}{T}, t') = |\dot{z}|_{max}$$

where t' is the length of the earthquake excitation. Other points of the same velocity spectra curve are obtained by using different natural periods of the system. The average spectra curves in Fig. 5.3 are based upon the "pseudo-velocity" spectrum values

$$S_{ps\text{-}vel}(\xi, \omega, t') = \omega |z|_{max} .$$

given damping factor, ξ, and earthquake excitation[9,14]:

$$SI_{\xi} = \int_{T=0.1}^{T=2.5} S_{vel}(\xi, \frac{2\pi}{T}, 30)\, dT$$

The numerical values of these three strength measurements of the earthquakes used to excite the nonlinear structure are listed in Table 6-3. Note that the spectrum intensities are given for $\xi = 0.0$.

In an attempt to find a means for predicting the magnitude of structural response, it is of interest to compare the earthquake strengths as defined above with the magnitudes of the corresponding structural responses. Unfortunately, it is seen that none of these three measurements correctly predicts the trend of the maximum absolute values of either the displacements or ductility factors of the structure.

The first two measurements, maximum absolute value and rms value of an earthquake, do not take any properties of the structure into consideration and, therefore, they should not be expected to accurately estimate the maximum absolute value of the various responses of a tall structure. However, if the same earthquake is used with two different amplitude scale factors, the response corresponding to the larger scale factor can be expected to be larger.

The period range chosen for the spectrum intensity includes many of the natural periods of a typical tall structure. However, because the modes of a particular structure are not specifically considered, the spectrum intensity should not be expected to accurately predict the amplitude of the response of that structure. Nevertheless,

the spectrum intensity may be indicative of the amplitude of the average response of numerous structures taken together.

Another method for estimating the maximum values of the horizontal displacement, $y_{j_{max}}$, in the structure is[9]

$$y_{j_{max}} = \left\{ \sum_{i=1}^{20} \left[\frac{\sum_{k=1}^{20} \phi_{ki} M_k}{\sum_{k=1}^{20} \phi_{ki}^2 M_k} \frac{\phi_i}{\omega_i} S_{ps\text{-}vel_i} \right]^2 \right\}^{\frac{1}{2}}$$

where

j : floor number,

i : mode number,

$S_{ps\text{-}vel_i}$: the pseudo-velocity spectrum value at T_i,

ϕ_{ki} : the k^{th} component of the i^{th} mode shape, and

ω_i : the i^{th} modal frequency.

Since this method takes the modes of the structure into account, it might be more successful in predicting the trends of the maximum absolute displacements of the structure as a function of the earthquakes. It is noted that this method does not consider the time dependence of the modes. From the results of Section 6.3, it appears necessary to take the time dependence into consideration in order to accurately predict the maximum values of the various responses. One such method of approximating the magnitude of a given response in a structure might use the maximum absolute value of a time dependent sum of the linear modal responses, such as shown in Fig. 6.8, with

each mode weighted according to its expected contribution to the response of interest. The abilities of these methods to predict the trends could be evaluated in a future study by applying them to the structure and earthquakes discussed in this chapter.

CHAPTER VII

CONCLUSIONS

The following is a summary of the conclusions reached for the investigation described in this report:

1) Since the one-component nonlinear beam model can have a different curvilinear or bilinear hysteresis loop at each end, it is more versatile than the two-component model which is restricted to bilinear hysteresis loops at the ends.

2) Of the three definitions of ductility factor presented, only one is adaptable to both bilinear and curvilinear hysteresis loops. In the computer studies, this one was used with the one-component model having bilinear hysteretic behavior.

3) As an overall check on the computer program for analyzing the response of a nonlinear multi-story structure subjected to earthquake excitation, several tests covering various aspects of the computations were made. The results of each test indicate that the program operates properly and provides the desired accuracy.

4) For a nonlinear structure, it is possible to find "equivalent" modal participation factors which describe the contributions of the various modes to the structural response. These can be obtained by expanding the displacement vector for the nonlinear structure in terms of the eigenvectors of the linear system.

Such an analysis indicates that more modes may contribute to the interfloor displacements than can be visually observed in the time history response plots, particularly in the upper portion of the structure.

5) The effects of yielding on the response of the A/20/2.2/2/6 frame subjected to the first four seconds of the El Centro (N-S) earthquake of 18 May 1940 are, generally,

   (a) to reduce the forces and moments to approximately 60% of the values for the linear system, and

   (b) to reduce the displacements, interfloor displacements, and total accelerations to about 80% of the corresponding values for the linear system.

   During severe yielding, the shapes of the response time history plots are significantly changed from the shapes of the corresponding responses for the linear structure.

6) In different computer runs, the nonlinear A/20/2.2/2/6 frame with 0.10 fraction of critical damping in the fundamental mode was subjected to the entire El Centro (N-S) earthquake of 18 May 1940 and to six pseudo-earthquakes. The time history plots for a given response at a given position in the structure were compared for the earthquakes used. Characteristic patterns of response behavior were noted which were in qualitative agreement with similar responses of linear elastic, shear-type, continuous cantilever beams. A brief summary of

these characteristic response patterns follows:

(a) For the overturning moment at the base, the fundamental mode dominates.

(b) For the horizontal displacements in the upper portion, the fundamental mode dominates; in the lower portion, the fundamental mode is important, but the 2^{nd} and 3^{rd} modes are also significant.

(c) For the interfloor displacements in the upper portion, as many as eight modes may be important. The fundamental mode decreases in importance approaching the roof. In the lower portion, the fundamental mode increases in importance approaching the base, but the 2^{nd}, 3^{rd} and 4^{th} modes also contribute significantly.

(d) For the total accelerations, frequencies of almost all modes are observed. This is partly a consequence of the relatively low damping in the higher modes.

(e) Results of the studies of the continuous systems indicate that, with increasing taper of the structure, the higher modes become more important for all of the responses throughout the structure.

The distribution of ductility factors in the girders are similar for the different earthquakes used. Column ductility factor distributions were also similar for the different earthquakes.

Because these results are consistent for all of the earthquakes used, it is believed that the characteristic patterns of

modal contributions to the responses and the shapes of the ductility factor distributions are a function more of the structure than of the earthquake.

The maximum absolute value of a particular structural response differs between earthquakes because this value is highly dependent upon:

(a) the extent to which individual modes are excited at a particular time, and

(b) the phase relationships between the contributing modes at that time.

These modal responses differ from one earthquake to another because the exact time histories of the earthquakes are different.

7) Comparing maximum responses of the yielding structure for the series of pseudo-earthquakes having similar statistics, the following percentage deviations were found:

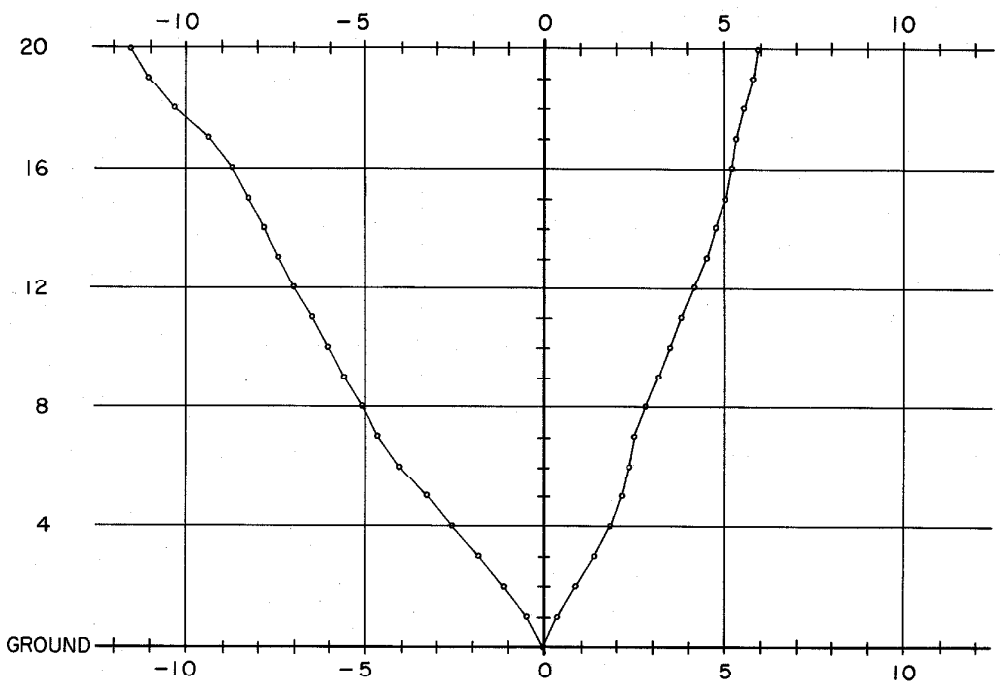
(a)	maximum absolute displacements	50%
(b)	maximum absolute interfloor displacements	40%
(c)	girder ductility factors	35%
(d)	interior column ductility factors	55%
(e)	exterior column ductility factors	15%

8) The effect of decreasing the fraction of critical damping from 0.10 to 0.05 in the fundamental mode of this particular structure excited by pseudo-earthquake number 6 is to increase the maximum values of the displacement response by 15% and of the ductility factor by 8%.

APPENDIX A

RESPONSE PLOTS FOR A NONLINEAR MULTI-STORY STRUCTURE SUBJECTED TO SEVERAL EARTHQUAKE EXCITATIONS

In this appendix response plots for the A/20/2.2/2/6 structure subjected to several earthquakes are presented. Sets of response plots consisting of the displacement envelope, distributions of ductility factors in the girders and in the columns, and time history response plots for various stations in the structure are given for seven structure-earthquake combinations. The positions in the structure of the stations indicated in the plots are shown in Fig. 6.1. The structural and earthquake parameters used are listed in the heading at the top of each plot. They are also tabulated in Table 6-3.



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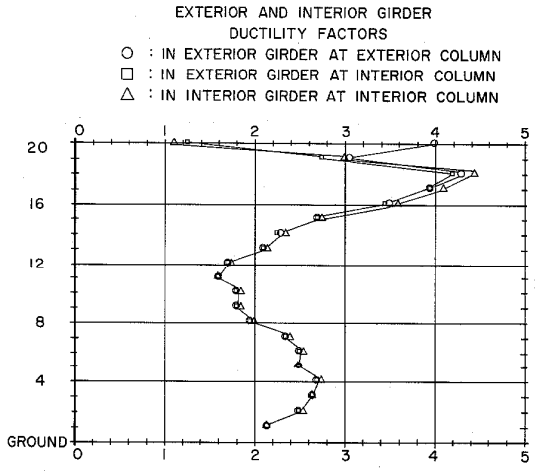


Fig. A.1.2.a

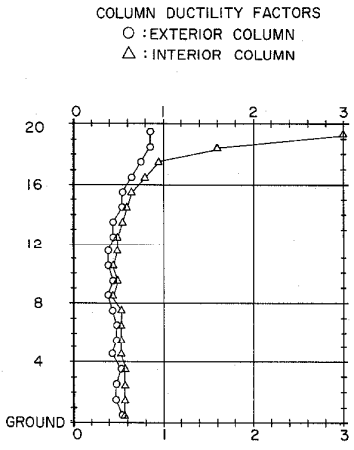
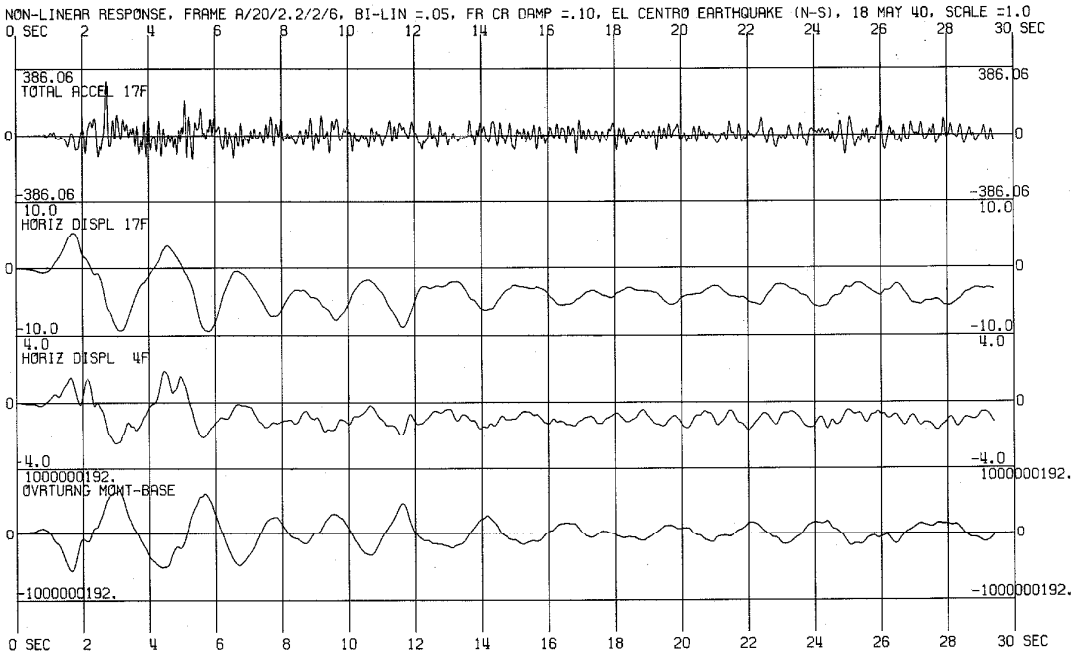


Fig. A.1.2.b



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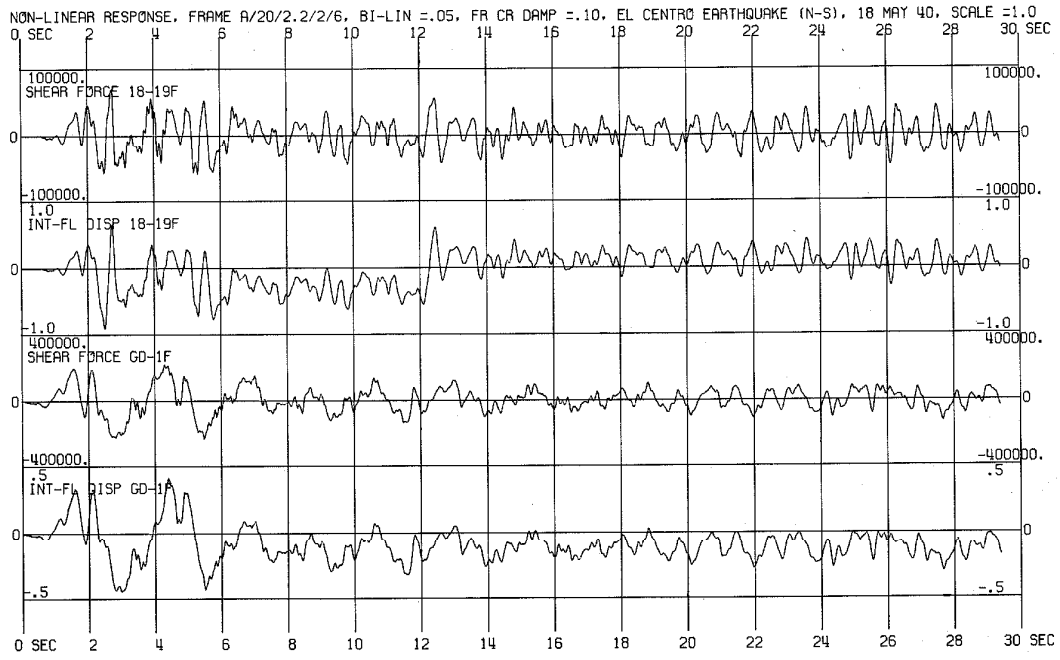


Fig. A.1.4

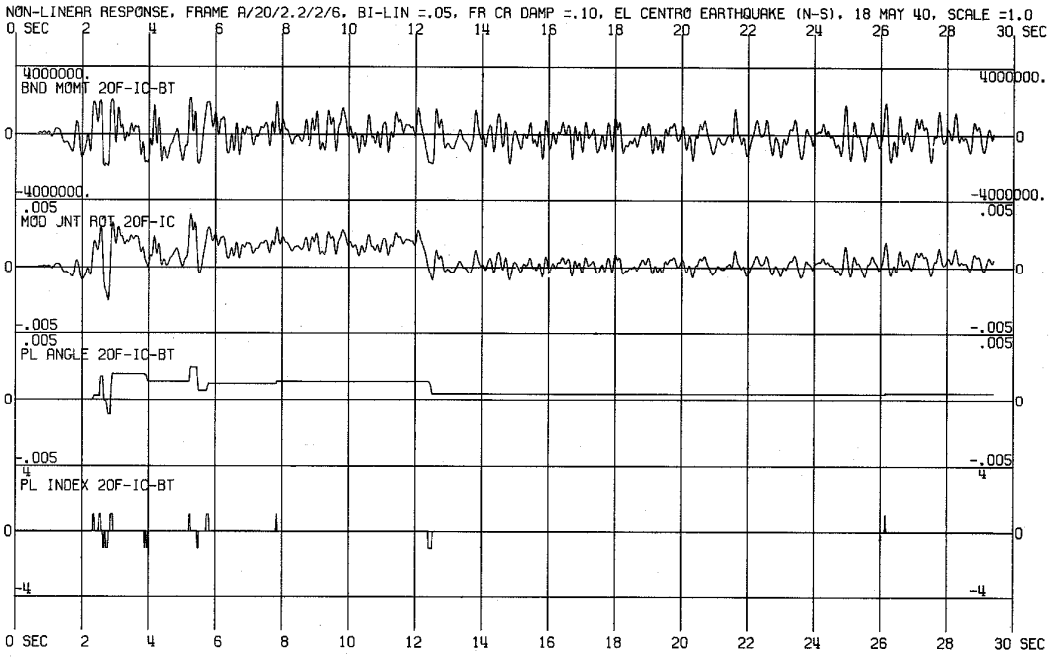
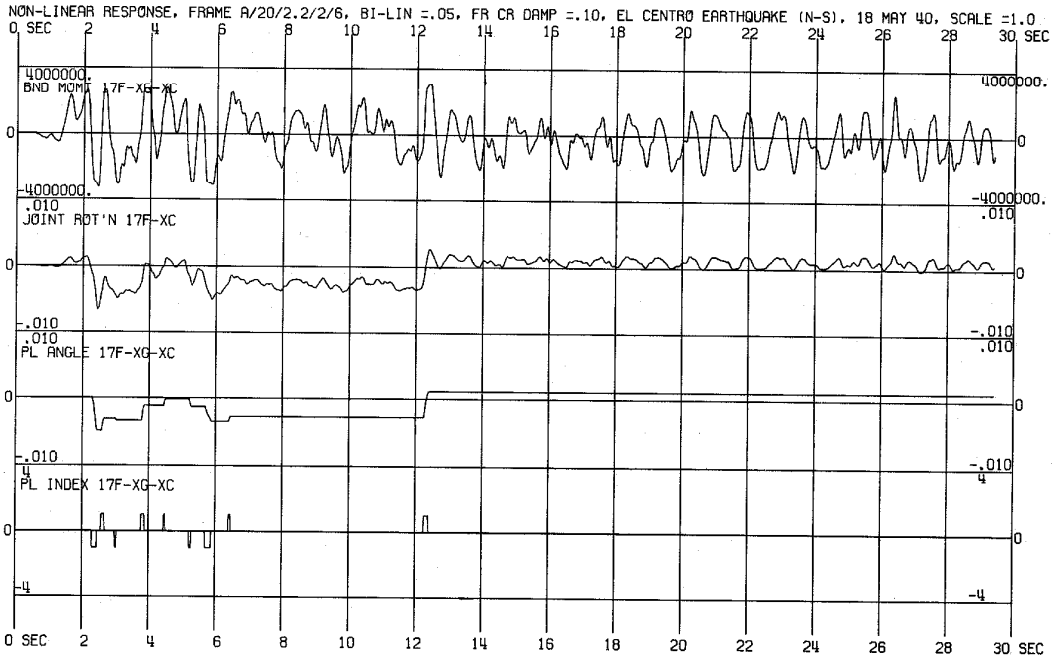


Fig. A.1.5 Station (d)



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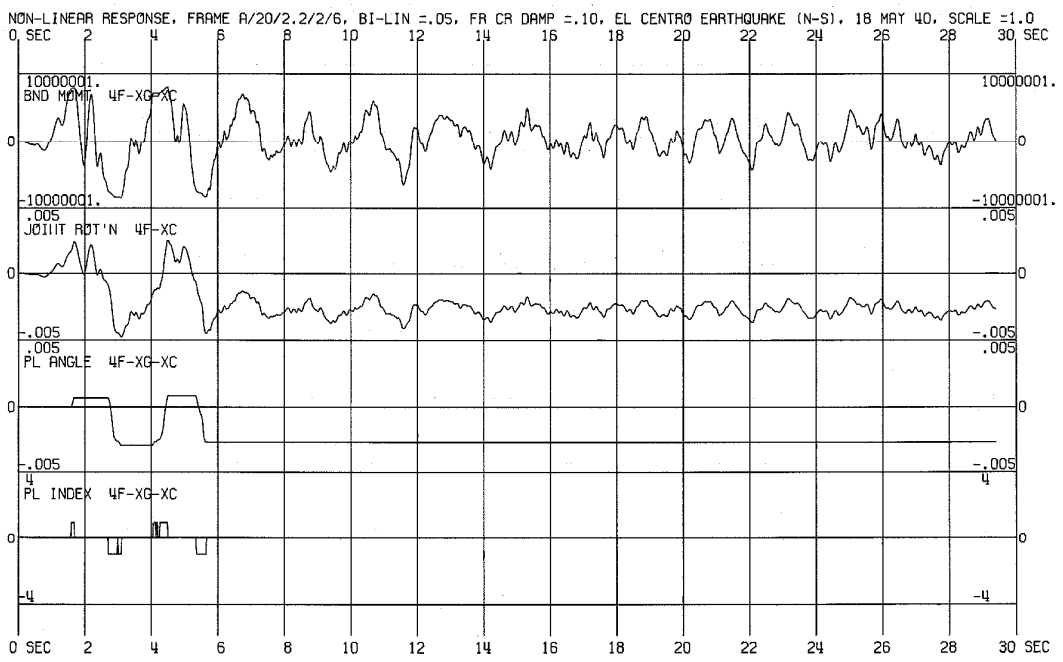

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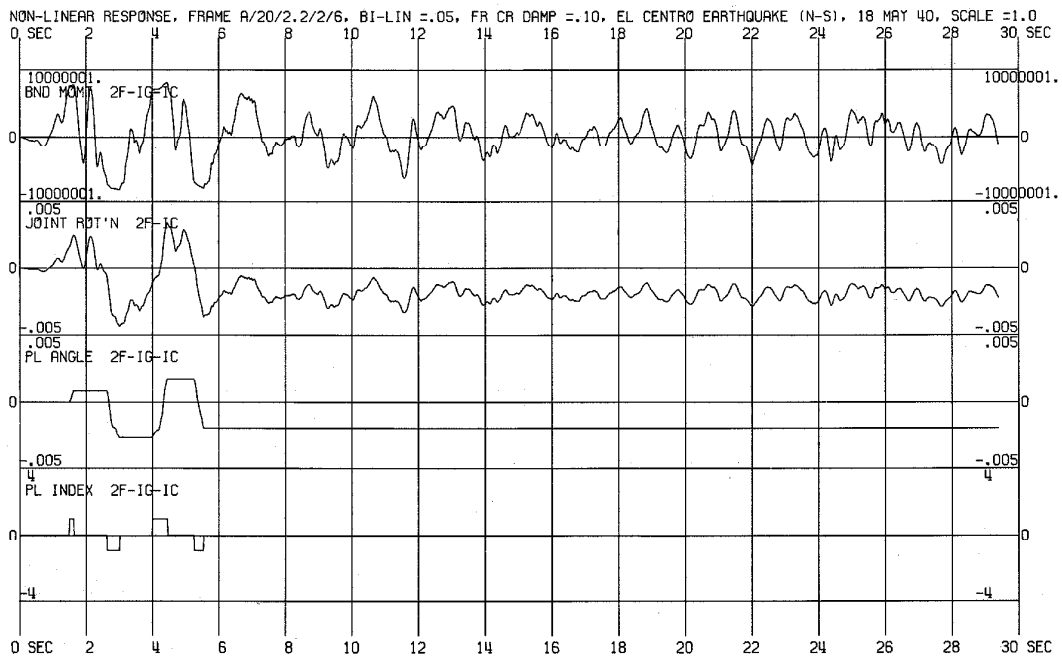

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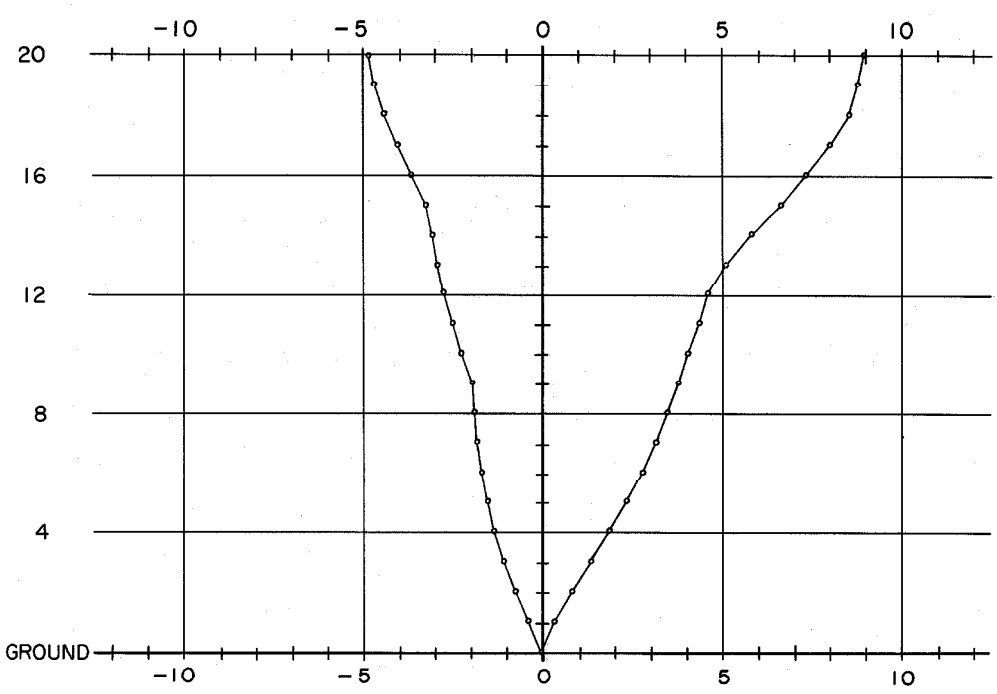

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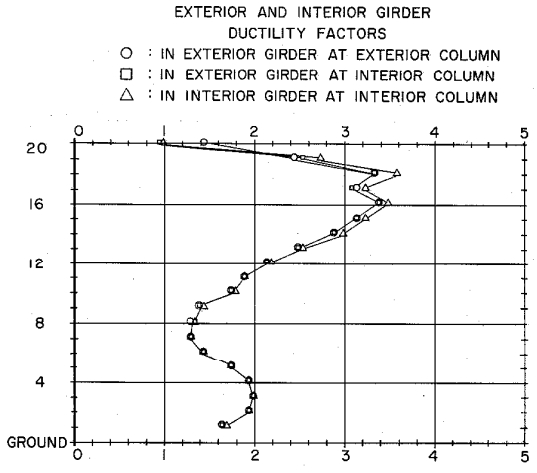

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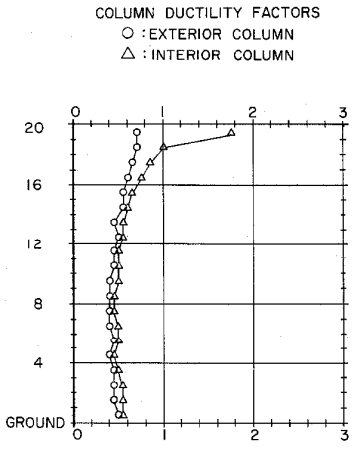

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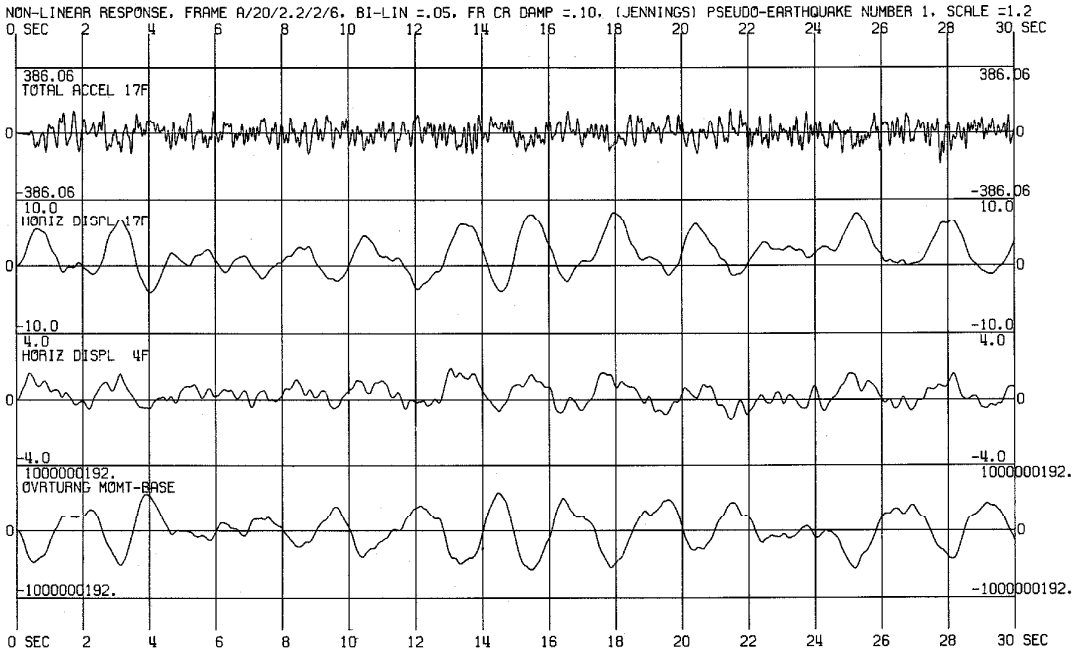

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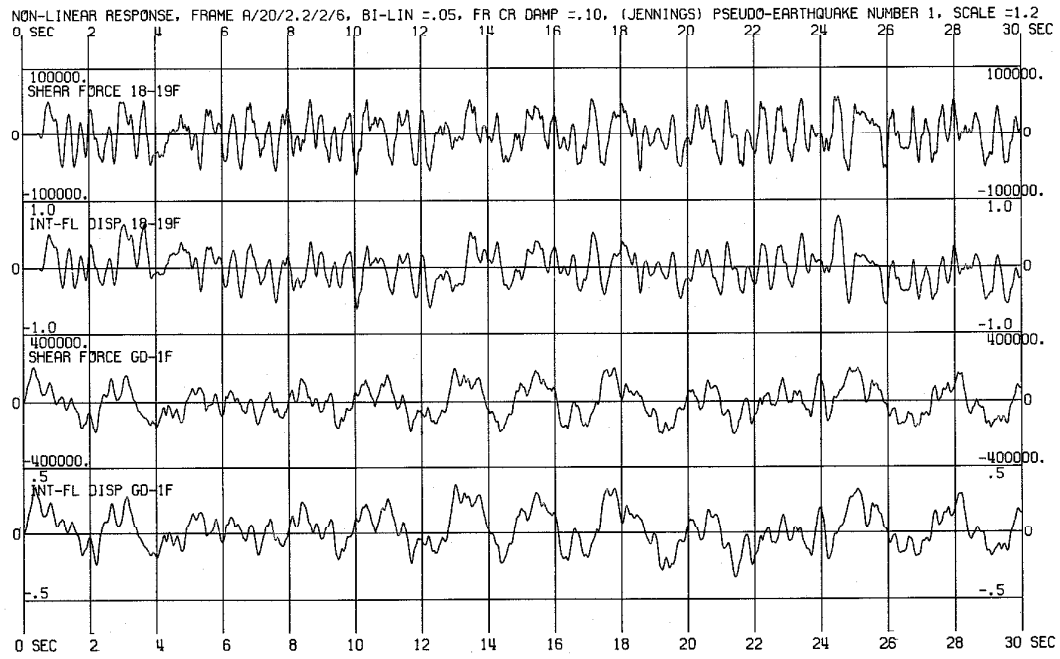

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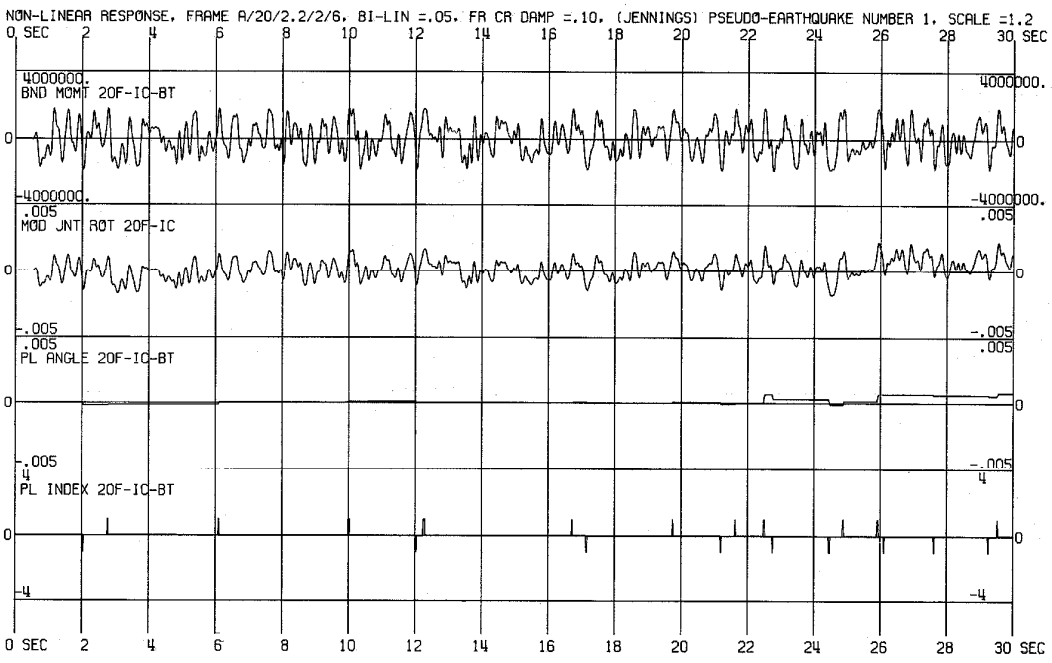

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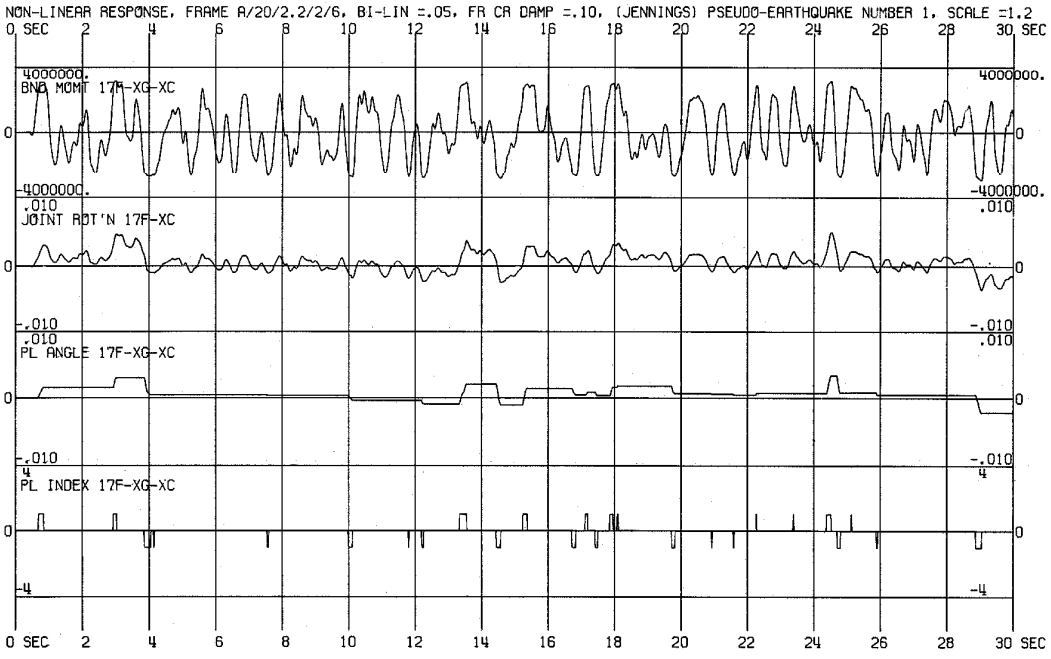

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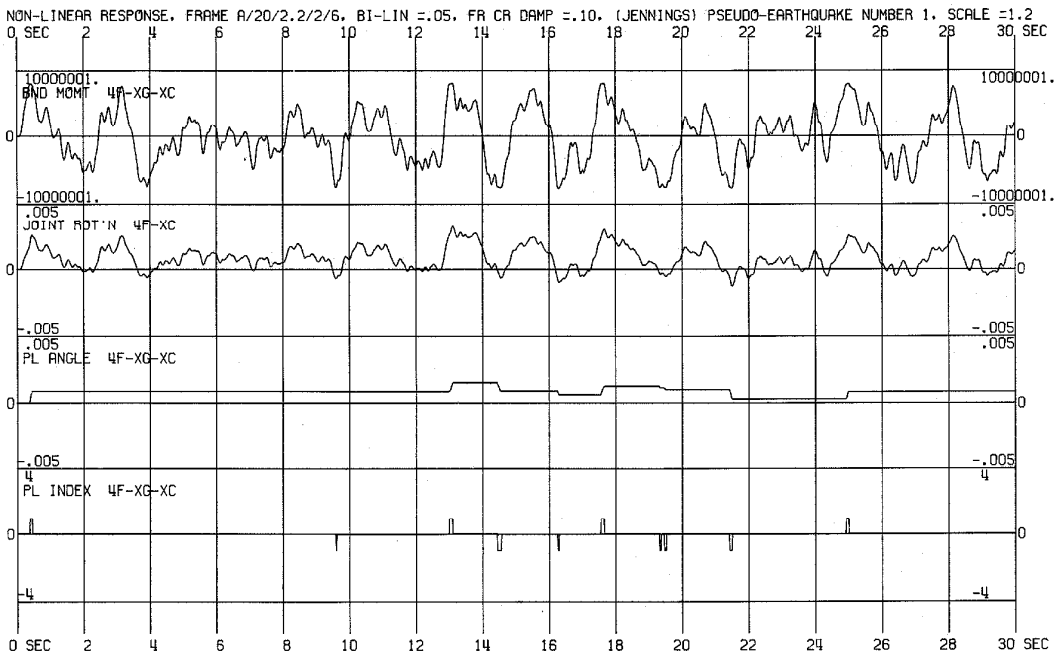

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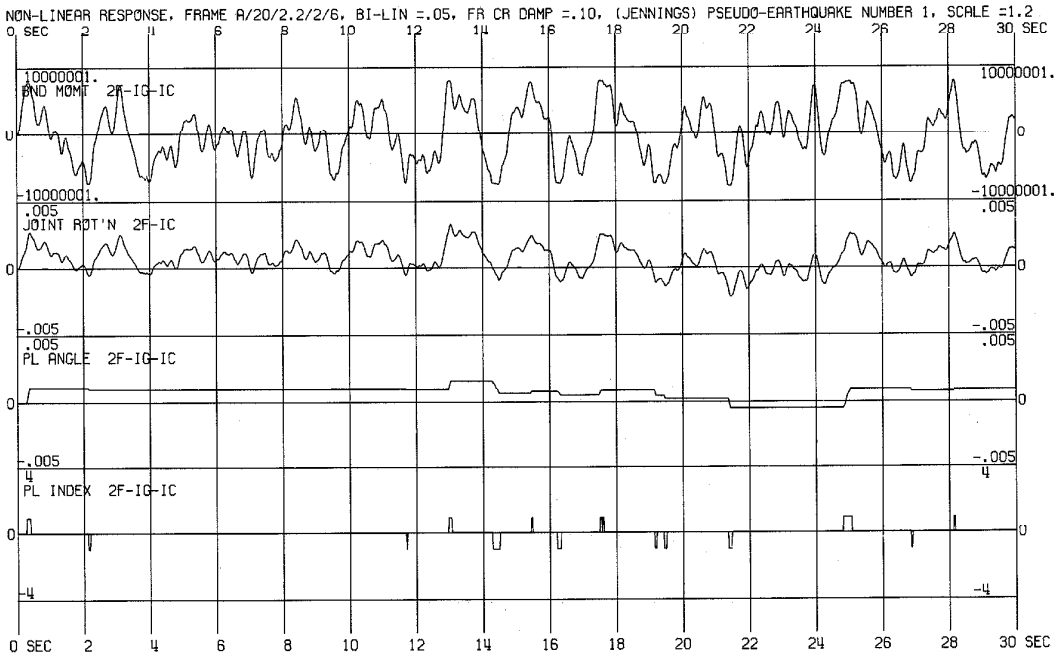


Fig. A.2.8 Station (h)

NONLINEAR RESPONSE OF FRAME A/20/2.2/2/6
BILINEARITY = 0.05, FRACTION OF CRITICAL DAMPING = 0.10
(JENNINGS) PSEUDO-EARTHQUAKE NUMBER 2 , SCALE = 1.2

MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INCHES

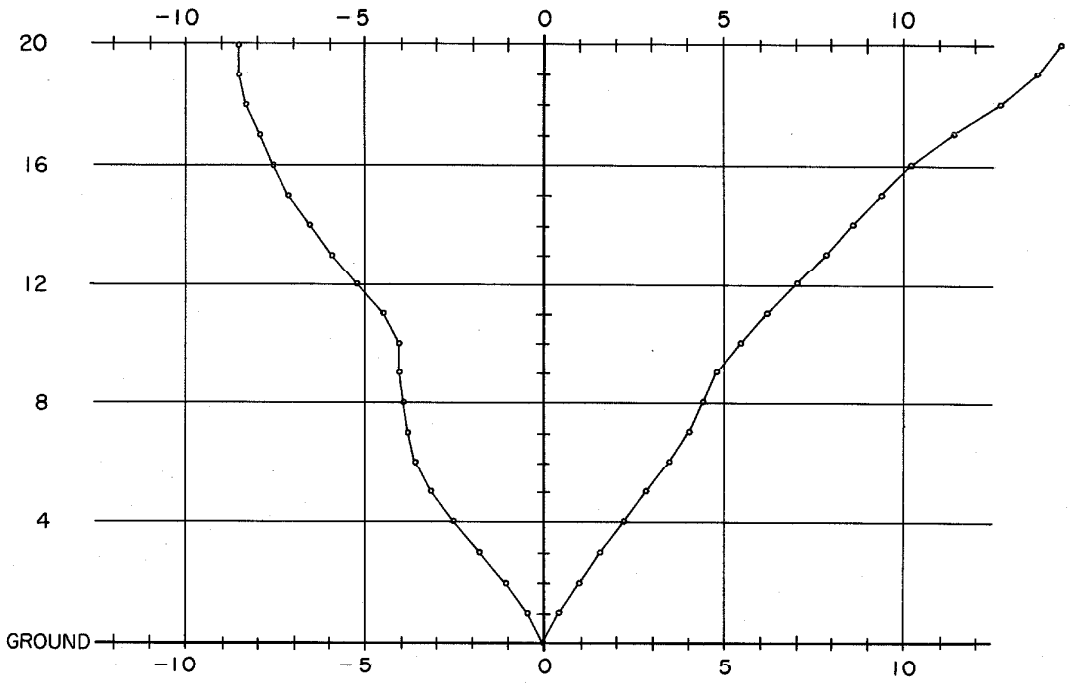


Fig. A.3.1

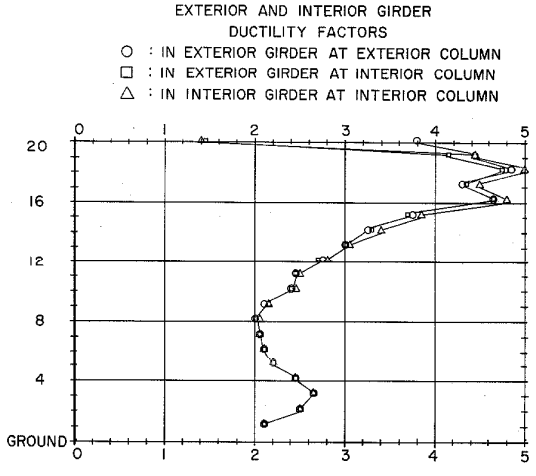


Fig. A.3.2.a

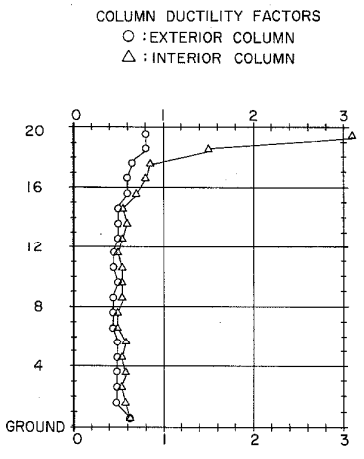


Fig. A.3.2.b

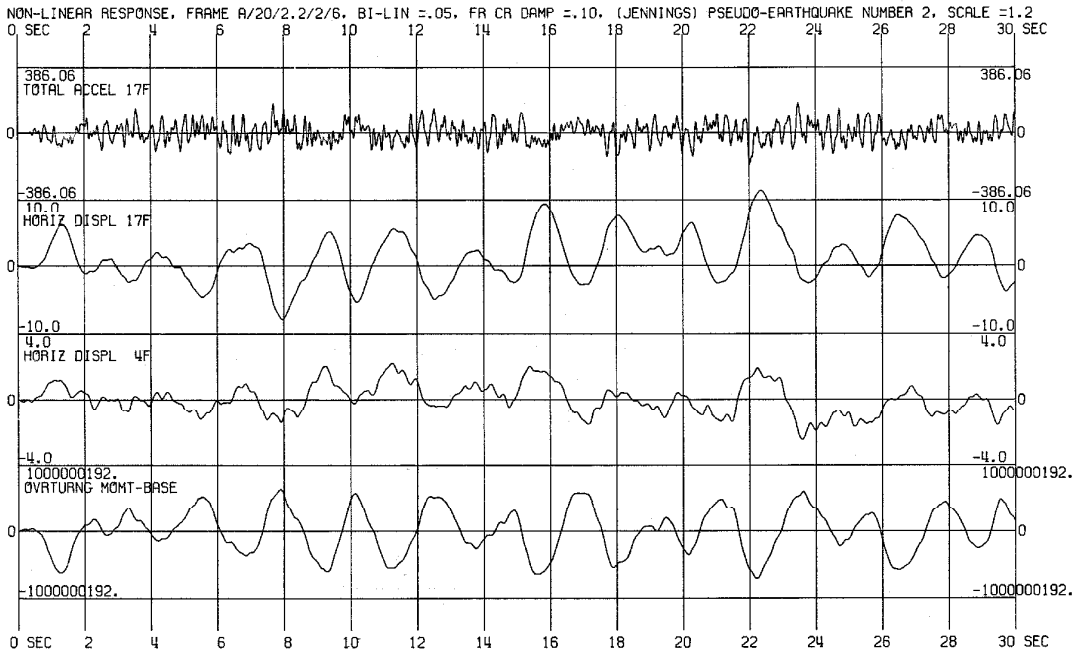
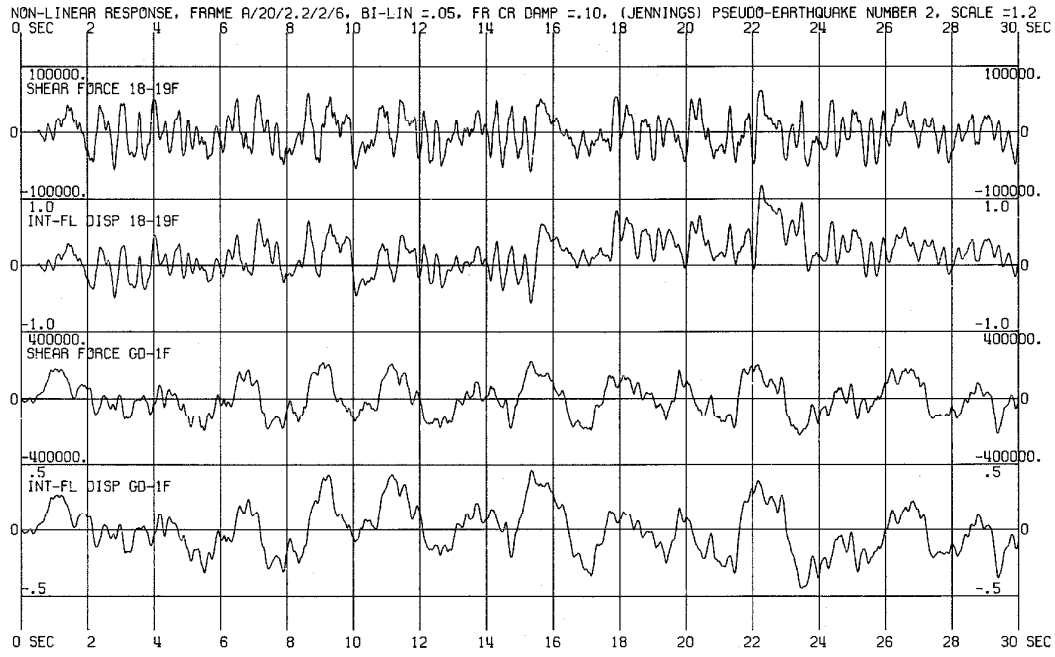


Fig. A.3.3



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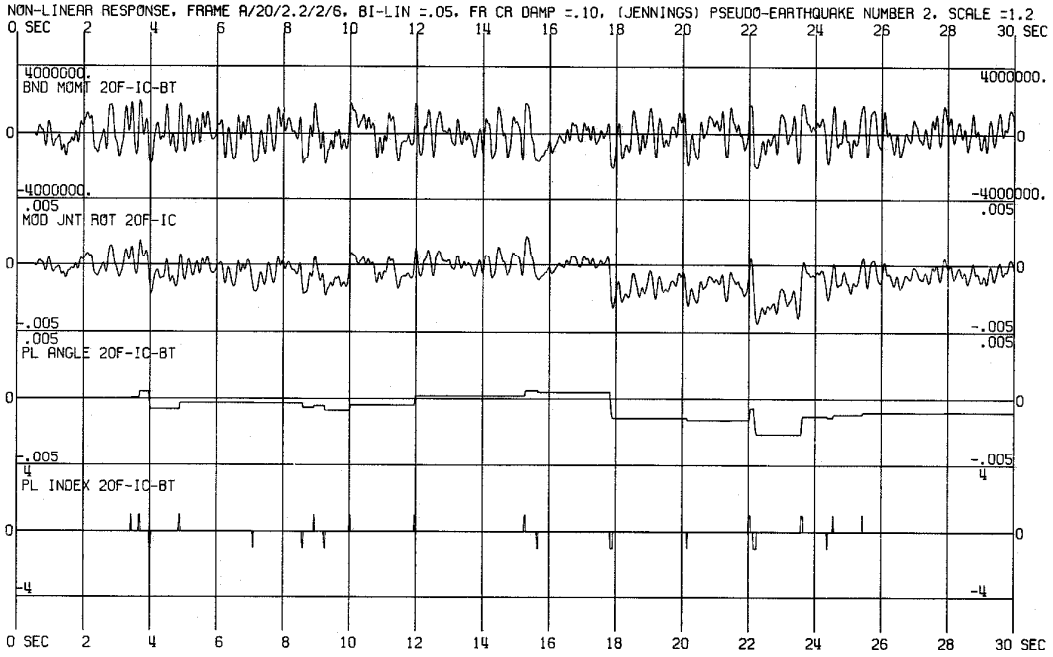

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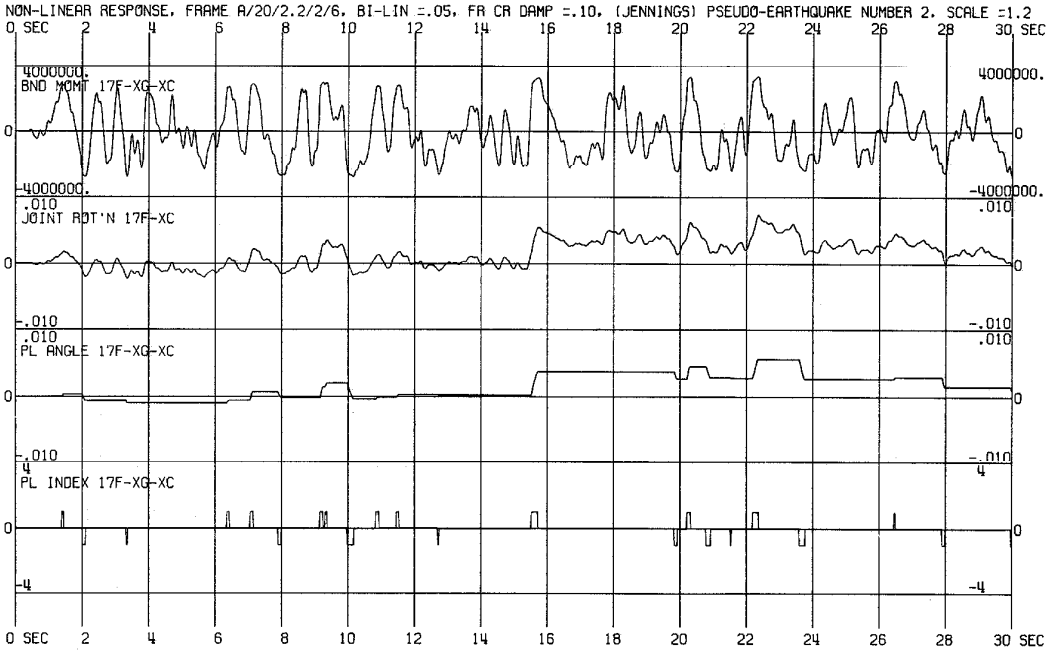

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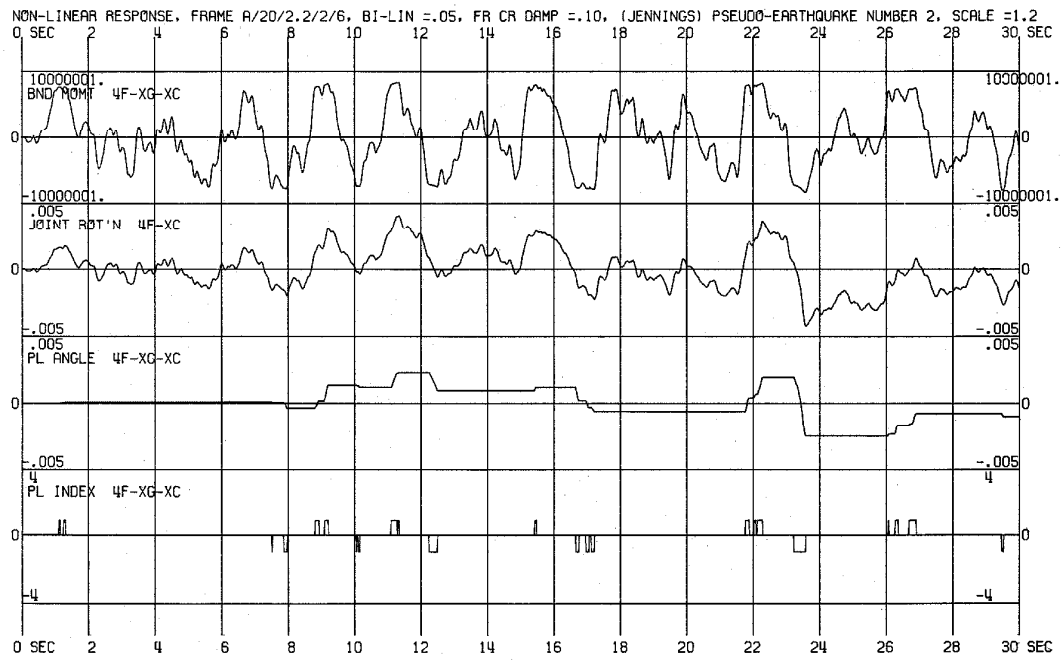

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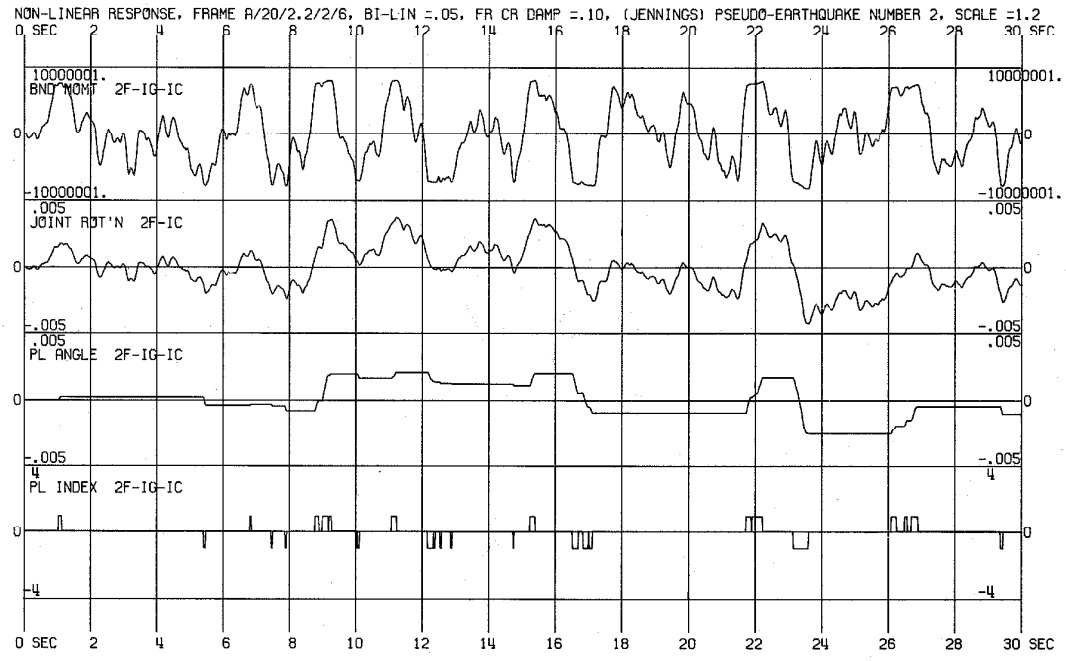


Fig. A.3.8 Station (h)

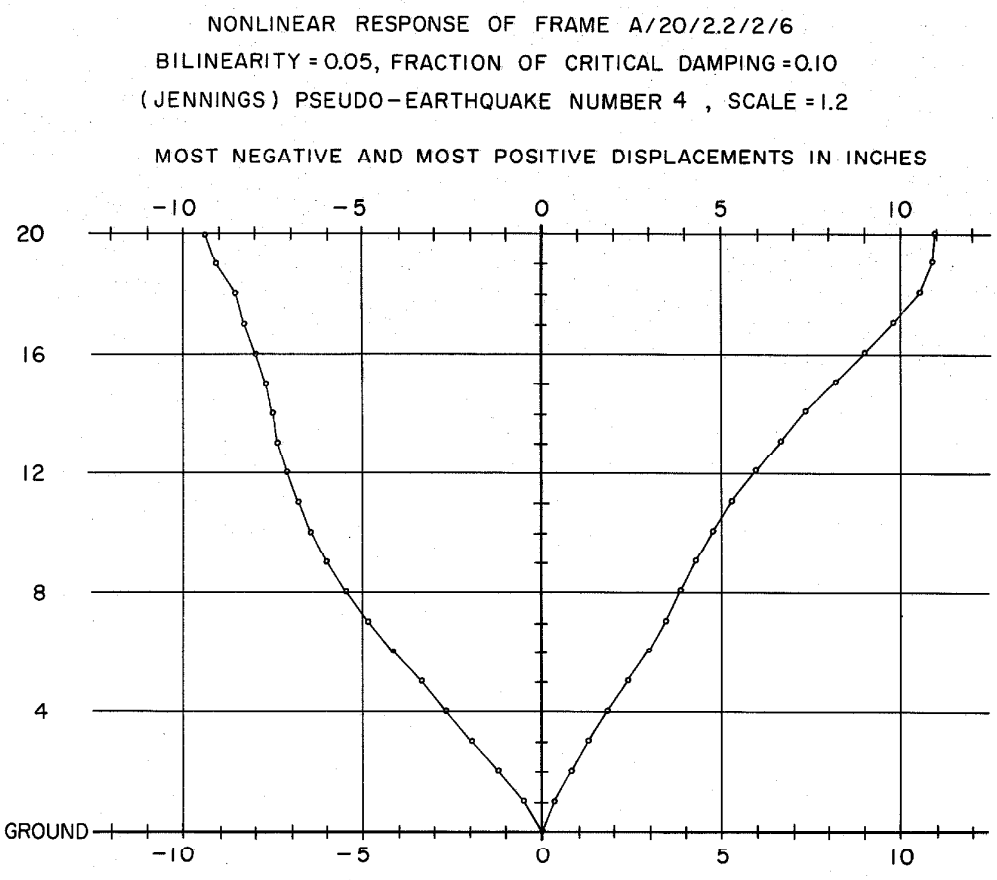
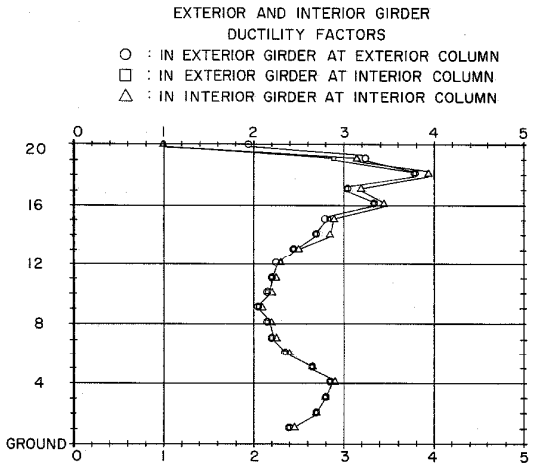


Fig. A.4.1



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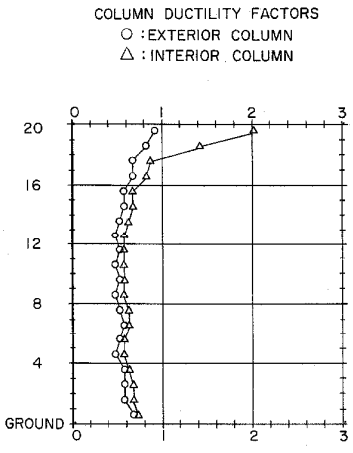

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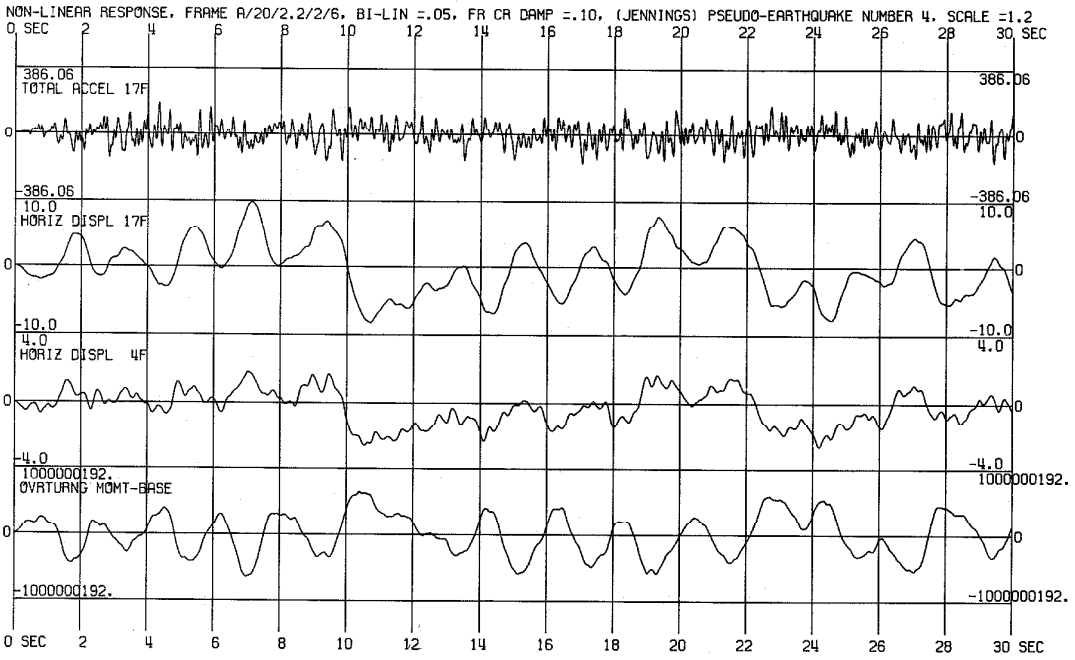


Fig. A.4.3

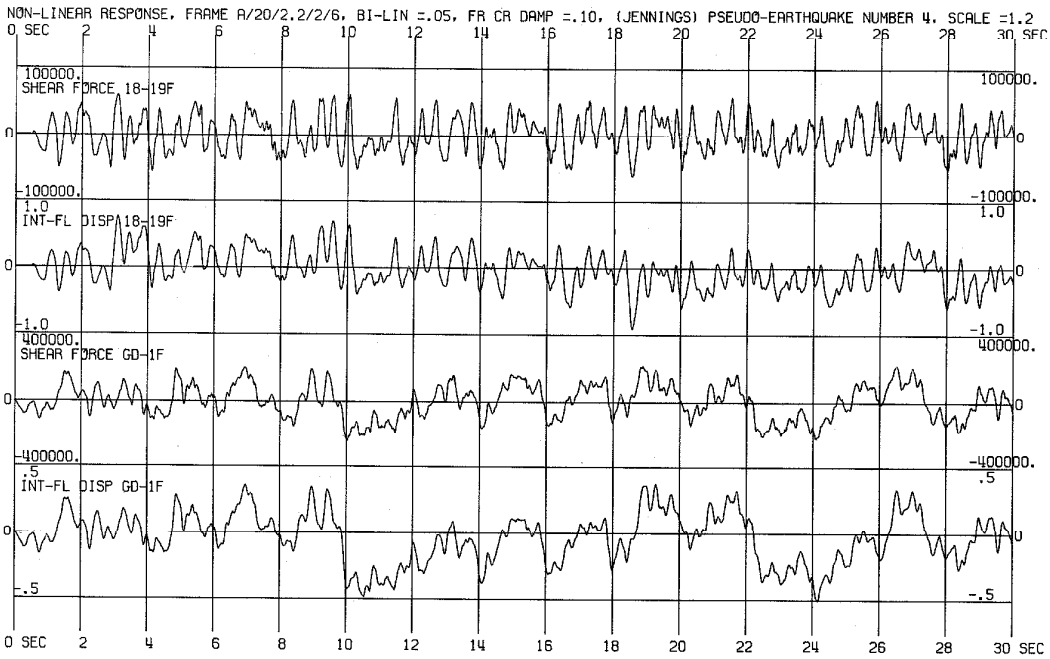


Fig. A.4.4

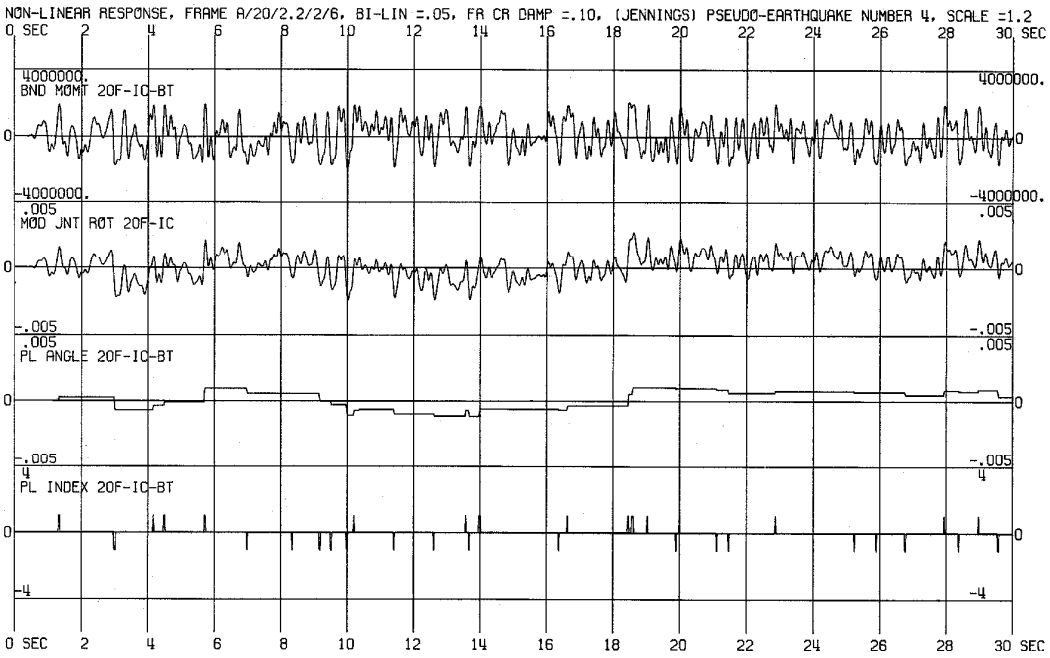


Fig. A.4.5 Station (d)

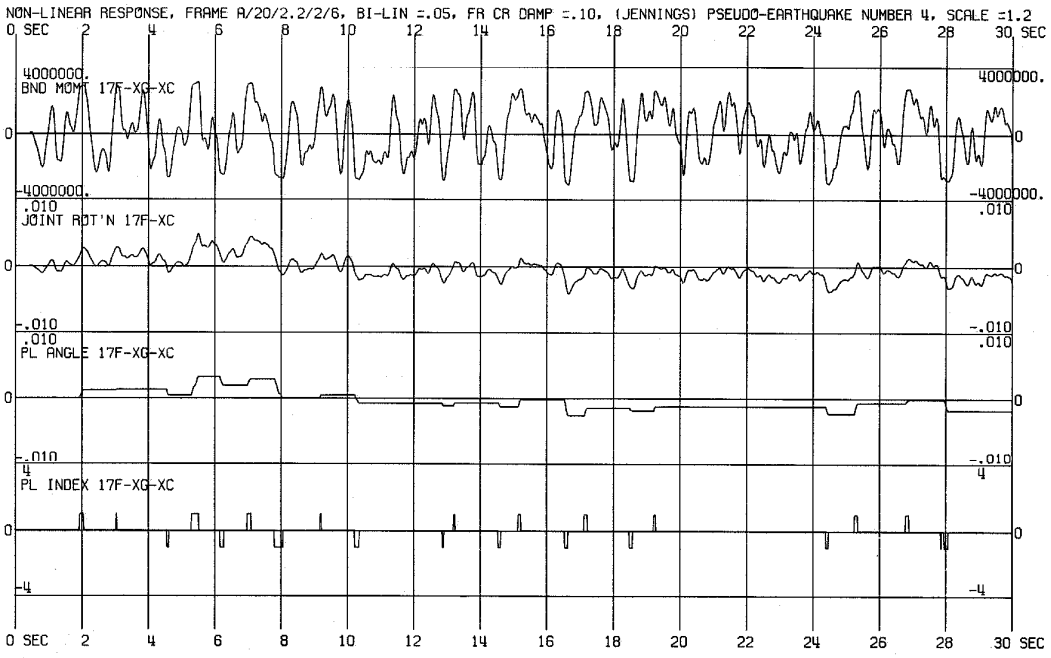


Fig. A.4.6 Station (e)

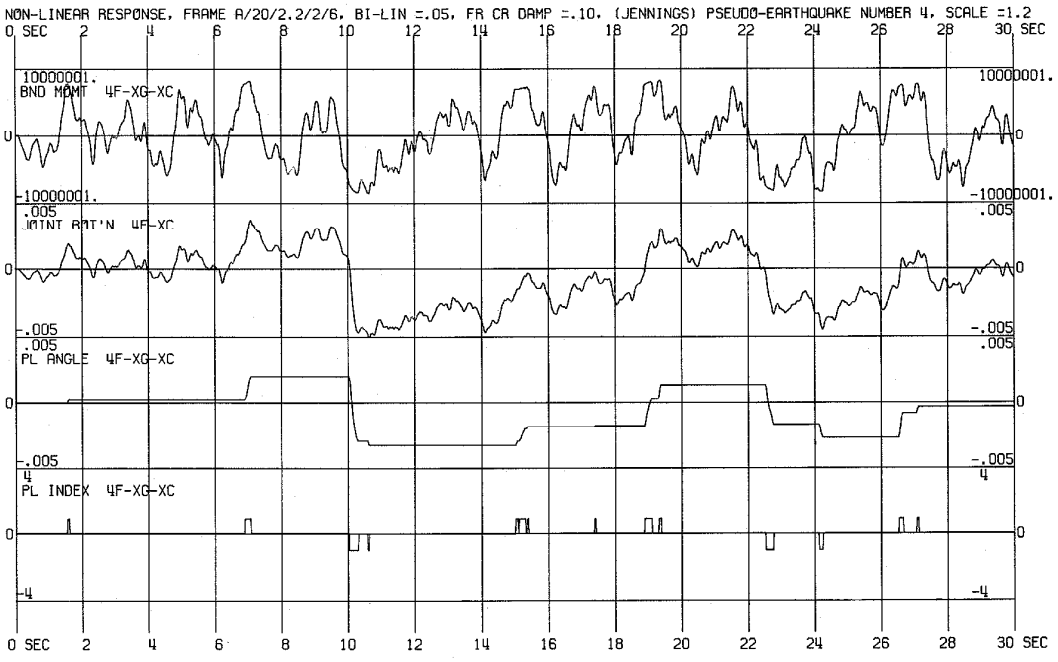


Fig. A.4.7 Station (f)

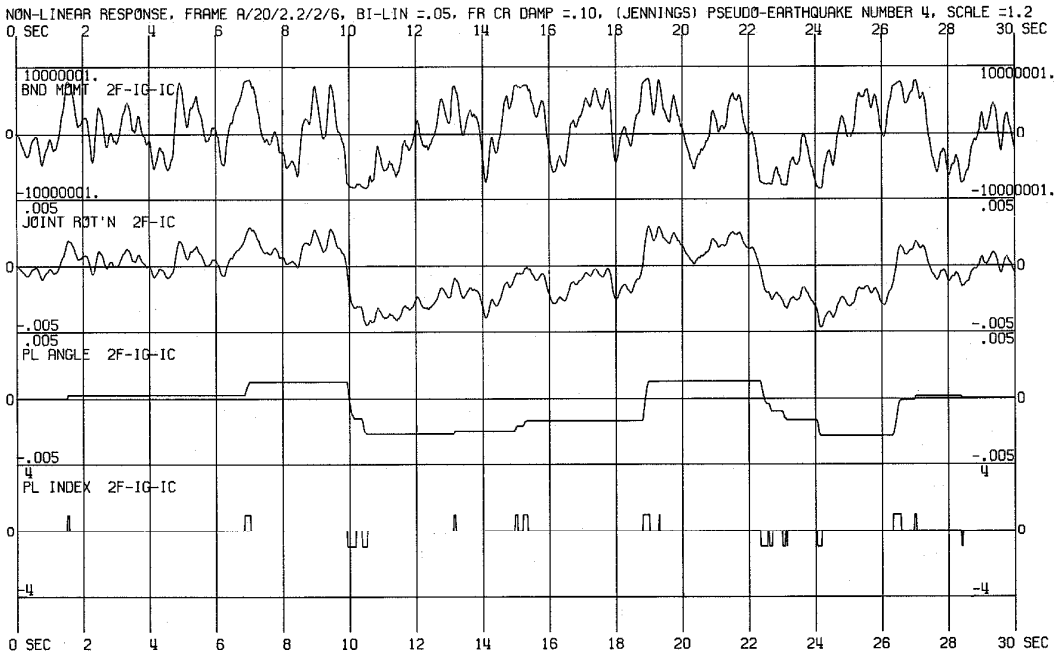
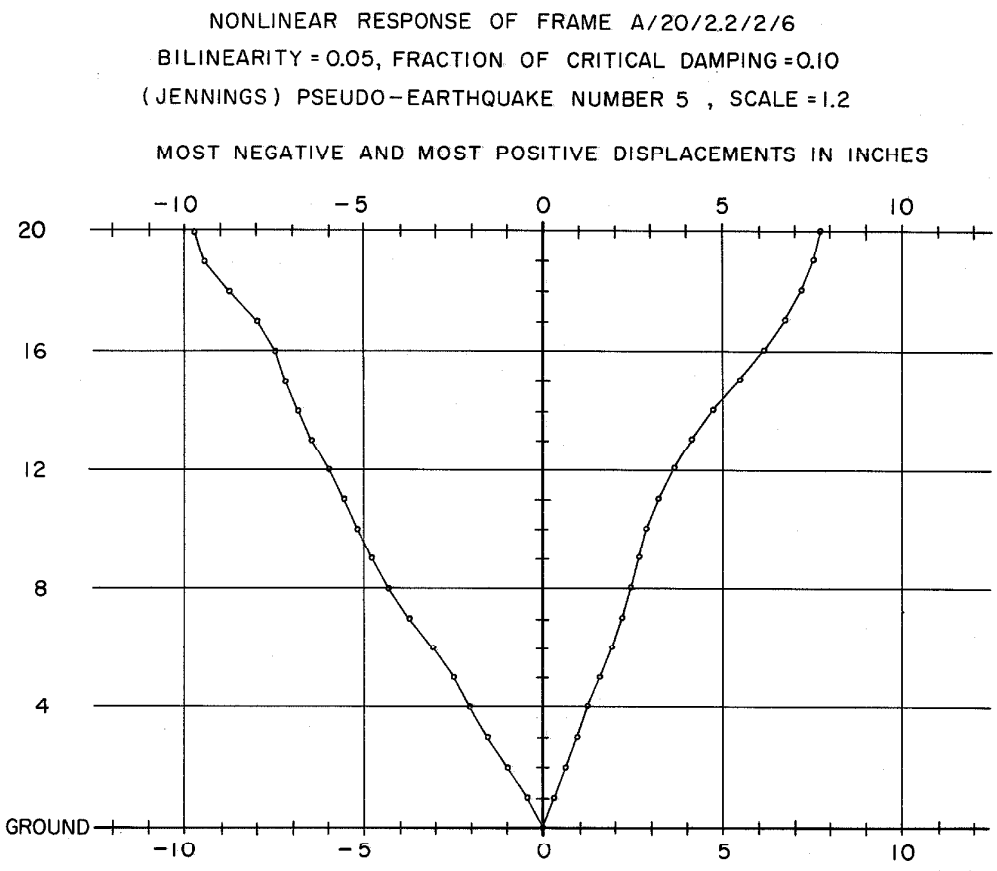


Fig. A.4.8 Station (h)



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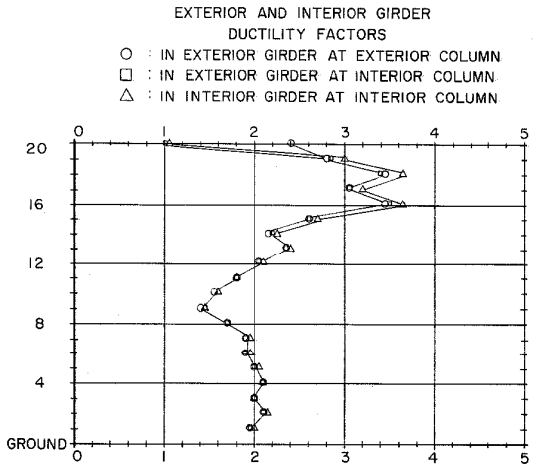

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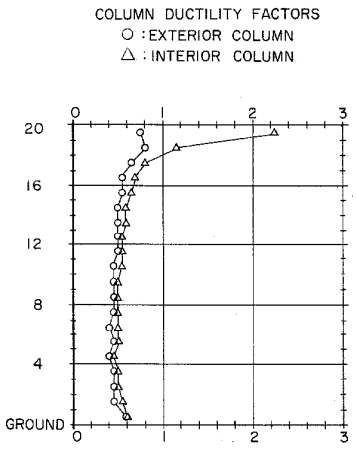


Fig. A.5.2.b

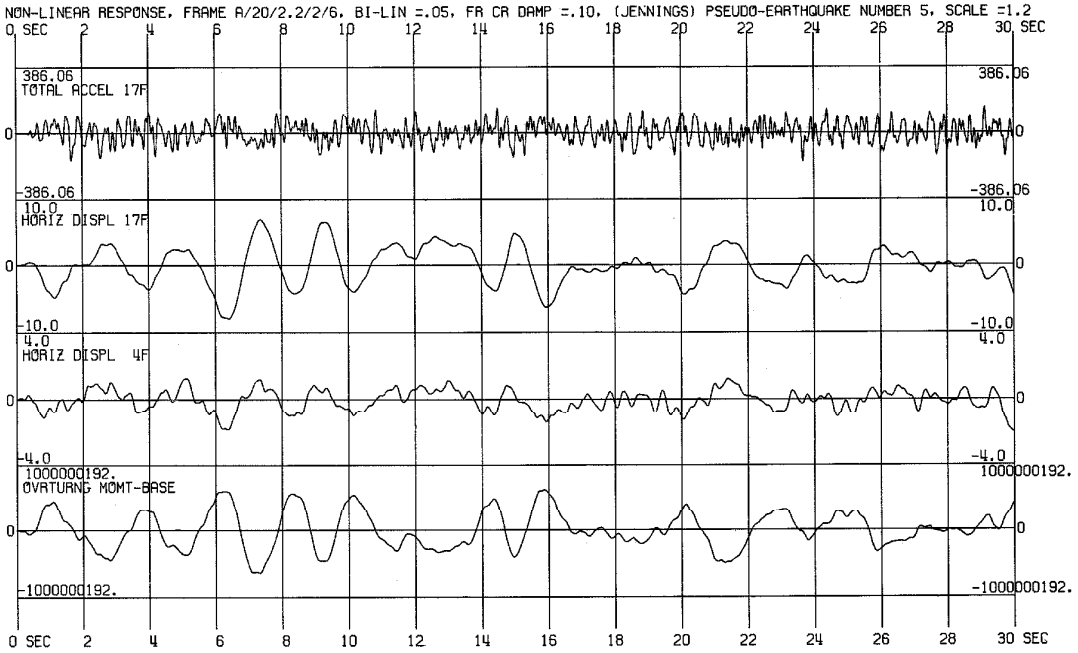


Fig. A.5.3

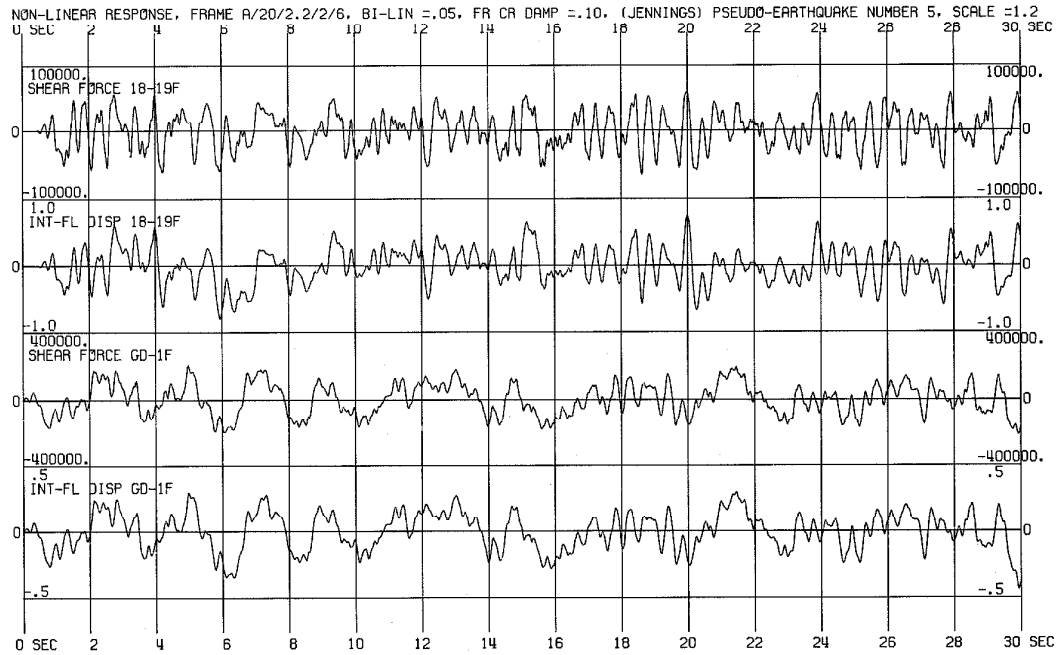


Fig. A.5.4

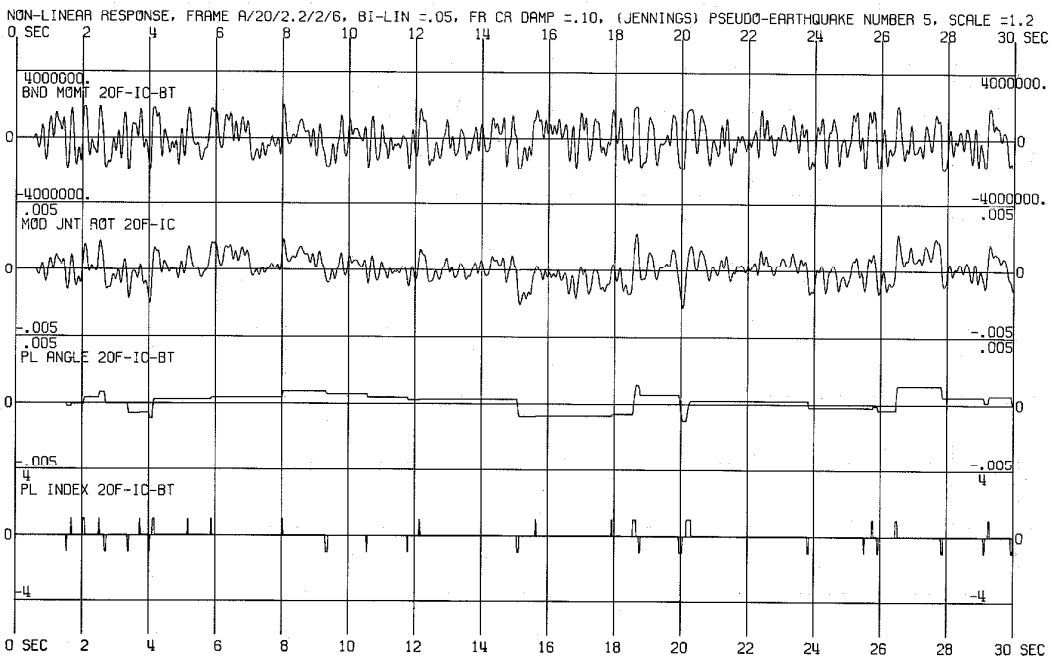


Fig. A.5.5 Station (d)

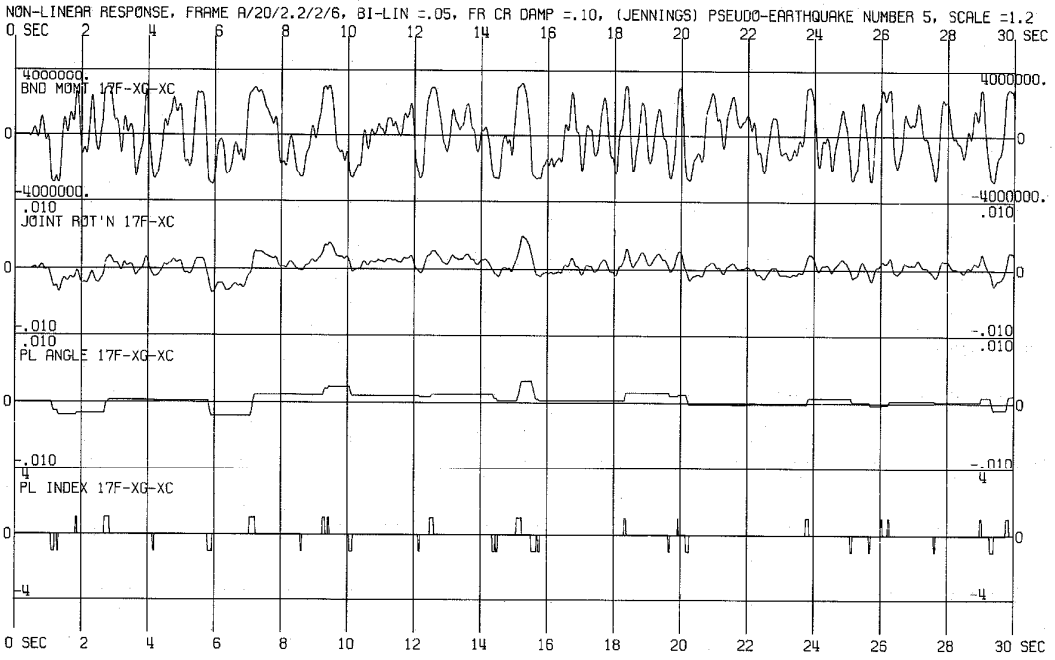


Fig. A.5.6 Station (e)

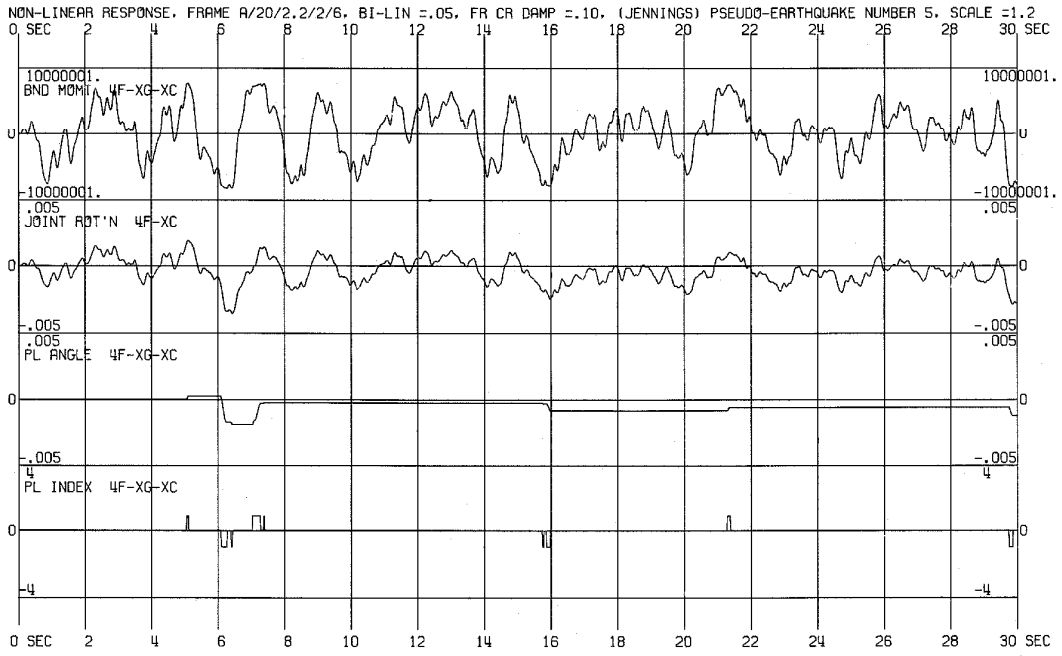


Fig. A.5.7 Station (f)

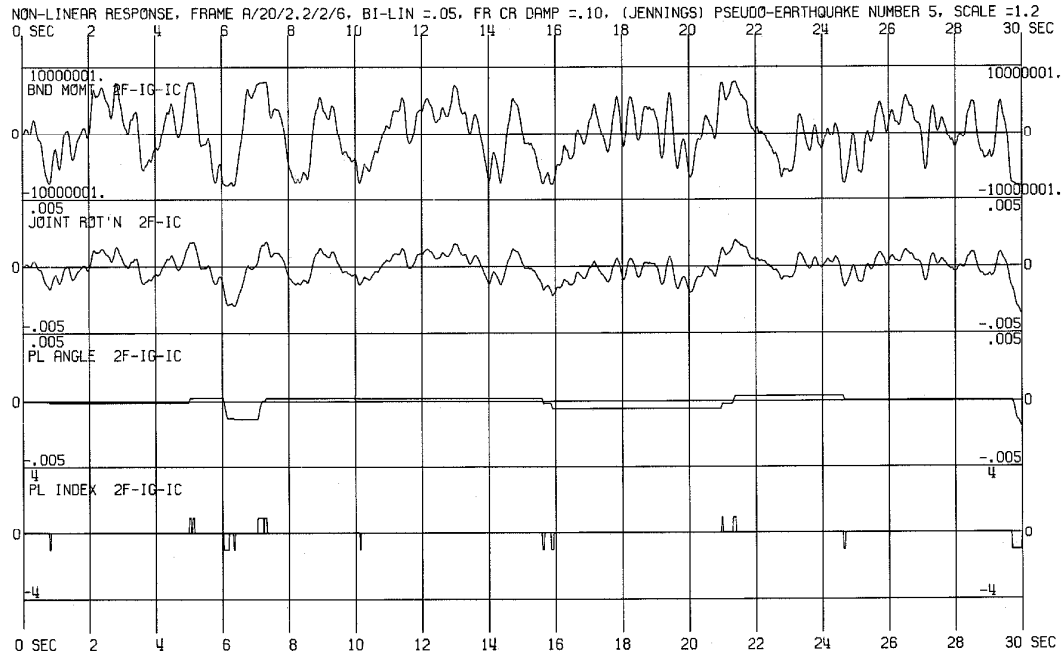


Fig. A.5.8 Station (h)

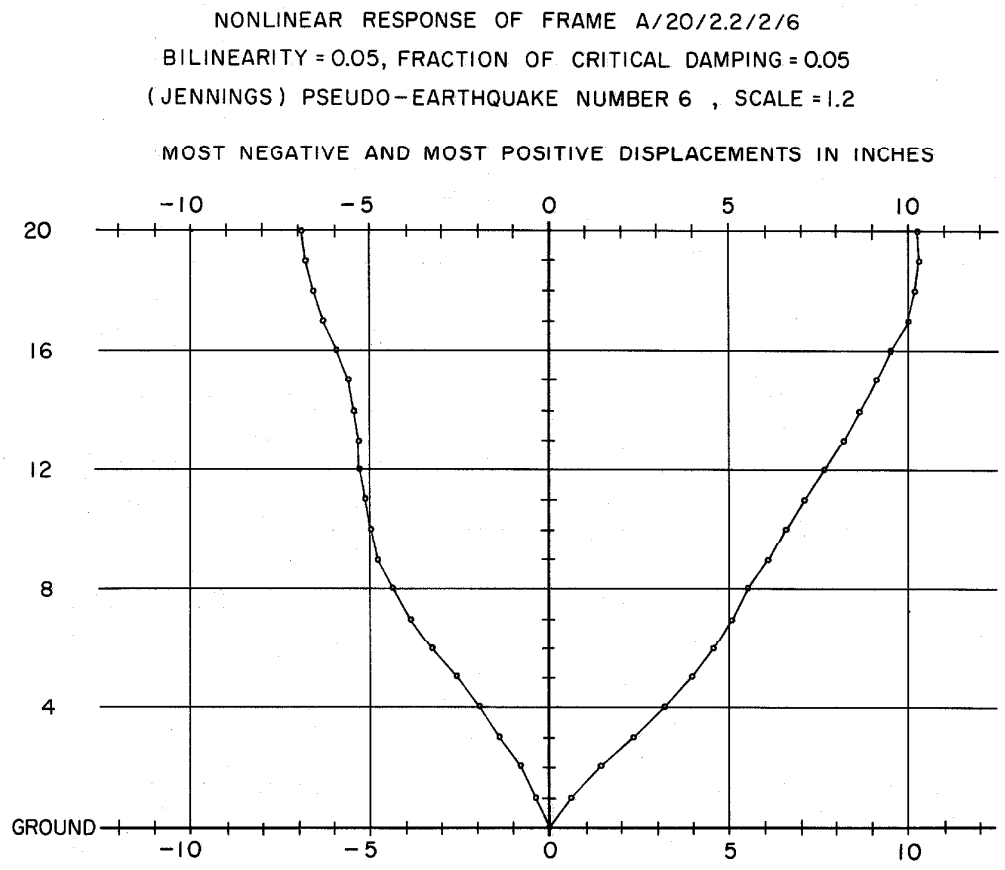


Fig. A.6.1

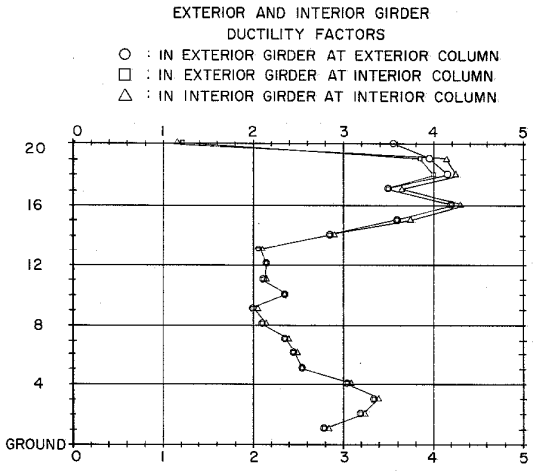


Fig. A.6.2.a

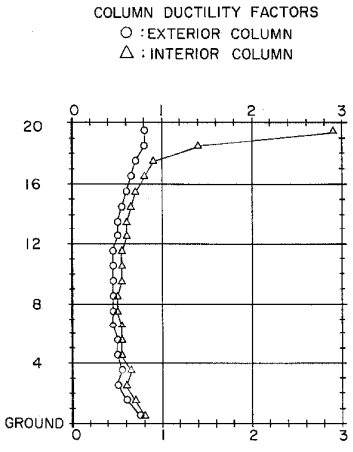


Fig. A.6.2.b

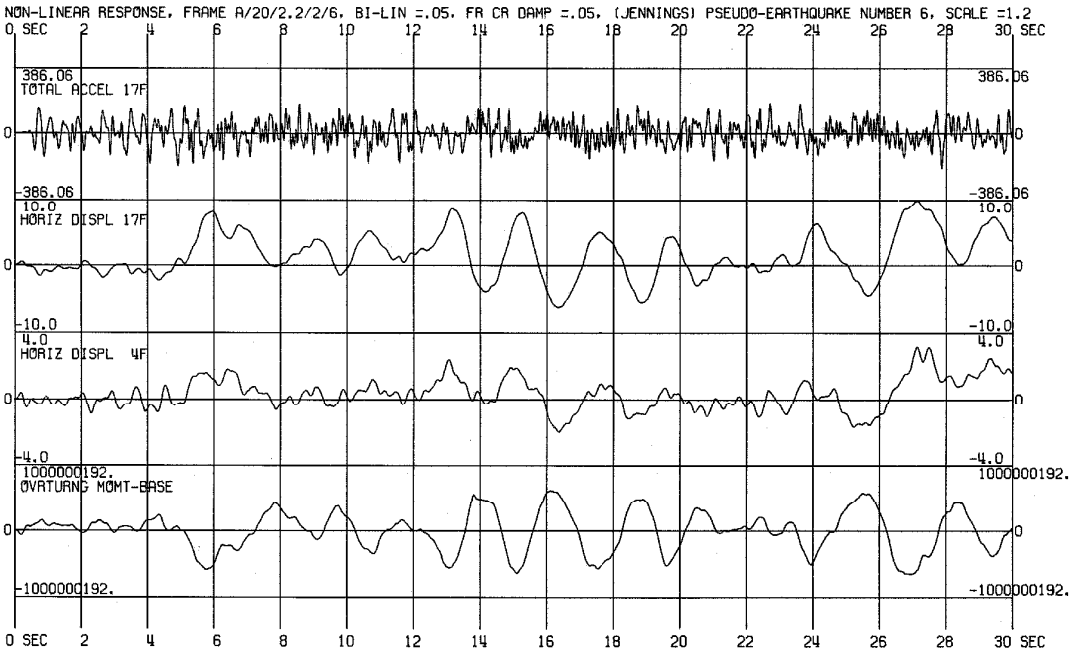


Fig. A.6.3

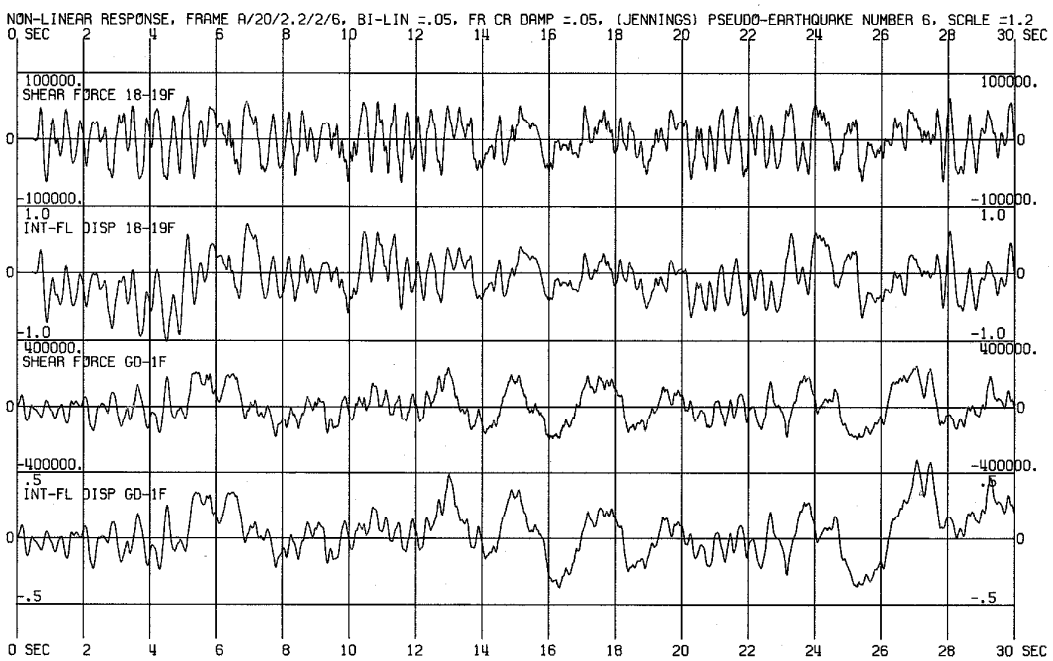


Fig. A.6.4

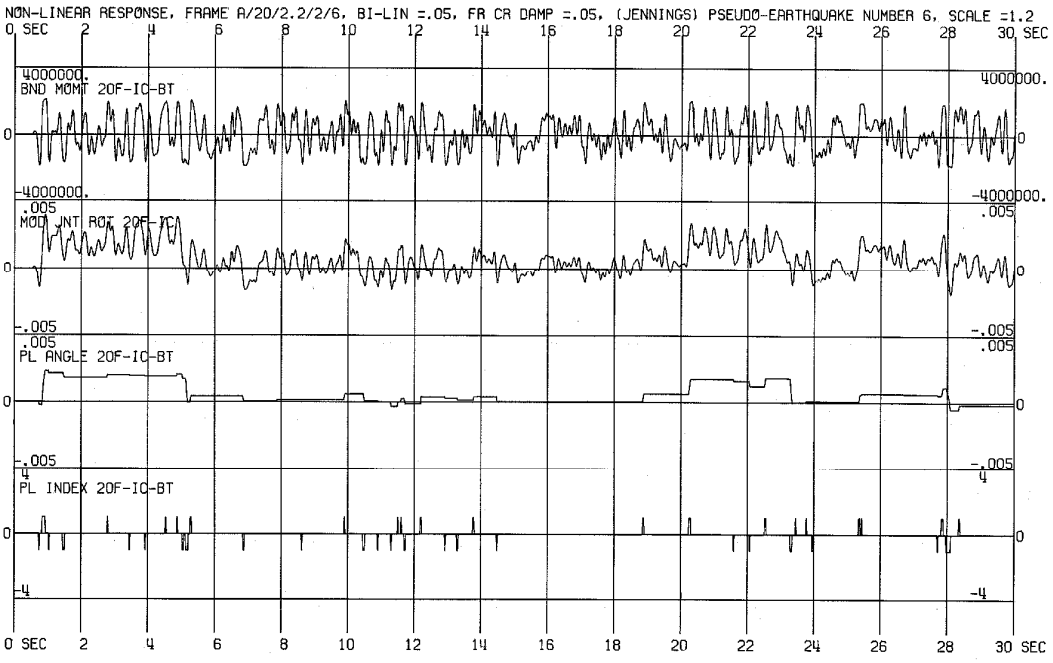


Fig. A.6.5 Station (d)

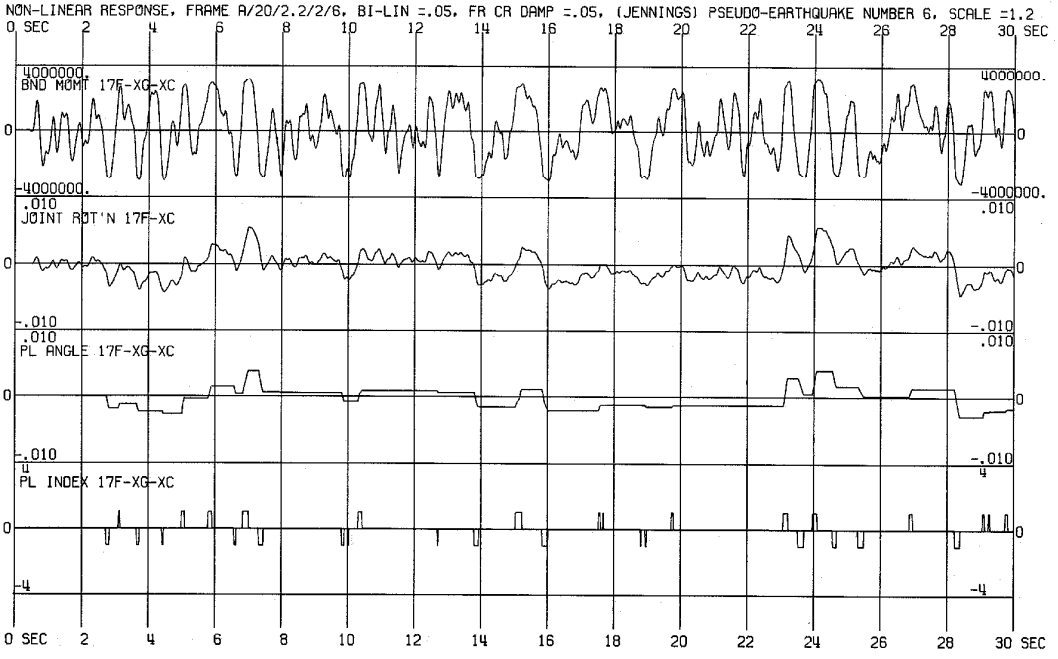


Fig. A.6.6 Station (e)

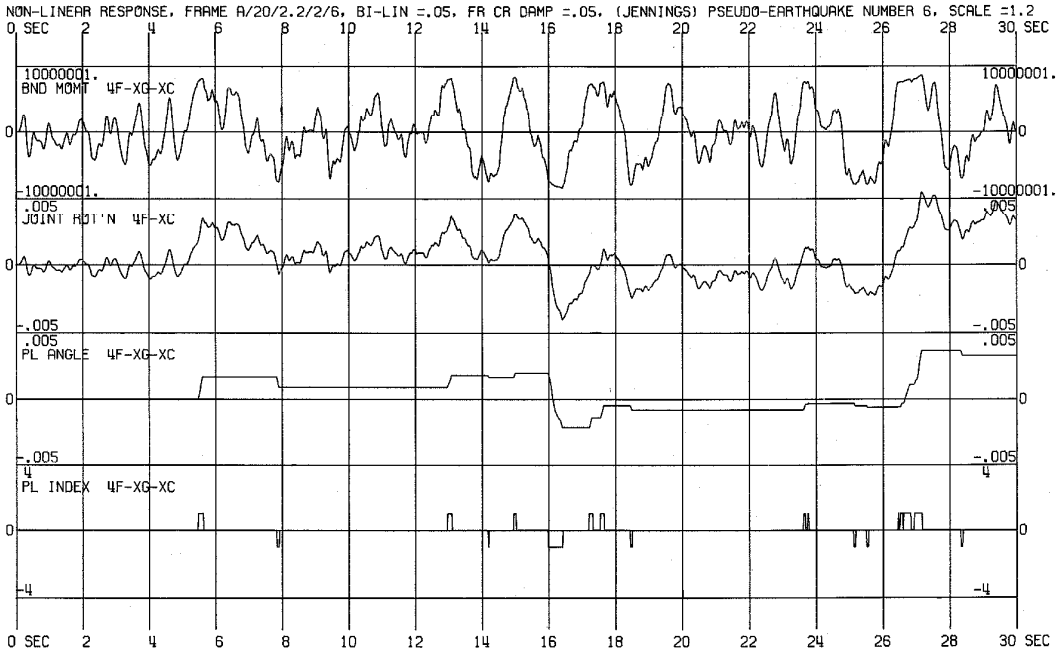


Fig. A.6.7 Station (f)

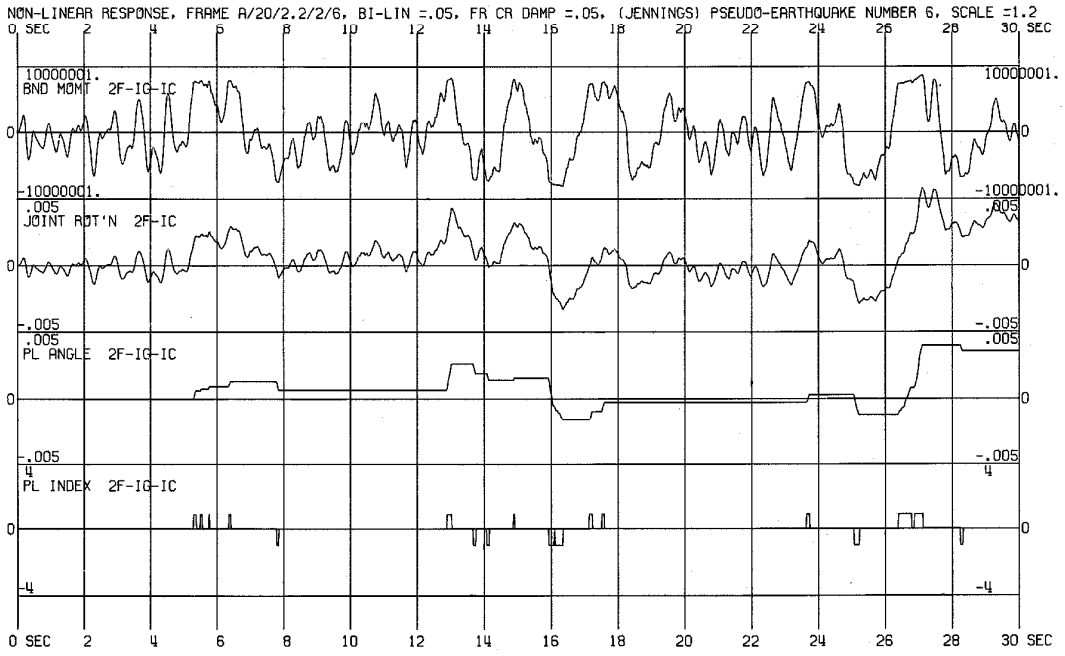


Fig. A.6.8 Station (h)

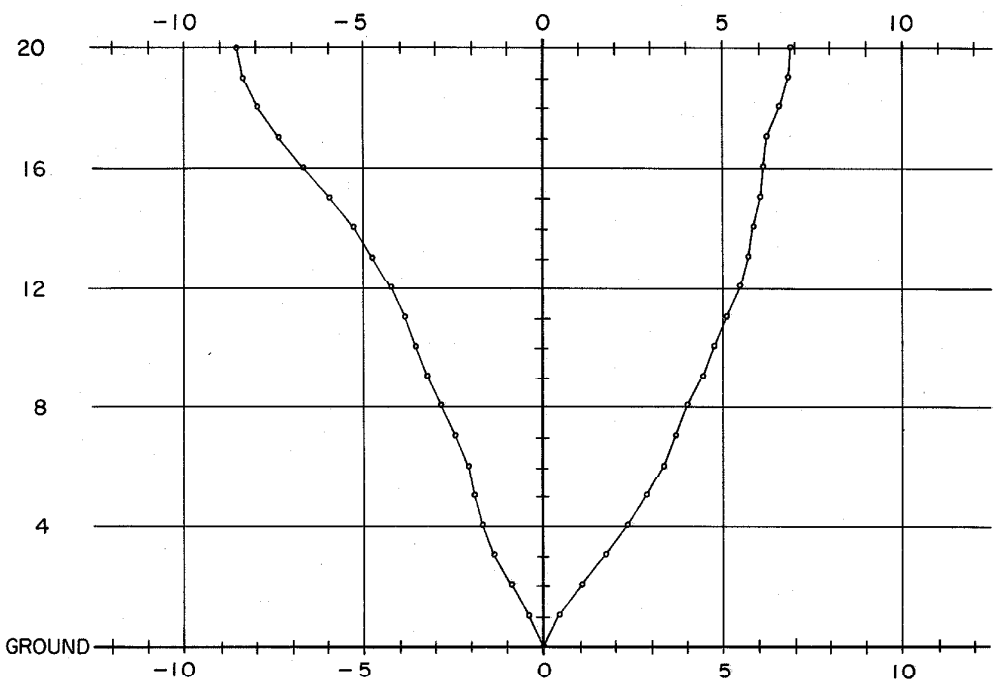


Fig. A.7.1

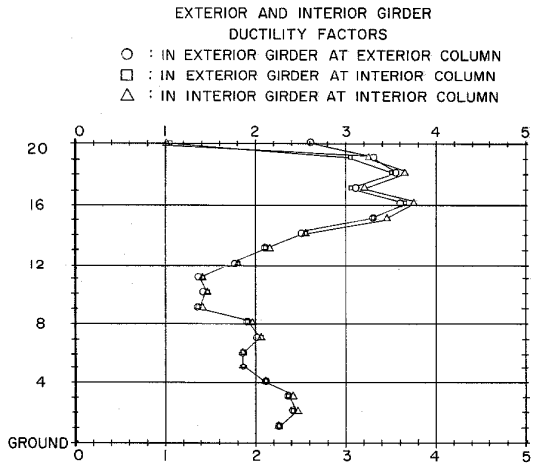


Fig. A.7.2.a

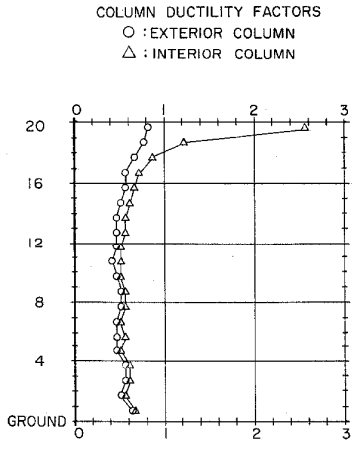


Fig. A.7.2.b

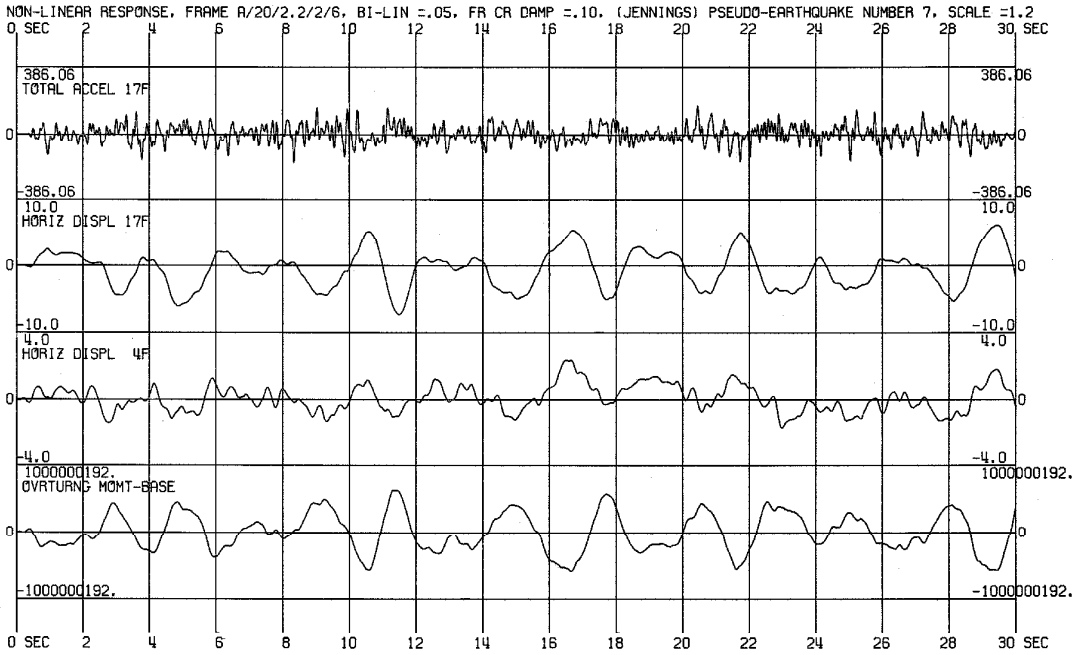


Fig. A.7.3

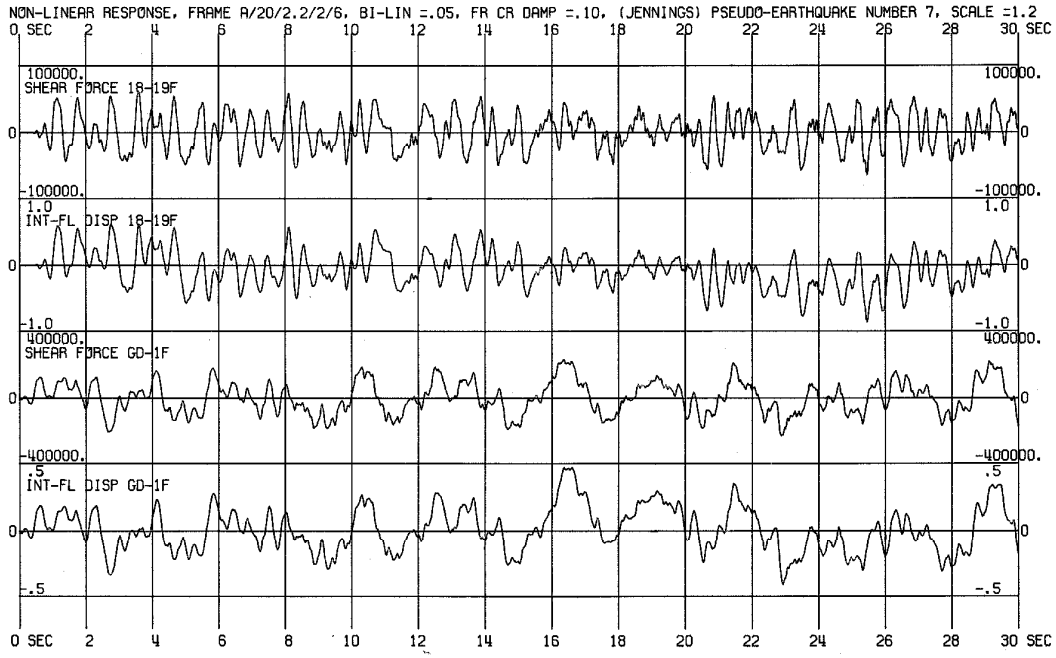


Fig. A.7.4

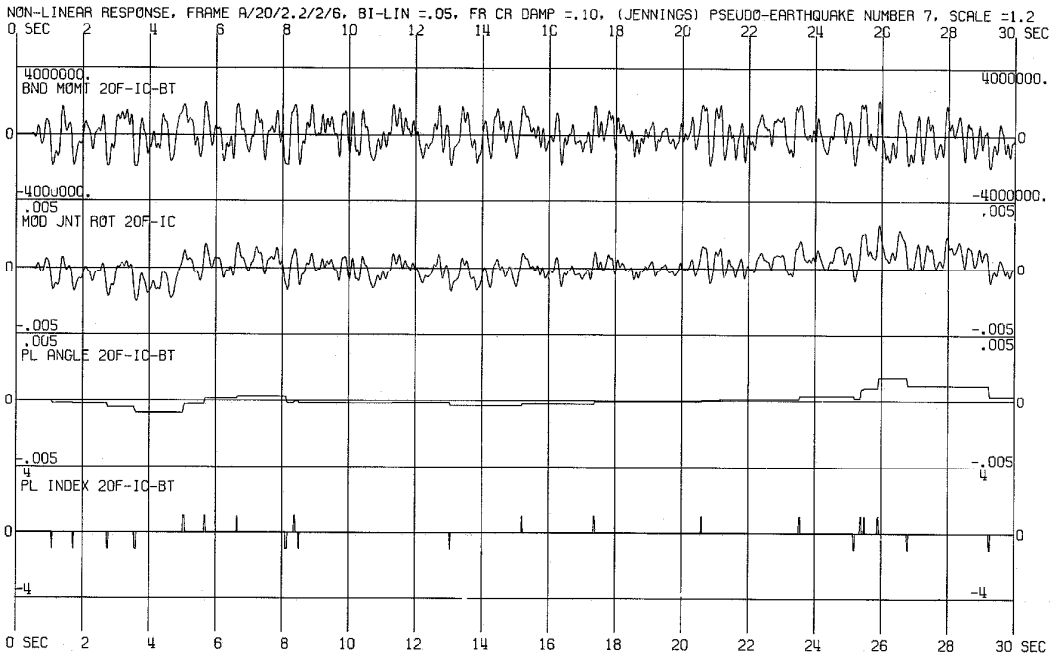


Fig. A.7.5 Station (d)

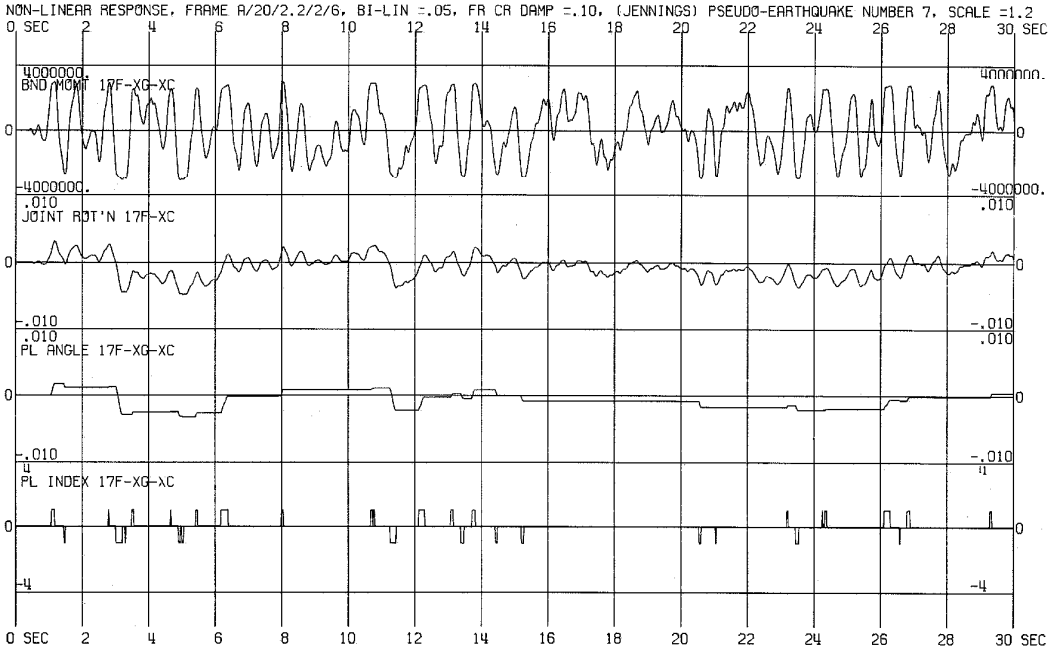


Fig. A.7.6 Station (e)

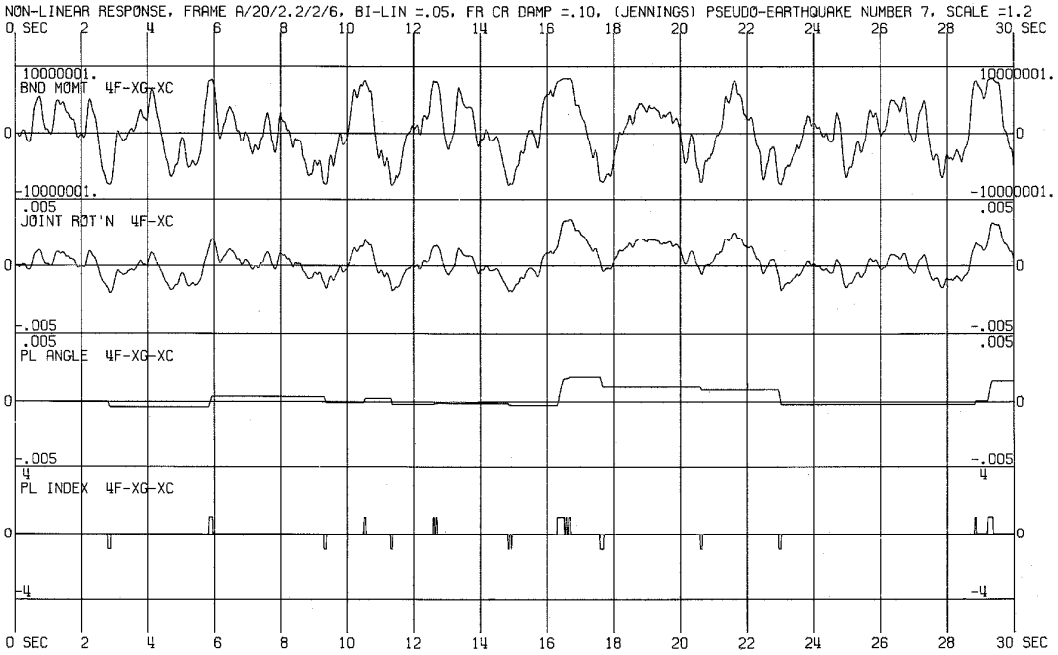


Fig. A.7.7 Station (f)

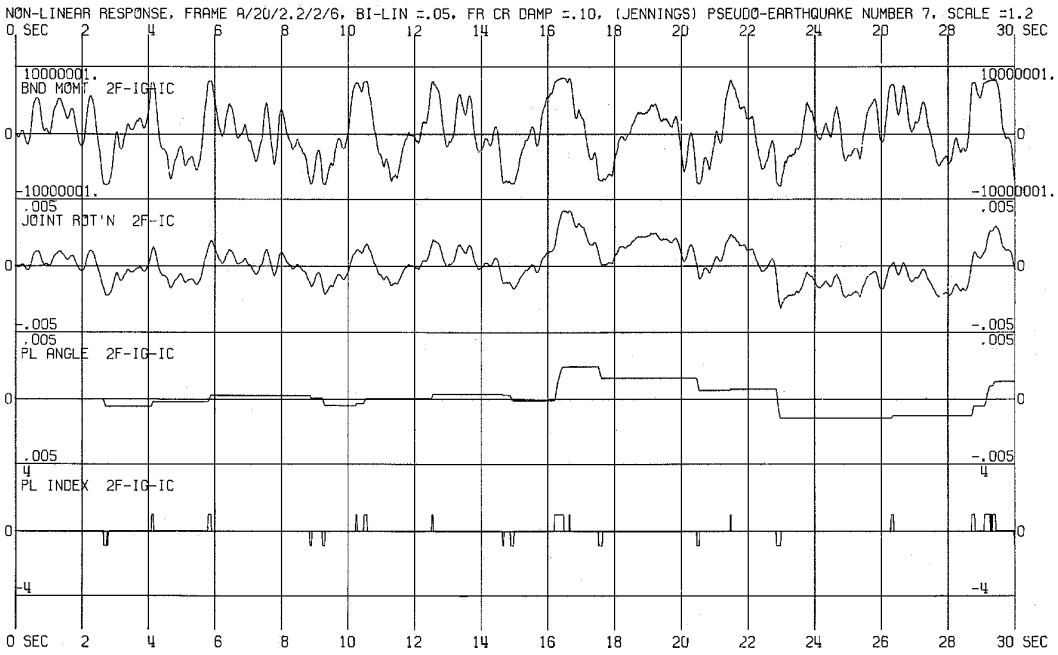


Fig. A.7.8 Station (h)

APPENDIX B

DISCUSSION OF THE YIELD CRITERIA USED IN THE FHA STUDY

The purpose of this appendix is to describe the special situation which causes the yield criteria used in the computer program for the FHA Study to lead to computational difficulties. Instead of the actual bending moment, this criteria compares a corresponding "test bending moment" with the yield bending moment in order to determine whether the end of a beam is in the linear range or in the yield range. By following the actual and corresponding test bending moments for a number of time increments along the paths shown in Fig. B.1a and Fig. B.1b, respectively, the special situation can be seen.

Usually, the bending moments are written as a function of the rotations and plastic angles of both ends of the beam; however, a simpler form in which the moment is a function of the rotation and plastic angle at one end only is sufficient for the present purpose. The equations written below are those for a simplified two-component beam. The usual moment-rotation equations for a two-component beam are described in Section 2.5.

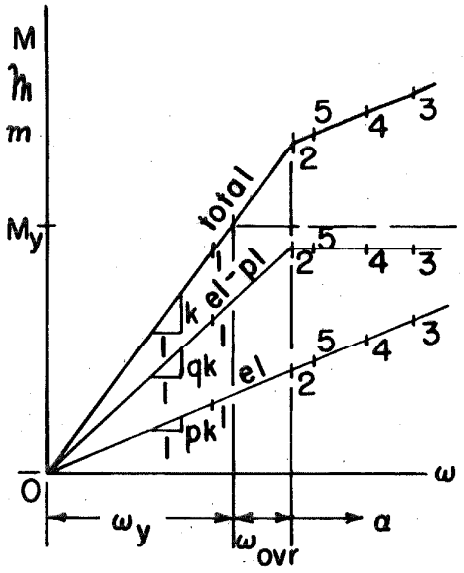


Fig. B.1a Actual Bending Moment

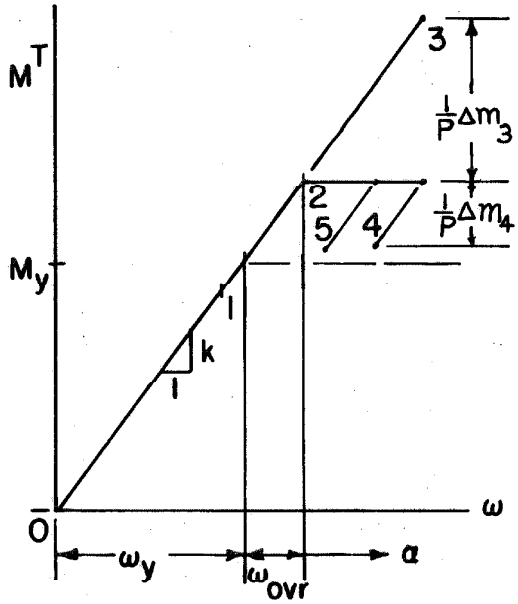


Fig. B.1b Test Bending Moment

Actual Bending Moment

(See Fig. B.1a)

General equations:

elasto-plastic component:

$$\mathbb{m}_t = qk(\omega_t - \alpha_t) ;$$

elastic component:

$$m_t = pk(\omega_t) ;$$

total actual bending moment:

$$M_t = k(\omega_t - q\alpha_t) ;$$

end rotation:

$$\omega_t = \omega_{t-\Delta t} + \Delta\omega_t ;$$

plastic angle:

$$\alpha_t = \alpha_{t-\Delta t} + \Delta\alpha_t ;$$

$t = 1, 2, 3, \ldots$ the integration step number, i.e., the time at the end of the increment; and

$p + q = 1$ (usually $p \approx 0.05$).

Test Bending Moment

(See Fig. B.1b)

General equations:

test bending moment:

$$M_t^T = k(\omega_t - \alpha_{t-\Delta t}) ;$$

however, with

$$\omega_t = \omega_{t-\Delta t} + \Delta\omega_t ,$$

$$M_t^T = k(\omega_{t-\Delta t} - \alpha_{t-\Delta t}) + k\Delta\omega_t ,$$

or

$$M_t^T = \frac{1}{q}\mathbb{m}_{t-\Delta t} + \frac{1}{p}\Delta m_t$$

where

$\mathbb{m}_t$, m_t, p, q, and t are the same as those for the "actual bending moment."

Follow the process for a number of time increments.

$t = 0$, initial point.

$\omega_o = 0$,

$\alpha_o = 0$,

$\mathbb{m}_o = 0$,

$t = 0$

Initially in the linear range; hence,

PLASTIC INDEX (1) = 0.

Actual Moment, cont.	Test Moment, cont.
$m_o = 0$, and $M_o = 0$.	

Integration step number 2,

calculate $\Delta\omega_1$.

$t = 1$ Since PLASTIC INDEX (1) = 0, $\Delta\alpha_1 = 0$. $\therefore\ \alpha_1 = 0$, $\mathbb{m}_1 = qk(\omega_1 - 0)$, $m_1 = pk\omega_1$, and $M_1 = k\omega_1$. $(M_1 = M_1^T)$	$t = 1$ $M_1^T = \frac{1}{q}\mathbb{m}_o + \frac{1}{p}\Delta m_1$, with $\mathbb{m}_o = 0$, $\Delta m_1 = pk\Delta\omega_1$, and $\Delta\omega_1 = \omega_1$, $M_1^T = k\omega_1$. Compare M_1^T with M_y. From point (t = 1) in Fig. B.1b, find that $M_1^T < M_y$. $\therefore$ PLASTIC INDEX (2) = 0, i.e., in the linear range for integration step number 2.

Integration step number 2,

calculate $\Delta\omega_2$.

Actual Moment, cont.

$t = 2$

Since PLASTIC INDEX (2) = 0,

$\Delta\alpha_2 = 0.$

$\therefore \alpha_2 = 0,$

$\mathbb{m}_2 = qk(\omega_2 - 0),$

$m_2 = pk\omega_2$, and

$M_2 = k\omega_2$.

$(M_2 = M_2^T)$

Test Moment, cont.

$t = 2$

$M_2^T = \frac{1}{q}\mathbb{m}_1 + \frac{1}{p}\Delta m_2$

With

$\mathbb{m}_1 = qk\omega_1$,

$\Delta\mathbb{m}_2 = pk\Delta\omega_2$,

and

$\Delta\omega_2 = \omega_2 - \omega_1$,

it follows that

$M_2^T = k\omega_2.$

Compare M_2^T with M_y. From point (t = 2) in Fig. B.1b, find that

$M_2^T > M$.

$\therefore$ PLASTIC INDEX (3) = 1, i.e., in the yield range for integration step number 3. From Fig. B.1b, note that

$M_2^T = M_y + k\omega_{ovr}$

where the overshoot ω_{ovr} is

$\omega_{ovr} = \omega_2 - \omega_y$.

Integration step number 3,

calculate $\Delta\omega_3$.

Actual Moment, cont.

$t = 3$

Since PLASTIC INDEX (3) = 1,

$\Delta\alpha_3 = \Delta\omega_3 = \omega_3 - \omega_2$.

Since $\alpha_3 = \alpha_2 + \Delta\alpha_3$,

and $\alpha_2 = 0$,

it follows that

$\alpha_3 = \omega_3 - \omega_2$.

Furthermore,

$\mathbb{m}_3 = qk\omega_2$,

or

$\mathbb{m}_3 = \mathbb{m}_2$;

$m_3 = pk\omega_3$;

and

$M_3 = k(\omega_3 - q\alpha_3)$.

Also,

$M_3 = M_2 + \Delta M_3$

or

$M_3 = M_2 + \Delta m_3$

where

$\Delta m_3 = pk\Delta\omega_3$.

Test Moment, cont.

$t = 3$

$M_3^T = \frac{1}{q}\mathbb{m}_2 + \frac{1}{p}\Delta m_3$

With

$\mathbb{m}_2 = qk\omega_2$,

$\Delta m_3 = pk\Delta\omega_3$,

it follows that

$M_3^T = k\omega_3$.

Compare M_3^T with M_y. From Fig. B.1b, find that $M_3^T > M_y$.

∴ PLASTIC INDEX (4) = 1, i.e., in the yield range for integration step number 4.

Note that

$M_3^T = M_2^T + k\Delta\omega_3$.

Since

$M_2^T = M_y + k\omega_{ovr}$,

$M_3^T = M_y + k(\omega_{ovr} + \Delta\omega_3)$.

If

$(\omega_{ovr} + \omega_3) > 0$,

then

$M_3^T > M_y$.

As seen in Fig. B.1b,

$\Delta\omega_3 = \omega_3 - \omega_2 > 0$.

In this instance, the criteria indicate that the system is in the

Actual Moment, cont.

Test Moment, cont.

yield range during the next time interval, as it should be. Hence,

PLASTIC INDEX (4) = 1.

Integration step number 4,

calculate $\Delta\omega_4$.

$t = 4$

Since PLASTIC INDEX (4) = 1,

$\Delta\alpha_4 = \Delta\omega_4 = \omega_4 - \omega_3$,

$\mathbb{m}_4 = \mathbb{m}_2$.

Also,

$M_4 = M_3 + \Delta m_4$

where

$\Delta m_4 = pk\Delta\omega_4$.

In this time increment "back-tracking" occurred, i.e.,

$\Delta\omega_4 < 0$,

as shown in Fig. B.1b.

Hence,

$\alpha_4 < \alpha_3$

and $M_4 < M_3$.

The first increment of back-tracking is unavoidable but additional ones are undesirable.

$t = 4$

$M_4^T = \frac{1}{q}\mathbb{m}_3 + \frac{1}{p}\Delta m_4$.

With

$\mathbb{m}_3 = \mathbb{m}_2 = qk\omega_2$

and

$\Delta m_4 = pk\Delta\omega_4$,

it follows that

$M_4^T = k\omega_2 + k\Delta\omega_4$.

Since

$k\omega_2 = M_y + k\omega_{ovr}$,

$M_4^T = M_y + k(\omega_{ovr} + \Delta\omega_4)$.

As shown in Fig. B.1b,

$\Delta\omega_4 < 0$,

but

$\omega_{ovr} + \Delta\omega_4 > 0$.

Consequently, the criteria for yielding:

$M_4^T > M_y$

Actual Moment, cont.	Test Moment, cont.
	is still satisfied indicating that the system continues in the yield range: PLASTIC INDEX (5) = 1.

As long as

$$\omega_{ovr} + \Delta\omega_t > 0$$

for

$$t = 4, 5, 6, \ldots ,$$

the criteria for being in the yield range will be satisfied even though

$$\Delta\omega_t < 0 \quad \text{for} \quad t = 4, 5, 6, \ldots .$$

In this special situation, the bending moment as well as the yield angle will continue to "backtrack," that is, decrease according to the equations for the yield range when they should have returned to the linear range. The system will <u>not</u> return to the linear range until some $\Delta\omega_t$ is sufficiently negative so that

$$\omega_{ovr} + \Delta\omega_t < 0 .$$

Fortunately, $\Delta\omega_t$ is usually sufficiently negative, or what is the same, ω_{ovr} is usually sufficiently small so that this problem with the yield criteria does not often cause serious difficulties.

APPENDIX C

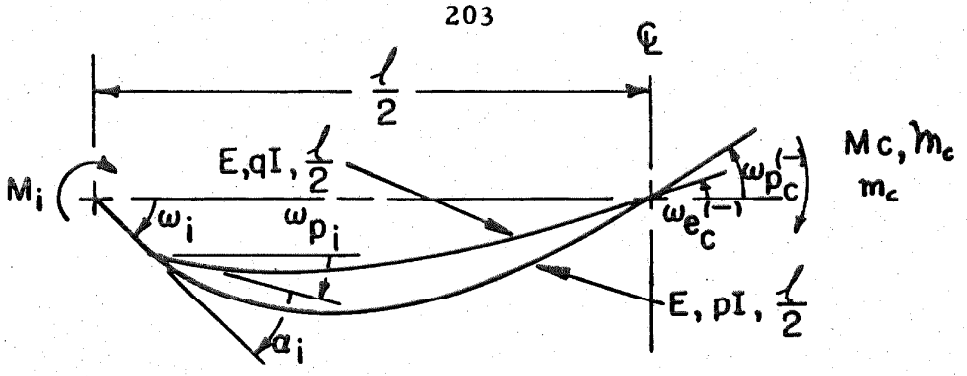
DISCUSSION OF THE TREATMENT OF SYMMETRY FOR THE TWO-COMPONENT BEAM MODEL

The purpose of this appendix is to discuss an approximate treatment of symmetry for the two-component model of the interior girder of a three-bay (four-column) structural frame. This approximation is convenient for the computer program used for the FHA Study[1]. In this treatment, the interior girder is cut at the centerline and the resulting "centerline" end, end (c), is treated the same as the end of any other beam. That is, the ends of both components of the girder are connected together and then pinned so that they rotate identically, as shown in Fig. C.1b. In this case, the sum of the bending moments of both components of the girder at end (c) is zero.

The correct treatment of symmetry is achieved by cutting the girder at the centerline and separately pinning the resulting end of each component so that they can rotate independently. In this case, the bending moment at end (c) of each component is zero, as shown in Fig. C.1a.

The nomenclature used in this appendix closely follows that for the two-component beam in Chapter II, with the usage for each system being indicated in Fig. C.1a and Fig. C.1b. The appropriate boundary conditions are also given in these figures.

The bending moment-end rotation-plastic angle equations for both the correct and approximate treatments of symmetry are given



left end:

$\Delta M_i = \Delta \mathfrak{M}_i + \Delta m_i$;

$\Delta \omega_i = \Delta \omega_{p_i} + \Delta \alpha_i$;

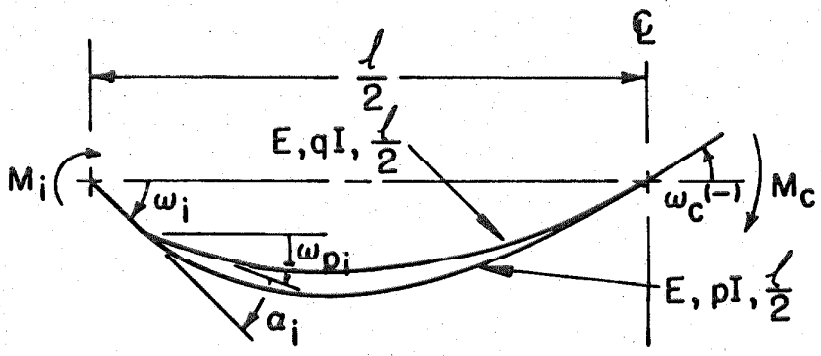
$\Delta \alpha_i \neq 0$.

"centerline" end:

$\Delta \mathfrak{M}_c = 0$; $\Delta m_c = 0$;

$\omega_{e_c} \neq \omega_{p_c}$; $\Delta \alpha_c = 0$.

Fig. C.1a Treatment of Symmetry of an Interior Girder



left end:

$\Delta M_i = \Delta \mathfrak{M}_i + \Delta m_i$;

$\Delta \omega_i = \Delta \omega_{p_i} + \Delta \alpha_i$;

$\Delta \alpha_i \neq 0$.

"centerline" end:

$\Delta M_c = \Delta \mathfrak{M}_c + \Delta m_c = 0$;

$\Delta \mathfrak{M}_c = -\Delta m_c$; $\Delta \alpha_c = 0$;

$\Delta \omega_c = \Delta \omega_{p_c} = \Delta \omega_{e_c}$.

Fig. C.1b Approximate Treatment of Symmetry of an Interior Girder

below. In both systems, the stiffness of the elastic component is:

$$pk' = p\left(\frac{EI}{\ell/2}\right) ;$$

and of the elasto-plastic component is:

$$qk' = q\left(\frac{EI}{\ell/2}\right) ;$$

where $p + q = 1$. In this case $k' = 2k$ where

$$k = \frac{EI}{\ell}$$

and ℓ is the length of the entire interior girder. Since no yielding occurs at end (c) for either system, only two yield cases are considered: end (i) is either linear or yielding.

Correct Treatment (See Fig. C.1a)	Approximate Treatment (See Fig. C.1b)
Boundary Conditions:	Boundary Conditions:
end (i):	end (i):
$\Delta M_i = \Delta \mathfrak{M}_i + \Delta m_i$,	$\Delta M_i = \Delta \mathfrak{M}_i + \Delta m_i$,
$\Delta \omega_{p_i} = \Delta \omega_i - \Delta \alpha_i$;	$\Delta \omega_{p_i} = \Delta \omega_i - \Delta \alpha_i$;
end (c):	end (c):
$\Delta \mathfrak{M}_c = 0$,	$\Delta M_c = \Delta \mathfrak{M}_c + \Delta m_c = 0$
$\Delta m_c = 0$,	or
$\Delta \omega_{p_c} \neq \Delta \omega_{e_c}$.	$\Delta \mathfrak{M}_c = - \Delta m_c$,
With these boundary conditions, the general moment-rotation	and
	$\Delta \omega_{p_c} = \Delta \omega_{e_c} = \Delta \omega_c$.

Correct Treatment, cont.

equations in incremental form are:

for the elasto-plastic component:

$$\Delta \mathfrak{M}_i = qk'[\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}\Delta\omega_{p_c}]$$

$$\Delta \mathfrak{M}_c = qk'[\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + \Delta\omega_{p_c}];$$

for the elastic component:

$$\Delta m_i = pk'[\Delta\omega_i + \tfrac{1}{2}\Delta\omega_{e_c}]$$

$$\Delta m_c = pk'[\tfrac{1}{2}\Delta\omega_i + \Delta\omega_{e_c}] .$$

Apply boundary conditions at end (c):

$$\Delta \mathfrak{M}_c = 0$$

and

$$\Delta m_c = 0 .$$

Hence,

$$\Delta\omega_{p_c} = -\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) ,$$

$$\Delta\omega_{e_c} = -\tfrac{1}{2}\Delta\omega_i .$$

For the total beam at end (i):

$$\Delta M_i = \Delta \mathfrak{M}_i + m_i .$$

Eliminating $\Delta\omega_{p_c}$ and $\Delta\omega_{e_c}$, this becomes

$$\Delta M_i = \tfrac{3}{4} k'(\Delta\omega_i - q\Delta\alpha_i) .$$

Approximate Treatment, cont.

With these boundary conditions, the general moment-rotation equations in incremental form are:

for the elasto-plastic component:

$$\Delta \mathfrak{M}_i = qk'[(\Delta\omega_i - \Delta\alpha_i) + \tfrac{1}{2}\Delta\omega_c]$$

$$\Delta \mathfrak{M}_c = qk'[\tfrac{1}{2}(\Delta\omega_i - \Delta\alpha_i) + \Delta\omega_c] ;$$

for the elastic component:

$$\Delta m_i = pk'[\Delta\omega_i + \tfrac{1}{2}\Delta\omega_c]$$

$$\Delta m_c = pk'[\tfrac{1}{2}\Delta\omega_i + \Delta\omega_c] .$$

Apply boundary conditions at end (c):

$$\Delta M_c = \Delta \mathfrak{M}_c + \Delta m_c = 0 .$$

Hence

$$\Delta\omega_c = -\tfrac{1}{2}(\Delta\omega_i - q\Delta\alpha_i).$$

For the total beam at end (i):

$$\Delta M_i = \Delta \mathfrak{M}_i + \Delta m_i .$$

Eliminating $\Delta\omega_c$, this becomes

$$\Delta M_i = \tfrac{3}{4} k'(\Delta\omega_i - q\Delta\alpha_i) .$$

Correct Treatment, cont.

Since

$k' = 2k$,

where $k = \frac{EI}{\ell}$ is the stiffness of the entire interior girder,

$\Delta M_i = 1.5(\Delta\omega_i - q\Delta\alpha_i)$.

The moment-rotation equation for the total beam at end (i) for the linear case

with

$\Delta\alpha_i = 0$,

is

$\Delta M_i = 1.5\, k\Delta\omega_i$.

Approximate Treatment, cont.

Since

$k' = 2k$,

where $k = \frac{EI}{\ell}$ is the stiffness of the entire interior girder,

$\Delta M = 1.5(\Delta\omega_i - q\Delta\alpha_i)$.

The moment-rotation equation for the total beam at end (i) for the linear case

with

$\Delta\alpha_i = 0$,

is

$\Delta M_i = 1.5\, k\Delta\omega_i$.

These equations for both systems are the same in the linear case.

Incremental plastic angle at end (i). During yielding, for the two-component beam (see Sec. 2.5),

$\Delta M_i = \Delta m_i$

or

$1.5k[\Delta\omega_i - q\Delta\alpha_i] = 1.5pk\Delta\omega_i$.

Incremental plastic angle at end (i). During yielding, for the the two-component beam (see Sec. 2.5),

$\Delta M_i = \Delta m_i$

or

$1.5k[\Delta\omega_i - q\Delta\alpha_i] = 1.5pk[\Delta\omega_i + \frac{q}{3}\Delta\alpha_i]$.

Correct Treatment, cont.

Hence

$\Delta\alpha_i = \Delta\omega_i$.

Moment-rotation equation for the total beam at end (i) during yielding. By eliminating $\Delta\alpha_i$, this equation becomes:

$\Delta M_i = 1.5\,pk\Delta\omega_i$.

Approximate Treatment, cont.

Hence

$\Delta\alpha_i = \left(\frac{3}{3+p}\right)\Delta\omega_i$.

Moment rotation equation for the total beam at end (i) during yielding. By eliminating $\Delta\alpha_i$, this equation becomes:

$\Delta M_i = \left(\frac{6}{3+p}\right) pk\Delta\omega_i$.

For $p = 0.05$, as used in this report,

$\Delta M_i = 1.97\,pk\Delta\omega_i$.

Comparing these two moment-rotation equations, it is observed that for the value of (p) used, the effective stiffness of the approximate technique during yielding is about 4/3 times that for the correct method. Because each joint of the frame is in the linear case about 90% of the time, and because the equations are the same for both systems in the linear case, only small differences in the structural responses occur. By comparing the computer results for the two methods with each other, these differences are seen to be on order of 1%, being small enough to justify the use of the approximate technique for treating symmetry.

APPENDIX D

FUTURE STUDIES

As a result of the investigation described in this report, several analyses are suggested which would provide additional understanding of how multi-story structures respond to earthquake excitation. These are briefly mentioned below.

1) Stiffness proportional damping in the form of Eq. 3.24 could be added to the computer program, thereby making it possible to observe the effect of this type of damping on the response. Combinations of the two types of damping would make it possible to match more closely the modal damping distribution in actual buildings.

2) An energy balance subroutine could be developed and added to the program. Such a balance might consist of the energy in (and out) at the base, dissipation by yielding, dissipation by damping mechanisms, kinetic energy and (recoverable) potential energy. Such an analysis would provide insight into the distribution of energy dissipation in a multi-story structure.

3) Instead of bilinear hysteresis loops which inherently have a sharp corner at the yield level, curvilinear hysteresis loops could be used with the one-component beam model. This would permit the effect of the sharp corner upon the response of the structure to be observed.

4) Analyses of nonlinear elastic, shear type, base excited, continuous cantilever beams might be considered. The excitation could be either pulse, sinusoidal, or earthquake.

5) Static moments could be applied to the ends of the girders to account for static loads in the structure. This would require the use of the entire frame because the yielding characteristics would no longer be symmetric with respect to the centerline.

APPENDIX E

LISTING OF THE COMPUTER PROGRAM

The computer program used for determining the response of nonlinear multi-story structures subjected to earthquake excitation is listed in this appendix. A flow chart is also provided. Numerous comment cards are used so that the program will be understandable to the reader with some background in Fortran II or Fortran IV.

This program was written in Fortran IV for use on the IBM 7090/7094 Computer at the Computer Center of the California Institute of Technology. The computer has a 32,000-cell memory core which is filled by this program and the necessary library subroutines. Alterations may be necessary to adapt this program to the computers at other installations.

A typical run in which the nonlinear A/20/2.2/2/6 frame was subjected to 30 seconds of pseudo-earthquake took 40 minutes of computer time. Since the time increment is 0.005 sec (earthquake time), there were 6000 integration steps. After every fourth time interval, most of the responses of the structure were written onto binary tape. After every second of earthquake time, the most positive and most negative (cumulative) values of all of the responses, along with the times at which they occurred, were written out on paper.

The resulting 2400 ft, 560 bit-per-inch binary tape was scanned to obtain the time histories of the responses to be plotted. The time history for each response was punched into a deck of cards.

The plots were made from these decks. The time necessary to scan a tape for 30 responses was about 10 minutes. Each page of four response plots took the computer 30 sec to set up; however, it took 7 minutes to actually plot.

When a modal analysis was made, the "equivalent" modal participation factors, the "equivalent" modal contributions to four displacements and to four interfloor displacements were found. This information was calculated and written out for every time increment. Such an analysis doubles the computer time required. One run of this nature was made for the first four seconds of the El Centro (N-S) earthquake of 18 May 1940.

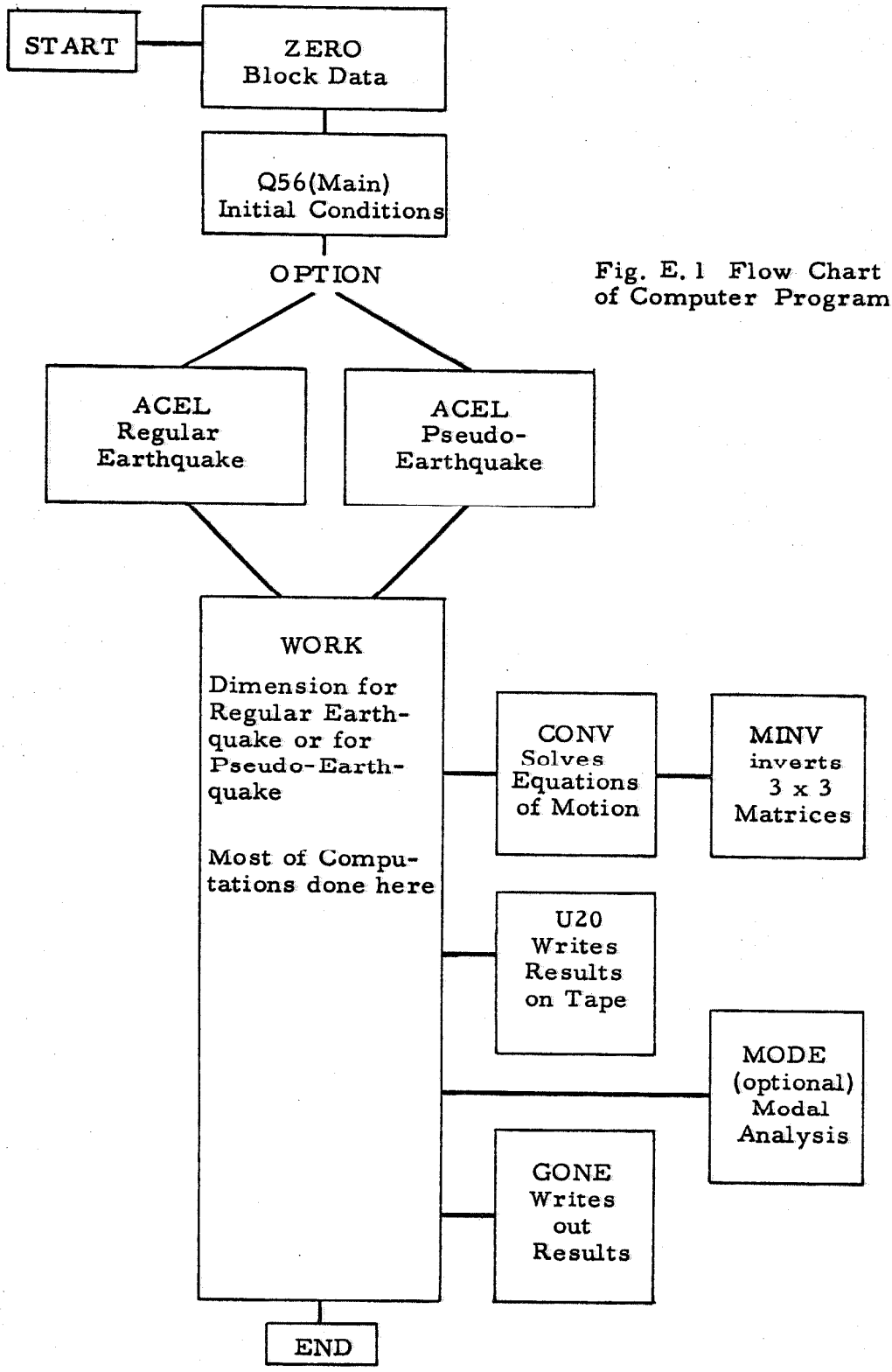


Fig. E.1 Flow Chart of Computer Program

```
$IBFTC Q56      DECK *INITIAL VALUES AND STRUCTURAL CONSTANTS*        Q56 MAIN   1
C                                                                       MAIN   2
C***********************************************************************MAIN   3
C                                                                       MAIN   4
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES     MAIN   5
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.  MAIN   6
C                                                                       MAIN   7
C     THE BILINEAR HYSTERETIC BENDING MOMONT - JOINT ROTATION - YIELD   MAIN   8
C     ANGLE RELATIONSHIP AT EACH END OF EACH BEAM (GIRDER AND COLUMN),  MAIN   9
C     THE DUCTILITY FACTOR, AND THE CRITERIA FOR ESTABLISHING THE       MAIN  10
C     STATE OF YIELD ARE BASED UPON THE 'SINGLE COMPONENT BEAM' MODEL    MAIN  11
C     BY GIBERSON.                                                      MAIN  12
C                                                                       MAIN  13
C     THE PURPOSE OF THE MAIN PROGRAM IS TO READ IN STRUCTURAL AND      MAIN  14
C     OTHER DATA, TO EVALUATE THE STRUCTURAL CONSTANTS, STIFFNESS       MAIN  15
C     PARAMETERS, AND INITIAL YIELD PARAMETERS.                         MAIN  16
C                                                                       MAIN  17
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE   MAIN  18
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-   MAIN  19
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                         MAIN  20
C                  LAST REVISION - 11 JANUARY 1967                      MAIN  21
C                                                                       MAIN  22
C***********************************************************************MAIN  23
C                                                                       MAIN  24
C     MUST USE 'ZCORE' OPTION ON IBJOB CARD.  THIS PLACES A '0' (ZERO)  MAIN  25
C     IN EACH MEMORY CELL OF CORE PRIOR TO LOADING.                     MAIN  26
C                                                                       MAIN  27
C***********************************************************************MAIN  28
C                                                                       MAIN  29
C                     CONTROL PARAMETERS                                MAIN  30
C                                                                       MAIN  31
C   NAMEQK     DESCRIPTION OF THE EARTHQUAKE EXCITATION RECORD          MAIN  32
C   TIMEND     TIME AT END OF INTEGRATION   (TIMEND .LE. ENDREC)        MAIN  33
C   P          RATIO OF SECOND SLOPE TO FIRST                           MAIN  34
C   F          =1.5*P/(1.0-P) , USED IN EQN 'DBM = F*DALP' WHEN YIELDING MAIN  35
C   IELPL      =0 FOR ELASTIC RESPONSE, =1 FOR NON-LINEAR RESPONSE      MAIN  36
C   LRTEST     WRITE RESULTS ON TAPE EVERY 'LRTEST' TIME INTERVALS      MAIN  37
C   NAMEST     DESCRIPTION OF THE STRUCTURE ANALYZED                    MAIN  38
C   20         NUMBER OF FLOORS IN STRUCTURE                            MAIN  39
C   N=2        TOP FLOOR                                                MAIN  40
C   N=21       FIRST FLOOR                                              MAIN  41
C   N=22       GROUND LEVEL                                             MAIN  42
C              DIMENSION OF THE ARRAYS IN THE VERTICAL DIRECTION IS  22 MAIN  43
C   NCARD      NUMBER OF  POINTS OF ACCELERATION RECORD READ IN         MAIN  44
C   DBDT       TIME INTERVAL (DOUBLE PRECISION) USED IN INTEGRATION     MAIN  45
C   ENDREC     END OF TIME OF ACCELERATION RECORD AS SET-UP BY SUBROU-   MAIN  46
C                 TINE 'ACEL'  (TIMEND .LE. ENDREC)                     MAIN  47
C   IOPTON     =0 FOR NO PRINT OUT OF ACCELEROGRAM, =1 FOR PRINT OUT    MAIN  48
C   SCALE      RATIO OF RMS OF ACCELEROGRAM USED TO ACCELEROGRAM OF     MAIN  49
C                 EL CENTRO (N-S), 18 MAY 1940, (CALTECH RECORD)        MAIN  50
C                                                                       MAIN  51
C***********************************************************************MAIN  52
C                                                                       MAIN  53
C                      UNITS OF VALUES READ IN                          MAIN  54
C                                                                       MAIN  55
C   ZETA            NONE              FRACT OF CRIT DAMPING MODE 1      MAIN  56
C   FUNDFQ          RAD/SEC           FUND FREQ OF LINEAR STRUCTURE     MAIN  57
C   CE,CI,GE,GI     IN**4             MOM OF INERTIA OF COLS AND GIRDS  MAIN  58
C   BMA(N,M,J)      KIP*IN            MAG OF ORIGINAL YIELD MOM         MAIN  59
C   ZM(N)           KIP*(SEC**2)/IN   1/2 OF THE MASS OF THE FLOOR      MAIN  60
C   HT(N)           INCHES            HEIGHT BETWEEN FLOORS             MAIN  61
C   E               KIPS/(IN**2)      YOUNGS MODULUS                    MAIN  62
C   ZL              INCHES            DIST BETWEEN COLS. IN FRAME       MAIN  63
C                                                                       MAIN  64
C           THE UNIT SYSTEM USED IN ALL COMPUTATIONS AND PRINT-OUTS IS  MAIN  65
C                    'INCH,LBS,SEC'.                                    MAIN  66
C                                                                       MAIN  67
C***********************************************************************MAIN  68
C                                                                       MAIN  69
C              MUST HAVE INPUT CARDS IN THIS  ORDER                     MAIN  70
C                                                                       MAIN  71
C              NAMEQK                                                   MAIN  72
C              TIMEND,P,IELPL,LRTEST                                    MAIN  73
C              ZETA,FUNDFQ                                              MAIN  74
C              NAMEST                                                   MAIN  75
C              E,ZL                                                     MAIN  76
C              STRUCTURAL CONSTANTS DECK                                MAIN  77
C              NCARD,DBDT,ENDREC,IOPTON,SCALE                           MAIN  78
C              TIME - ACCELERATION DECK                                 MAIN  79
C              MODAL ANALYSIS DECK - OPTIONAL                           MAIN  80
C                 EIGENVECTORS AND EIGENVALUES COME FROM PROGRAM 'Q33'   MAIN  81
C                 FOR EIGENVECTOR NORMALIZATION, SEE SUBROUTINE 'MODE'   MAIN  82
C                                                                       MAIN  83
C***********************************************************************MAIN  84
C                                                                       MAIN  85
      DIMENSION CE(22),CI(22),GE(22),GI(22)                             MAIN  86
C                                                                       MAIN  87
      COMMON/MNWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4),      MAIN  88
     2 GA(22,3),CA(22,3),HT(22),H(22),ZM(22),A(5),S(8),P,F,ZETA,FUNDFQ, MAIN  89
     3 IL,AL,TIMENC,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                     MAIN  90
C                                                                       MAIN  91
      COMMON/MNGONE/ YIELD(22,3,4)                                      MAIN  92
C                                                                       MAIN  93
C..... READ IN DATA, AND EVALUATE OTHER CONSTANTS .....                 MAIN  94
C                                                                       MAIN  95
      READ(5,1010) NAMEQK                                               MAIN  96
1010  FORMAT(13A6)                                                      MAIN  97
C                                                                       MAIN  98
      READ(5,1020)  TIMEND,P,IELPL,LRTEST                               MAIN  99
1020  FORMAT(10X,2F10.5,2I10)                                           MAIN 100
C                                                                       MAIN 101
      READ(5,1030) ZETA,FUNDFQ                                          MAIN 102
1030  FORMAT(10X,2F10.0)                                                MAIN 103
      READ(5,1011) NAMEST                                               MAIN 104
1011  FORMAT(10A6)                                                      MAIN 105
      READ(5,1030) E,ZL                                                 MAIN 106
      READ(5,1050)  (CE(N),BMA(N,2,4),CI(N),BMA(N,3,4),GE(N),BMA(N,2,3),MAIN 107
     2  GI(N),BMA(N,3,3),HT(N),ZM(N), N=2,21)                           MAIN 108
1050  FORMAT(8F8.0,2F6.0,4X)                                            MAIN 109
      ZKIP=1000.0                                                       MAIN 110
      AL=1.0/ZL                                                         MAIN 111
      E=E*ZKIP                                                          MAIN 112
      EL=E*AL                                                           MAIN 113
      ALAMDA = ZETA*FUNDFQ                                              MAIN 114
      BLAMDA = 2.0*ZETA*FUNDFQ                                          MAIN 115
C                                                                       MAIN 116
C..... EVALUATE YIELD MOMENTS, GIRDER AND COLUMN CONSTANTS, AND .....  MAIN 117
C..... INCIPIENT YIELD ANGLES.....                                     MAIN 118
C                                                                       MAIN 119
      DO 50 N=2,21                                                      MAIN 120
      N1=N+1                                                            MAIN 121
      ZM(N)=ZM(N)*ZKIP                                                  MAIN 122
      H(N) = 1.0/HT(N)                                                  MAIN 123
      BMA(N,2,4)=ZKIP*BMA(N,2,4)                                        MAIN 124
      BMA(N,3,4)=ZKIP*BMA(N,3,4)                                        MAIN 125
      BMA(N,2,3)=ZKIP*BMA(N,2,3)                                        MAIN 126
      BMA(N,3,3)=ZKIP*BMA(N,3,3)                                        MAIN 127
      BMA(N,3,1)=BMA(N,2,3)                                             MAIN 128
      GA(N,2)=4.0*EL*GE(N)                                              MAIN 129
      GA(N,3)=4.0*EL*GI(N)                                              MAIN 130
      EH=E/HT(N)                                                        MAIN 131
      CA(N,2)=4.0*EH*CE(N)                                              MAIN 132
      CA(N,3)=4.0*EH*CI(N)                                              MAIN 133
      BMA(N1,2,2)=BMA(N,2,4)                                            MAIN 134
      BMA(N1,3,2)=BMA(N,3,4)                                            MAIN 135
      DO 40 M=2,3                                                       MAIN 136
      YIELD(N,M,3)= BMA(N,M,3)/(1.5*GA(N,M))                            MAIN 137
      YIELD(N,M,4)= BMA(N,M,4)/(1.5*CA(N,M))                            MAIN 138
      YIELD(N1,M,2)=YIELD(N,M,4)                                        MAIN 139
40    CONTINUE                                                          MAIN 140
```

```
      YIELD(N,3,1)=YIELD(N,2,3)                                          MAIN 141
50    CONTINUE                                                           MAIN 142
C                                                                        MAIN 143
      DO 60 M=1,3                                                        MAIN 144
      DO 60 J=1,4                                                        MAIN 145
      DO 60 N=2,22                                                       MAIN 146
      BMB(N,M,J) = -BMA(N,M,J)                                           MAIN 147
60    CONTINUE                                                           MAIN 148
C                                                                        MAIN 149
C..... EVALUATE BILINEAR STIFFNESS PARAMETERS 'F, A, S' ..............MAIN 150
C                                                                        MAIN 151
      F = 1.5*P/(1.0-P)                                                  MAIN 152
C                                                                        MAIN 153
      A(1) = 1.0/(1.0 + F)                                               MAIN 154
      A(2) = 0.5*A(1)                                                    MAIN 155
      D = 1.0 + 2.6666667*F + 1.3333333*F*F                              MAIN 156
      A(3) = (1.0 + 1.3333333*F)/D                                       MAIN 157
      A(4) = 0.66666667*F/D                                              MAIN 158
      A(5) = (1.0 + 2.0*F)/D                                             MAIN 159
C                                                                        MAIN 160
      S(3) = F*A(1)                                                      MAIN 161
      S(4) = F*A(2)                                                      MAIN 162
      S(5) = (0.75 + F)/(1.0 + F)                                        MAIN 163
      S(6) = F*A(3)                                                      MAIN 164
      S(7) = F*A(4)                                                      MAIN 165
      S(8) = F*A(5)                                                      MAIN 166
C                                                                        MAIN 167
C..... WRITE OUT INPUT DATA AND INITIAL CONDITIONS ....................MAIN 168
C                                                                        MAIN 169
      WRITE(6,1069) A,S,F,P,D                                            MAIN 170
1069  FORMAT((10X,1PE17.7))                                              MAIN 171
C                                                                        MAIN 172
      WRITE(6,1003) NAMES1,P,ZETA,FUNDFQ,TIMEND,NAMEQK                   MAIN 173
1003  FORMAT(1H1 ,10A6,15H,BILINEARITY =  ,F5.3,16H,FRAC CR DAMP =   ,   MAIN 174
     2 F7.5,13H,FUND FREQ =  ,F7.5, 8H RAD/SEC  ,//,                    MAIN 175
     3 5X, 15HUSING THE FIRST ,F10.5, 15H SECONDS OF THE , 13A6,2X,//)   MAIN 176
      WRITE(6,1060)                                                      MAIN 177
1060  FORMAT( 132H STORY   CE       BMA       YIELD        CI       BMA  MAIN 178
     2 YIELD        GE       BMA       YIELD        GI       BMA       YIELD  MAIN 179
     3   HT    ZM         )                                             MAIN 180
      NN=21                                                              MAIN 181
      DO 70 N=2,22                                                       MAIN 182
      NN = NN-1                                                          MAIN 183
      WRITE(6,1070) NN,CE(N),BMA(N,2,4),YIELD(N,2,4),CI(N),BMA(N,3,4),   MAIN 184
     2 YIELD(N,3,4),GE(N),BMA(N,2,3),YIELD(N,2,3),GI(N),BMA(N,3,3),      MAIN 185
     3 YIELD(N,3,3),HT(N),ZM(N)                                          MAIN 186
1070  FORMAT(1X,I3,4(0PF8.0,0PF11.0,1PE10.3), 0P2F6.0)                   MAIN 187
70    CONTINUE                                                           MAIN 188
C                                                                        MAIN 189
      WRITE(6,1002)                                                      MAIN 190
1002  FORMAT(1H0)                                                        MAIN 191
      WRITE(6,1080)                                                      MAIN 192
1080  FORMAT(12X,7HCA(N,2), 10X,7HCA(N,3), 10X,7HGA(N,2), 10X,7HGA(N,3))MAIN 193
      NN=21                                                              MAIN 194
      DO 80 N=2,22                                                       MAIN 195
      NN = NN-1                                                          MAIN 196
      WRITE(6,1090)NN,CA(N,2),CA(N,3),GA(N,2),GA(N,3)                    MAIN 197
1090  FORMAT(2X,I3,1P4E17.7)                                             MAIN 198
80    CONTINUE                                                           MAIN 199
C                                                                        MAIN 200
      CALL ACEL                                                          MAIN 201
      CALL WORK                                                          MAIN 202
      STOP                                                               MAIN 203
      END                                                                MAIN 204
```

```
$IBFTC ACEL    DECK *ACCELEROGRAM FROM REGULAR EARTHQUAKE*          Q56 ACRG   1
      SUBROUTINE ACEL                                                ACRG   2
C                                                                    ACRG   3
C*********************************************************************ACRG   4
C                                                                    ACRG   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES  ACRG   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.ACRG   7
C                                                                    ACRG   8
C     EACH OF THE  TWO  'ACEL' SUBROUTINES SETS-UP AN ACCELEROGRAM   ACRG   9
C     WITH UNITS OF 'FRACTIONS OF G' AT EQUAL TIME INTERVALS 'DBDT'  ACRG  10
C     FOR USE BY SUBROUTINE 'WORK'.  THE DIFFERENCE BETWEEN THEM IS  ACRG  11
C     THE FORMAT OF THE INPUT ACCELEROGRAM.                          ACRG  12
C        THE '(JENNINGS) PSEUDO-EARTHQUAKE' IS GIVEN WITH VALUES OF  ACRG  13
C        ACCELERATION IN UNITS OF 'FT/(SEC**2)' AT EQUAL TIME INTER- ACRG  14
C        VALS OF 0.025 SEC.  THE CORRESPONDING TIMES MUST BE GENERATED.ACRG 15
C        THE REGULAR EARTHQUAKE ACCELEROGRAMS ARE GIVEN IN           ACRG  16
C        'TIME-ACCELERATION' PAIRS AT UNEQUAL TIME INTERVALS AND THE ACRG  17
C        UNITS OF ACCELERATION ARE 'FRACTIONS OF G'.                 ACRG  18
C                                                                    ACRG  19
C     THIS 'ACEL' SUBROUTINE HANDLES THE 'REGULAR EARTHQUAKES'.      ACRG  20
C                                                                    ACRG  21
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE ACRG  22
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-ACRG  23
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                      ACRG  24
C                  LAST REVISION - 4 DECEMBER 1966                   ACRG  25
C                                                                    ACRG  26
C*********************************************************************ACRG  27
C                                                                    ACRG  28
      DOUBLE PRECISION DBTIME,DBDT,ED,DBTIM3,DBZK                    ACRG  29
C                                                                    ACRG  30
      COMMON/ACEWOR/  DBTIME,DBDT,ED,DBTIM3,IOPTON,                  ACRG  31
     2 TIM3(601),DDY3(601),DDY1(6011), KEND                         ACRG  32
C                                                                    ACRG  33
      READ(5,1110)  NCARD,DBDT,ENDREC,IOPTON,SCALE                   ACRG  34
1110  FORMAT(I10,D10.0,F10.0,I10,F10.0)                              ACRG  35
      READ(5,1120) (TIM3(I),DDY3(I),I=1,NCARD)                       ACRG  36
1120  FORMAT(10X,2F10.0)                                             ACRG  37
      ED = DBLE(ENDREC)/DBDT                                         ACRG  38
      DT = SNGL(DBDT)                                                ACRG  39
      KEND = SNGL(ED) + 1.5                                          ACRG  40
      INC = KEND - 1                                                 ACRG  41
      WRITE(6,1130) NCARD,DT,ENDREC,INC                              ACRG  42
1130  FORMAT(1H1, 25H NO. TIME-ACCEL. PAIRS =  ,I4,//,               ACRG  43
     2  26H    EACH TIME INCREMENT = ,F12.8, 5H SEC. ,//,           ACRG  44
     3 26H          END OF RECORD =  ,F12.8, 5H SEC. , //,         ACRG  45
     4 26H    NO. TIME INCREMENTS =  ,I4  )                         ACRG  46
      TIME = TIM3(1)                                                 ACRG  47
      DBTIM3 = DBLE(TIM3(1))                                         ACRG  48
      DBTIME = DBTIM3                                                ACRG  49
      L = 1                                                          ACRG  50
      K = 1                                                          ACRG  51
110   L = L+1                                                        ACRG  52
      DDT = TIM3(L) - TIM3(L-1)                                      ACRG  53
      DDP = DDY3(L) - DDY3(L-1)                                      ACRG  54
      IF(DDT.LE.0.0)  GO TO 110                                      ACRG  55
      SLOPE = DDP/DDT                                                ACRG  56
120   IF(TIME.GT.TIM3(L))  GO TO 110                                 ACRG  57
      DDY1(K) =(DDY3(L-1) + (TIME - TIM3(L-1))*SLOPE)*SCALE          ACRG  58
      IF(IOPTON.EQ.0)  GO TO 125                                     ACRG  59
      WRITE(6,1140)   K,TIME,DDY1(K)                                 ACRG  60
1140  FORMAT(   8X,I4,2X,2E15.8)                                     ACRG  61
125   CONTINUE                                                       ACRG  62
      ZK = K                                                         ACRG  63
      DBZK = DBLE(ZK)                                                ACRG  64
      DBTIME = DBTIM3 + DBZK*DBDT                                    ACRG  65
      TIME = SNGL(DBTIME)                                            ACRG  66
      K = K+1                                                        ACRG  67
      IF(KEND.GE.K)  GO TO 120                                       ACRG  68
130   CONTINUE                                                       ACRG  69
      RETURN                                                         ACRG  70
      END                                                            ACRG  71
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$IBFTC ACEL    DECK *ACCELEROGRAM FROM PSEUDO-EARTHQUAKE*          Q56 ACPS   1
      SUBROUTINE ACEL                                                ACPS   2
C                                                                    ACPS   3
C*********************************************************************ACPS   4
C                                                                    ACPS   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES  ACPS   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION. ACPS  7
C                                                                    ACPS   8
C     EACH OF THE  TWO  'ACEL' SUBROUTINES SETS-UP AN ACCELEROGRAM   ACPS   9
C     WITH UNITS OF 'FRACTIONS OF G' AT EQUAL TIME INTERVALS 'DBDT'  ACPS  10
C     FOR USE BY SUBROUTINE 'WORK'.  THE DIFFERENCE BETWEEN THEM IS  ACPS  11
C     THE FORMAT OF THE INPUT ACCELEROGRAM.                          ACPS  12
C        THE '(JENNINGS) PSEUDO-EARTHQUAKE' IS GIVEN WITH VALUES OF  ACPS  13
C        ACCELERATION IN UNITS OF 'FT/(SEC**2)' AT EQUAL TIME INTER- ACPS  14
C        VALS OF 0.025 SEC.  THE CORRESPONDING TIMES MUST BE GENERATED. ACPS 15
C        THE REGULAR EARTHQUAKE ACCELEROGRAMS ARE GIVEN IN           ACPS  16
C        'TIME-ACCELERATION' PAIRS AT UNEQUAL TIME INTERVALS AND THE ACPS  17
C        UNITS OF ACCELERATION ARE 'FRACTIONS OF G'.                 ACPS  18
C                                                                    ACPS  19
C    THIS 'ACEL' SUBROUTINE HANDLES THE '(JENNINGS) PSEUDO-EARTHQUAKES'.ACPS 20
C                                                                    ACPS  21
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE ACPS  22
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- ACPS  23
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                      ACPS  24
C                  LAST REVISION - 4 DECEMBER 1966                   ACPS  25
C                                                                    ACPS  26
C*********************************************************************ACPS  27
C                                                                    ACPS  28
      EQUIVALENCE (DDY1(4E11),DDY3(1))                               ACPS  29
C                                                                    ACPS  30
      DOUBLE PRECISION DBTIME,DBDT,ED,DBTIM3,DBZK                    ACPS  31
      DOUBLE PRECISION DBDT3,DBZII                                   ACPS  32
C                                                                    ACPS  33
      DIMENSION NAME(13),TIM3(1201),DDY3(1201)                       ACPS  34
C                                                                    ACPS  35
      COMMON/ACEWOR/  DBTIME,DBDT,ED,DBTIM3,IOPTON,                  ACPS  36
     2 DDY1(6011), KEND                                              ACPS  37
C                                                                    ACPS  38
      READ(5,1110)  NCARD,DBDT,ENDREC,IOPTON,SCALE                   ACPS  39
1110  FORMAT(I10,D10.0,F10.0,I10,F10.0)                              ACPS  40
C     READ(5,1120) (TIM3(I),DDY3(I),I=1,NCARD)                       ACPS  41
C1120 FORMAT(10X,2F10.0)                                             ACPS  42
C                                                                    ACPS  43
C .....  THE FOLLOWING STATEMENTS READ IN THE PSUEDO-EARTHQUAKES .....ACPS 44
C                                                                    ACPS  45
      READ(5,1123) NAME                                              ACPS  46
1123  FORMAT(13A6)                                                   ACPS  47
      READ(5,1125) DDY3                                              ACPS  48
1125  FORMAT(6F11.7,14X)                                             ACPS  49
      DBDT3 = 0.025D+00                                              ACPS  50
      SSCALE = 3.16*SCALE                                            ACPS  51
      DO 100 I = 1,NCARD                                             ACPS  52
      ZII = I - 1                                                    ACPS  53
      DBZII = DBLE(ZII)                                              ACPS  54
      TIM3(I) = SNGL(DBZII*DBDT3)                                    ACPS  55
100   CONTINUE                                                       ACPS  56
      WRITE(6,1123) NAME                                             ACPS  57
      WRITE(6,1127) (I,TIM3(I),DDY3(I),I=1,NCARD)                    ACPS  58
1127  FORMAT(10X,I5,F12.6,F12.7)                                     ACPS  59
      DO 105 I=1,NCARD                                               ACPS  60
      DDY3(I) = DDY3(I)*SSCALE/32.171667                             ACPS  61
105   CONTINUE                                                       ACPS  62
      DDT = SNGL(DBDT3)                                              ACPS  63
C                                                                    ACPS  64
      ED = DBLE(ENDREC)/DBDT                                         ACPS  65
      DT = SNGL(DBDT)                                                ACPS  66
      KEND = SNGL(ED) + 1.5                                          ACPS  67
      INC = KEND - 1                                                 ACPS  68
      WRITE(6,1130) NCARD,DT,ENDREC,INC                              ACPS  69
1130  FORMAT(1H1, 25H NO. TIME-ACCEL. PAIRS =  ,I4,//,               ACPS  70
     2  26H    EACH TIME INCREMENT = ,F12.8, 5H SEC. ,//,            ACPS  71
     3 26H          END OF RECORD =  ,F12.8, 5H SEC. , //,          ACPS  72
     4 26H    NO. TIME INCREMENTS =  ,I4  )                         ACPS  73
      TIME = TIM3(1)                                                 ACPS  74
      DBTIM3 = DBLE(TIM3(1))                                         ACPS  75
      DBTIME = DBTIM3                                                ACPS  76
      L = 1                                                          ACPS  77
      K = 1                                                          ACPS  78
110   L = L+1                                                        ACPS  79
C     DDT = TIM3(L) - TIM3(L-1)                                      ACPS  80
      DDP = DDY3(L) - DDY3(L-1)                                      ACPS  81
      IF(DDT.LE.0.0)  GO TO 110                                      ACPS  82
      SLOPE = DDP/DDT                                                ACPS  83
120   IF(TIME.GT.TIM3(L))  GO TO 110                                 ACPS  84
      DDY1(K) = DDY3(L-1) + (TIME - TIM3(L-1))*SLOPE                 ACPS  85
      IF(IOPTON.EQ.0)  GO TO 125                                     ACPS  86
      WRITE(6,1140)   K,TIME,DDY1(K), TIM3(L),DDY3(L)                ACPS  87
1140  FORMAT(   8X,I4,2X,1P4E15.8)                                   ACPS  88
125   CONTINUE                                                       ACPS  89
      ZK = K                                                         ACPS  90
      DBZK = DBLE(ZK)                                                ACPS  91
      DBTIME = DBTIM3 + DBZK*DBDT                                    ACPS  92
      TIME = SNGL(DBTIME)                                            ACPS  93
      K = K+1                                                        ACPS  94
      IF(KEND.GE.K)  GO TO 120                                       ACPS  95
130   CONTINUE                                                       ACPS  96
      RETURN                                                         ACPS  97
      END                                                            ACPS  98
```

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$IBFTC WORK     DECK     ** MOST OF THE WORK IS DONE HERE**          Q56 WORK   1
      SUBROUTINE WORK                                                WORK   2
C                                                                    WORK   3
C*********************************************************************WORK   4
C                                                                    WORK   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES  WORK   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.WORK   7
C                                                                    WORK   8
C     THE PURPOSE OF SUBROUTINE 'WORK' IS TO EVALUATE THE PARAMETERS WORK   9
C     NEEDED BY SUBROUTINE 'CONV' IN ORDER TO SOLVE THE EQUATIONS OF WORK  10
C     MOTION FOR EACH TIME INTERVAL.  THEN THE VARIOUS RESPONSES ARE WORK  11
C     MONITORED CUMULATIVELY FROM TIME ZERO FOR THE MOST POSITIVE AND WORK  12
C     MOST NEGATIVE VALUES AND THE TIMES THEY OCCUR.  AFTER EVERY    WORK  13
C     SECOND OF REAL TIME AND AT THE END OF THE RUN THESE MAXIMA AND WORK  14
C     TIMES ARE WRITTEN OUT BY SUBROUTINE 'GONE'.                    WORK  15
C                                                                    WORK  16
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE WORK  17
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-WORK  18
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                      WORK  19
C                  LAST REVISION - 11 JANUARY 1967                   WORK  20
C                                                                    WORK  21
C*********************************************************************WORK  22
C                                                                    WORK  23
      DOUBLE PRECISION DBU,DBUU                                      WORK  24
      DOUBLE PRECISION DBTIME,DBDT,ED,DBTIM3,DBZJJ                   WORK  25
C                                                                    WORK  26
      DIMENSION   TACC(22),AXCLD(22,3),GSR(22,3),SHEAR(22),          WORK  27
     2 L(22,3,4),ALP(22,3,4),BM(22,3,4),BMLAST(22,3,4),BMT(22,3),    WORK  28
     3 OTMOM(22),SRT(22)                                             WORK  29
C                                                                    WORK  30
      COMMON/WRKGON/UMAX(22),UMIN(22),TMXU(22),TMNU(22),             WORK  31
     2 VMAX(22),VMIN(22),TMXV(22),TMNV(22),                          WORK  32
     3 UUMAX(22),UUMIN(22),TMXUU(22),TMNUU(22),                      WORK  33
     4 SRMAX(22),SRMIN(22),TMXSR(22),TMNSR(22),                      WORK  34
     5 WMAX(22,3),WMIN(22,3),TMXW(22,3),TMNW(22,3),                  WORK  35
     6 ALPMAX(22,3,4),ALPMIN(22,3,4),TMXALP(22,3,4),TMNALP(22,3,4),  WORK  36
     7 BMMAX(22,3,4),BMMIN(22,3,4),TMXBM(22,3,4),TMNBM(22,3,4),      WORK  37
     8 TACCMX(22),TACCMN(22),TMXTAC(22),TMNTAC(22),                  WORK  38
     9 GSRMX(22,3),GSRMN(22,3),TMXGSR(22,3),TMNGSR(22,3),           WORK  39
     1 CLDMAX(22,3),CLDMIN(22,3),TMXCLD(22,3),TMNCLD(22,3),          WORK  40
     2 OTMMX(22),OTMMN(22),TMXOTM(22),TMNOTM(22),                    WORK  41
     3 ABSBMT(22,3),TABSBM(22,3),ABSSRT(22),TABSSR(22)               WORK  42
C                                                                    WORK  43
      COMMON/MNWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4),   WORK  44
     2 GA(22,3),CA(22,3),HT(22),H(22),ZM(22),A(5),S(8),P,F,ZETA,FUNDFQ,WORK  45
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                  WORK  46
C                                                                    WORK  47
C....... THERE ARE TWO OPTIONS FOR THE SECOND CARD OF 'COMMON/ACEWOR/ 'WORK  48
C.......  ONE FOR '(JENNINGS) PSEUDO-EARTHQUAKES, THE OTHER FOR REG- WORK  49
C.......  ULAR EARTHQUAKES.  SEE EITHER SUBROUTINE 'ACEL' FOR FURTHER WORK  50
C.......  EXPLANATION.....................................................WORK  51
C                                                                    WORK  52
      COMMON/ACEWOR/  DBTIME,DBDT,ED,DBTIM3,IOPTON,                  WORK  53
     2 DDY1(6011), JJEND                                             WORK  54
C     2 TIM3(601),DDY3(601),DDY1(6011),JJEND          (USED FOR REG EQKS) WORK  55
C                                                                    WORK  56
      COMMON/WKCONV/SCA(22,3),SCB(22,3),SCC(22,3),SCABH(22,3),       WORK  57
     2 SCBCH(22,3),SCDHH(22),SGA(22,3),SGB(22,3),SGC(22,3),          WORK  58
     3 DU(22),U(22),UU(22),DUUH(22),UUH(22),DV(22),V(22),            WORK  59
     4 DACC(22),ACC(22),DW(22,3),W(22,3),       DDDY,DT,KC,N22       WORK  60
C                                                                    WORK  61
      COMMON/CONMOD/ DBU(22),DBUU(22)                                WORK  62
C                                                                    WORK  63
      COMMON/WKGNIN/LLX,LLI,LLB,LLS,LLD,LLP, LGD(2),LINE(129)        WORK  64
C                                                                    WORK  65
      ZG=386.06                                                      WORK  66
C                                                                    WORK  67
      JJ = 1                                                         WORK  68
      ITIME = 1                                                      WORK  69
      ZTIME = ITIME                                                  WORK  70
      DT = SNGL(DBDT)                                                WORK  71
      DBTIME = DBTIM3                                                WORK  72
      TIM1 = SNGL(DBTIME)                                            WORK  73
C                                                                    WORK  74
      DDY1ZG = DDY1(1)*ZG                                            WORK  75
      DO 5 N=2,21                                                    WORK  76
5     ACC(N) = -DDY1ZG                                               WORK  77
C                                                                    WORK  78
      WRITE(20) NAMEST,NAMEQK                                        WORK  79
C                                                                    WORK  80
      WRITE(20)   JJ,L1,L,SHEAR,BM,OTMOM,AXCLD,ACC,TACC,V,           WORK  81
     2 DBU,DBUU,W,UUH,ALP,TIM1,DT,DDY1(JJ)                          WORK  82
C                                                                    WORK  83
      WRITE(6,1001)                                                  WORK  84
 1001 FORMAT(1H1)                                                    WORK  85
C                                                                    WORK  86
      WRITE(6,1075) DT                                               WORK  87
1075  FORMAT(5H DT =  ,1PE15.7)                                      WORK  88
C                                                                    WORK  89
      WRITE(6,1076)                                                  WORK  90
1076  FORMAT( 48H   JJ    TIM1         DDY1            U(2)        L1  )WORK  91
C                                                                    WORK  92
      WRITE(6,1077)  JJ,TIM1,DDY1(JJ),U(2),L1                        WORK  93
1077  FORMAT( 1X,I4,1F10.6,1P2E15.7,I3)                              WORK  94
C                                                                    WORK  95
C........ BEGIN NEW TIME INTERVAL ...................................WORK  96
C                                                                    WORK  97
10    CONTINUE                                                       WORK  98
      ZJJ = JJ                                                       WORK  99
      DBZJJ = DBLE(ZJJ)                                              WORK 100
      DBTIME = DBTIM3 + DBZJJ*DBDT                                   WORK 101
      TIM1 = SNGL(DBTIME)                                            WORK 102
      JJM1 = JJ                                                      WORK 103
      JJ = JJ+1                                                      WORK 104
      DDDY = (DDY1(JJ) - DDY1(JJM1))*ZG                              WORK 105
C     IF(ABS(DT).GT.0.000001)  GO TO 80                              WORK 106
C     DO 70 N=2,21                                                   WORK 107
C     ACC(N) = ACC(N) - DDDY                                         WORK 108
C 70  CONTINUE                                                       WORK 109
C     WRITE(6,1111)  LLX,TIM1,JT,DDDY                                WORK 110
C1111 FORMAT(2X,A1,2X,2F12.5,13X,1PE18.7)                            WORK 111
C     GO TO 10                                                       WORK 112
C 80  CONTINUE                                                       WORK 113
C                                                                    WORK 114
C........ EVALUATE 'EFFECTIVE STIFFNESSES  SCA, SCB, SCC' FOR COLUMNS ...WORK 115
C                                                                    WORK 116
      DO 150 M=2,3                                                   WORK 117
      DO 145 N=2,21                                                  WORK 118
      CANM = CA(N,M)                                                 WORK 119
      IF(L(N,M,4).NE.0)  GO TO 120                                   WORK 120
      IF(L(N+1,M,2).NE.0)  GO TO 110                                 WORK 121
C-------------- NO HINGES IN COLUMN ----------------------------------WORK 122
      SCA(N,M) = CANM                                                WORK 123
      SCB(N,M) = 0.5*CANM                                            WORK 124
      SCC(N,M) = CANM                                                WORK 125
      GO TO 140                                                      WORK 126
C------------- HINGE IN COLUMN AT (N+1,M,2) ----------------------------WORK 127
110   SCA(N,M) = S(5)*CANM                                           WORK 128
      SCB(N,M) = S(4)*CANM                                           WORK 129
      SCC(N,M) = S(3)*CANM                                           WORK 130
      GO TO 140                                                      WORK 131
120   IF(L(N+1,M,2).NE.0) GO TO 130                                  WORK 132
C--------------HINGE IN COLUMN AT (N,M,4) ----------------------------WORK 133
      SCA(N,M) = S(3)*CANM                                           WORK 134
      SCB(N,M) = S(4)*CANM                                           WORK 135
      SCC(N,M) = S(5)*CANM                                           WORK 136
      GO TO 140                                                      WORK 137
C--------------HINGES IN COLUMN AT BOTH ENDS ------------------------WORK 138
130   SCA(N,M) = S(6)*CANM                                           WORK 139
      SCB(N,M) = S(7)*CANM                                           WORK 140
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      SCC(N,M) = SCA(N,M)                                                    WORK 141
140   CONTINUE                                                               WORK 142
      SCABH(N,M)=(SCA(N,M)+SCB(N,M))*H(N)                                    WORK 143
      SCBCH(N,M)=(SCB(N,M)+SCC(N,M))*H(N)                                    WORK 144
C.............. SCDHH(N) IS EVALUATED BELOW IN STMT 195 ..................WORK 145
145   CONTINUE                                                               WORK 146
150   CONTINUE                                                               WORK 147
C                                                                            WORK 148
C....... EVALUATE 'EFFECTIVE STIFFNESSES  SGA, SGB, SGC' FOR GIRDERS ...WORK 149
C                                                                            WORK 150
      DO 200 N=2,21                                                          WORK 151
      GAN2 = GA(N,2)                                                         WORK 152
      IF(L(N,2,3).NE.0) GO TO 170                                            WORK 153
      IF(L(N,3,1).NE.0) GO TO 160                                            WORK 154
C--------------NO HINGES IN GIRDER ------------------------------------WORK 155
      SGA(N,2) = GAN2                                                        WORK 156
      SGB(N,2) = 0.5*GAN2                                                    WORK 157
      SGC(N,2) = GAN2                                                        WORK 158
      GO TO 185                                                              WORK 159
C-------------- HINGE IN GIRDER AT (N,3,1) ----------------------------WORK 160
160   SGA(N,2) = S(5)*GAN2                                                   WORK 161
      SGB(N,2) = S(4)*GAN2                                                   WORK 162
      SGC(N,2) = S(3)*GAN2                                                   WORK 163
      GO TO 185                                                              WORK 164
170   IF(L(N,3,1).NE.0)   GO TO 180                                          WORK 165
C-------------- HINGE IN GIRDER AT (N,2,3) ----------------------------WORK 166
      SGA(N,2) = S(3)*GAN2                                                   WORK 167
      SGB(N,2) = S(4)*GAN2                                                   WORK 168
      SGC(N,2) = S(5)*GAN2                                                   WORK 169
      GO TO 185                                                              WORK 170
C-------------- HINGE AT BOTH ENDS OF GIRDER --------------------------WORK 171
180   SGA(N,2) = S(6)*GAN2                                                   WORK 172
      SGB(N,2) = S(7)*GAN2                                                   WORK 173
      SGC(N,2) = S(6)*GAN2                                                   WORK 174
185   IF(L(N,3,3).NE.0)  GO TO 190                                           WORK 175
C....... SGA(N,3) CONTAINS SGB(N,3) DUE TO SYMMETRY OF THE MIDDLE BAY...WORK 176
      SGA(N,3) = 1.5*GA(N,3)                                                 WORK 177
      GO TO 195                                                              WORK 178
190   SGA(N,3) = S(8)*GA(N,3)                                                WORK 179
195   SCDHH(N) =(SCABH(N,2) + SCBCH(N,2) + SCABH(N,3) + SCBCH(N,3))*H(N)WORK 180
200   CONTINUE                                                               WORK 181
C                                                                            WORK 182
      CALL CONV                                                              WORK 183
C                                                                            WORK 184
C....... SUBROUTINE 'CONV' SOLVES THE EQUATIONS OF MOTION FOR THE .....WORK 185
C......... VARIABLES 'DACC,DV,DU,DUUH,DW' AND OTHERS ....................WORK 186
C                                                                            WORK 187
C....... MONITOR FOR MAXIMUM AND MINIMUM   'JOINT ROTATIONS - W ' .....WORK 188
C                                                                            WORK 189
      DO 220 M=2,3                                                           WORK 190
      DO 215 N=2,21                                                          WORK 191
      IF(WMAX(N,M).GE.W(N,M))  GO TO 210                                     WORK 192
      WMAX(N,M)=W(N,M)                                                       WORK 193
      TMX=(N,M)=TIM1                                                         WORK 194
      GO TO 211                                                              WORK 195
210   CONTINUE                                                               WORK 196
      IF(WMIN(N,M).LE.W(N,M))  GO TO 211                                     WORK 197
      WMIN(N,M)=W(N,M)                                                       WORK 198
      TMNW(N,M)=TIM1                                                         WORK 199
211   CONTINUE                                                               WORK 200
215   CONTINUE                                                               WORK 201
220   CONTINUE                                                               WORK 202
C                                                                            WORK 203
      IF(L1.EQ.0)  GO TO 305                                                 WORK 204
C                                                                            WORK 205
C....... EVALUATE 'YIELD ANGLES'  IN COLUMNS FOR THIS TIME INTERVAL ....WORK 206
C                                                                            WORK 207
      DO 250 M=2,3                                                           WORK 208
      DO 245 N=2,21                                                          WORK 209
      N1 = N+1                                                               WORK 210
      IF( L(N,M,4).NE.0)  GO TO 235                                          WORK 211
      IF( L(N1,M,2).NE.0)  GO TO 230                                         WORK 212
C-------------- NO HINGES IN COLUMN ------------------------------------WORK 213
      GO TO 245                                                              WORK 214
C-------------- HINGE IN COLUMN AT (N+1,M,2) --------------------------WORK 215
230   ALP(N1,M,2) = ALP(N1,M,2) + A(2)*(DW(N,M)-DUUH(N)) +                   WORK 216
     2                A(1)*(DW(N1,M)-DUUH(N))                               WORK 217
      GO TO 245                                                              WORK 218
235   IF( L(N1,M,2).NE.0)  GO TO 240                                         WORK 219
C--------------HINGE IN COLUMN AT (N,M,4) ----------------------------WORK 220
      ALP(N,M,4) = ALP(N,M,4) + A(1)*(DW(N,M)-DUJH(N)) +                     WORK 221
     2             A(2)*(DW(N1,M)-DUUH(N))                                   WORK 222
      GO TO 245                                                              WORK 223
C--------------HINGES IN COLUMN AT BOTH ENDS -------------------------WORK 224
240   ALP(N,M,4) = ALP(N,M,4) + A(3)*(DW(N,M)-DUJH(N)) +                     WORK 225
     2             A(4)*(DW(N1,M)-DUUH(N))                                   WORK 226
      ALP(N1,M,2) = ALP(N1,M,2) + A(4)*(DW(N,M)-DUUH(N)) +                   WORK 227
     2             A(3)*(DW(N1,M)-DUUH(N))                                   WORK 228
245   CONTINUE                                                               WORK 229
250   CONTINUE                                                               WORK 230
C                                                                            WORK 231
C....... EVALUATE 'YIELD ANGLES'  IN GIRDERS FOR THIS TIME INTERVAL ....WORK 232
C                                                                            WORK 233
      DO 300 N=2,21                                                          WORK 234
      IF( L(N,2,3).NE.0)  GO TO 270                                          WORK 235
      IF( L(N,3,1).NE.0)  GO TO 260                                          WORK 236
C--------------NO HINGES IN GIRDER ------------------------------------WORK 237
      GO TO 285                                                              WORK 238
C-------------- HINGE IN GIRDER AT (N,3,1) ----------------------------WORK 239
260   ALP(N,3,1) = ALP(N,3,1) + A(2)*DW(N,2) + A(1)*DW(N,3)                  WORK 240
      GO TO 265                                                              WORK 241
270   IF( L(N,3,1).NE.0)  GO TO 280                                          WORK 242
C-------------- HINGE IN GIRDER AT (N,2,3) ----------------------------WORK 243
      ALP(N,2,3) = ALP(N,2,3) + A(1)*DW(N,2) + A(2)*DW(N,3)                  WORK 244
      GO TO 285                                                              WORK 245
C-------------- HINGE AT BOTH ENDS OF GIRDER --------------------------WORK 246
280   ALP(N,2,3) = ALP(N,2,3) + A(3)*DW(N,2) + A(4)*DW(N,3)                  WORK 247
      ALP(N,3,1) = ALP(N,3,1) + A(4)*DW(N,2) + A(3)*DW(N,3)                  WORK 248
C.............. EVALUATE              'ALP' FOR GIRDERS OF MIDDLE BAY ....WORK 249
285   IF( L(N,3,3).EQ.0)  GO TO 300                                          WORK 250
C.............. HINGES AT BOTH ENDS OF GIRDER  'IF(L(N,3,3)=1)'  ..........WORK 251
      ALP(N,3,3) = ALP(N,3,3) + A(5)*DW(N,3)                                 WORK 252
300   CONTINUE                                                               WORK 253
305   CONTINUE                                                               WORK 254
C                                                                            WORK 255
C.......  SAVE VALUES OF BENDING MOMENTS AT BEGINNING OF THIS 'DT'.....WORK 256
C                                                                            WORK 257
      DO 310 M=2,3                                                           WORK 258
      DO 310 J=1,4                                                           WORK 259
      DO 310 N=2,22                                                          WORK 260
      BMLAST(N,M,J) = BM(N,M,J)                                              WORK 261
310   CONTINUE                                                               WORK 262
C ..... EVALUATE 'BENDING MOMENTS' IN COLUMNS AT END OF THIS 'DT'.....WORK 263
C                                                                            WORK 264
      DO 345 M=2,3                                                           WORK 265
      DO 340 N=2,21                                                          WORK 266
      N1=N+1                                                                 WORK 267
      BM(N,M,4) = CA(N,M)*((W(N,M) -UUH(N) - ALP(N,M,4)) +                   WORK 268
     2  (0.5*(W(N1,M) -UUH(N) -ALP(N1,M,2))))                               WORK 269
      BM(N1,M,2) = CA(N,M)*((W(N1,M) -UUH(N)- ALP(N1,M,2)) +                 WORK 270
     2 (0.5*(W(N,M) -UUH(N) - ALP(N,M,4))))                                  WORK 271
340   CONTINUE                                                               WORK 272
345   CONTINUE                                                               WORK 273
C                                                                            WORK 274
C ..... EVALUATE 'BENDING MOMENTS' IN GIRDERS AT END OF THIS 'DT'.....WORK 275
C                                                                            WORK 276
      DO 350 N=2,21                                                          WORK 277
      BM(N,2,3) = GA(N,2)*((W(N,2) - ALP(N,2,3)) +                           WORK 278
     2  (0.5*(W(N,3) - ALP(N,3,1))))                                         WORK 279
      BM(N,3,1) = GA(N,2)*((W(N,3) - ALP(N,3,1)) +                           WORK 280
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```
     2  (0.5*(W(N,2) - ALP(N,2,3))))                                          WORK 281
      BM(N,3,3) = 1.5*GA(N,3)*(W(N,3) - ALP(N,3,3))                           WORK 282
350   CONTINUE                                                                WORK 283
C                                                                             WORK 284
C..... MASTER CONTROL SECTION FOR EVALUATING THE ' STATE OF YIELD '  .WORK 285
C                                                                             WORK 286
      IF(IELPL.EQ.0)  GO TO 336                                               WORK 287
      LL1 = L1                                                                WORK 288
      L1 = 0                                                                  WORK 289
      DO 335 M=2,3                                                            WORK 290
      DO 334 J=1,4                                                            WORK 291
      DO 330 N=2,22                                                           WORK 292
      IF((M+J).EQ.3)  GO TO 334                                               WORK 293
      DBM = BM(N,M,J) - BMLAST(N,M,J)                                         WORK 294
      IF(L(N,M,J).EQ.0)  GO TO 315                                            WORK 295
      BMA(N,M,J) = BMA(N,M,J) + DBM                                           WORK 296
      BMB(N,M,J) = BMB(N,M,J) + DBM                                           WORK 297
315   CONTINUE                                                                WORK 298
C                                                                             WORK 299
C..... UPDATE THE  'PLASTIC INDEX - L(N,M,J)' FOR NEXT TIME INTERVAL .WORK 300
C                                                                             WORK 301
      IF(BM(N,M,J).GE.BMA(N,M,J) .AND. DBM.GT.0.0)  GO TO 320                 WORK 302
      IF(BM(N,M,J).LE.BMB(N,M,J) .AND. DBM.LT.0.0)  GO TO 325                 WORK 303
      L(N,M,J) = 0                                                            WORK 304
      GO TO 330                                                               WORK 305
320   L(N,M,J) = +1                                                           WORK 306
      L1 = 1                                                                  WORK 307
      GO TO 330                                                               WORK 308
325   L(N,M,J) = -1                                                           WORK 309
      L1 = 1                                                                  WORK 310
330   CONTINUE                                                                WORK 311
334   CONTINUE                                                                WORK 312
335   CONTINUE                                                                WORK 313
336   CONTINUE                                                                WORK 314
C                                                                             WORK 315
C.............. END OF MASTER CONTROL SECTION ..............................WORK 316
C                                                                             WORK 317
      DDY1ZG = DDY1(JJ)*ZG                                                    WORK 318
      DO 450 N=2,22                                                           WORK 319
C                                                                             WORK 320
C..... MONITOR FOR   MAXIMUM AND MINIMUM  'TOTAL ACCELERATION - TACC'.WORK 321
C                                                                             WORK 322
      TACC(N) = ACC(N) + DDY1ZG                                               WORK 323
      IF(TACCMX(N).GE.TACC(N))  GO TO 355                                     WORK 324
      TACCMX(N)=TACC(N)                                                       WORK 325
      TMXTAC(N)=TIM1                                                          WORK 326
      GO TO 360                                                               WORK 327
355   CONTINUE                                                                WORK 328
      IF(TACCMN(N).LE.TACC(N))  GO TO 360                                     WORK 329
      TACCMN(N)=TACC(N)                                                       WORK 330
      TMNTAC(N)=TIM1                                                          WORK 331
360   CONTINUE                                                                WORK 332
C                                                                             WORK 333
C..... MONITOR FOR   MAXIMUM AND MINIMUM  'VELOCITY - V '.............WORK 334
C                                                                             WORK 335
      IF(VMAX(N).GE.V(N))  GO TO 365                                          WORK 336
      VMAX(N)=V(N)                                                            WORK 337
      TMXV(N)=TIM1                                                            WORK 338
      GO TO 370                                                               WORK 339
365   CONTINUE                                                                WORK 340
      IF(VMIN(N).LE.V(N))  GO TO 370                                          WORK 341
      VMIN(N)=V(N)                                                            WORK 342
      TMNV(N)=TIM1                                                            WORK 343
370   CONTINUE                                                                WORK 344
C                                                                             WORK 345
C..... MONITOR FOR   MAXIMUM AND MINIMUM  'DISPLACEMENT - U ' .........WORK 346
C                                                                             WORK 347
      IF(UMAX(N).GE.U(N))  GO TO 375                                          WORK 348
      UMAX(N)=U(N)                                                            WORK 349
      TMXU(N)=TIM1                                                            WORK 350
      GO TO 380                                                               WORK 351
375   CONTINUE                                                                WORK 352
      IF(UMIN(N).LE.U(N))  GO TO 380                                          WORK 353
      UMIN(N)=U(N)                                                            WORK 354
      TMNU(N)=TIM1                                                            WORK 355
380   CONTINUE                                                                WORK 356
C                                                                             WORK 357
C.............. EVALUATE 'SHEAR' AND ..................................WORK 358
C..... MONITOR FOR MAX AND MIN  INTER-FLOOR SHEAR 'SHEAR' (2 COL) ....WORK 359
C                                                                             WORK 360
      IF(N.EQ.22)  GO TO 390                                                  WORK 361
      N1=N+1                                                                  WORK 362
      SHEAR(N) = -((BM(N,2,4) + BM(N,3,4) + BM(N1,2,2)+BM(N1,3,2))*H(N))WORK 363
      IF(SRMAX(N).GE.SHEAR(N))  GO TO 385                                     WORK 364
      SRMAX(N)=SHEAR(N)                                                       WORK 365
      TMXSR(N)=TIM1                                                           WORK 366
      GO TO 390                                                               WORK 367
385   CONTINUE                                                                WORK 368
      IF(SRMIN(N).LE.SHEAR(N))  GO TO 390                                     WORK 369
      SRMIN(N)=SHEAR(N)                                                       WORK 370
      TMNSR(N)=TIM1                                                           WORK 371
390   CONTINUE                                                                WORK 372
C                                                                             WORK 373
C..... MONITOR FOR   MAXIMUM AND MINIMUM  'INTER-FLOOR DISP. - UU' ...WORK 374
C                                                                             WORK 375
      IF(UUMAX(N).GE.UU(N))  GO TO 395                                        WORK 376
      UUMAX(N)=UU(N)                                                          WORK 377
      TMXUU(N)=TIM1                                                           WORK 378
      GO TO 400                                                               WORK 379
395   CONTINUE                                                                WORK 380
      IF(UUMIN(N).LE.UU(N))  GO TO 400                                        WORK 381
      UUMIN(N)=UU(N)                                                          WORK 382
      TMNUU(N)=TIM1                                                           WORK 383
400   CONTINUE                                                                WORK 384
C                                                                             WORK 385
C..... EVALUATE ' CHECK-SUM' OF SHEAR FORCES ON EACH FLOOR .........WORK 386
C                                                                             WORK 387
      SRT(N) = ZM(N)*(TACC(N) + BLAMDA*V(N)) + SHEAR(N) - SHEAR(N-1)          WORK 388
      IF(ABSSRT(N).GE.ABS(SRT(N)))  GO TO 445                                 WORK 389
      ABSSRT(N) = ABS(SRT(N))                                                 WORK 390
      TABSSR(N) = TIM1                                                        WORK 391
445   CONTINUE                                                                WORK 392
450   CONTINUE                                                                WORK 393
C                                                                             WORK 394
      DO 500 M=2,3                                                            WORK 395
      DO 495 J=1,4                                                            WORK 396
      DO 490 N=2,22                                                           WORK 397
      IF(LL1.EQ.0)  GO TO 480                                                 WORK 398
C                                                                             WORK 399
C.....MONITOR FOR   MAXIMUM AND MINIMUM  'YIELD ANGLE - ALP ' .....WORK 400
C                                                                             WORK 401
      IF(ALPMAX(N,M,J).GE.ALP(N,M,J))  GO TO 470                              WORK 402
      ALPMAX(N,M,J)=ALP(N,M,J)                                                WORK 403
      TMXALP(N,M,J)=TIM1                                                      WORK 404
      GO TO 475                                                               WORK 405
470   CONTINUE                                                                WORK 406
      IF(ALPMIN(N,M,J).LE.ALP(N,M,J))  GO TO 475                              WORK 407
      ALPMIN(N,M,J)=ALP(N,M,J)                                                WORK 408
      TMNALP(N,M,J)=TIM1                                                      WORK 409
475   CONTINUE                                                                WORK 410
480   CONTINUE                                                                WORK 411
C                                                                             WORK 412
C..... MONITOR FOR   MAXIMUM AND MINIMUM  'BENDING MOMENT - BM ' .....WORK 413
C                                                                             WORK 414
      IF(BMMAX(N,M,J).GE.BM(N,M,J))  GO TO 485                                WORK 415
      BMMAX(N,M,J)=BM(N,M,J)                                                  WORK 416
      TMXBM(N,M,J)=TIM1                                                       WORK 417
      GO TO 490                                                               WORK 418
485   CONTINUE                                                                WORK 419
      IF(BMMIN(N,M,J).LE.BM(N,M,J)) GO TO 490                                 WORK 420
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      BMMIN(N,M,J)=BM(N,M,J)                                                    WORK 421
      TMNBM(N,M,J)=TIM1                                                         WORK 422
490   CONTINUE                                                                  WORK 423
495   CONTINUE                                                                  WORK 424
500   CONTINUE                                                                  WORK 425
C                                                                               WORK 426
C.............. EVALUATE  'GIRDER SHEAR FORCES, COLUMN AXIAL FORCES, AND .....WORK 427
C........................ OVERTURNING MOMENTS' ..............................WORK 428
C                                                                               WORK 429
      DO 510 N=2,22                                                             WORK 430
      GSR(N,2)  = -((BM(N,2,3) + BM(N,3,1)) *AL)                                WORK 431
      GSR(N,3)  = -(2.0*BM(N,3,3)*AL)                                           WORK 432
      AXCLD(N,2) = AXCLD(N-1,2) +  GSR(N,2)                                     WORK 433
      AXCLD(N,3) = AXCLD(N-1,3) + (GSR(N,3) -  GSR(N,2))                        WORK 434
      OTMOM(N) = ZL*(AXCLD(N-1,3) + 3.0*AXCLD(N-1,2))                           WORK 435
     2           + 2.0*(BM(N,2,2) + BM(N,3,2))                                  WORK 436
C                                                                               WORK 437
C....... MONITOR FOR MAXIMUM AND MINIMUM 'OVERTURNING MOMENTS - OTMOM' .....WORK 438
C                                                                               WORK 439
      IF(OTMMX(N).GE.OTMOM(N)) GO TO 505                                        WORK 440
      OTMMX(N) = OTMOM(N)                                                       WORK 441
      TMXOTM(N) = TIM1                                                          WORK 442
      GO TO 510                                                                 WORK 443
505   CONTINUE                                                                  WORK 444
      IF(OTMMN(N).LE.OTMOM(N))  GO TO 510                                       WORK 445
      OTMMN(N) = OTMOM(N)                                                       WORK 446
      TMNOTM(N) = TIM1                                                          WORK 447
510   CONTINUE                                                                  WORK 448
C                                                                               WORK 449
C.............. EVALUATE  'CHECK-SUM'  OF BENDING MOMENTS AT EACH JOINT ..WORK 450
C                                                                               WORK 451
      DO 550 M=2,3                                                              WORK 452
      DO 545 N=2,22                                                             WORK 453
      BMT(N,M) = 0.0                                                            WORK 454
      DO 520 J=1,4                                                              WORK 455
      BMT(N,M) = BMT(N,M) + BM(N,M,J)                                           WORK 456
520   CONTINUE                                                                  WORK 457
      IF(ABSBMT(N,M).GE.ABS(BMT(N,M)) )  GO TO 525                              WORK 458
      ABSBMT(N,M) = ABS(BMT(N,M))                                               WORK 459
      TABSBM(N,M) = TIM1                                                        WORK 460
525   CONTINUE                                                                  WORK 461
C                                                                               WORK 462
C.......... MONITOR FOR MAXIMUM AND MINIMUM  'GIRDER SHEAR - GSR '.......WORK 463
C                                                                               WORK 464
      IF(GSRMX(N,M).GE.GSR(N,M))  GO TO 530                                     WORK 465
      GSRMX(N,M)=GSR(N,M)                                                       WORK 466
      TMXGSR(N,M)=TIM1                                                          WORK 467
      GO TO 535                                                                 WORK 468
530   CONTINUE                                                                  WORK 469
      IF(GSRMN(N,M).LE.GSR(N,M))  GO TO 535                                     WORK 470
      GSRMN(N,M)=GSR(N,M)                                                       WORK 471
      TMNGSR(N,M)=TIM1                                                          WORK 472
535   CONTINUE                                                                  WORK 473
C                                                                               WORK 474
C.......... MONITOR FOR MAXIMUM AND MINIMUM  'AXIAL COLUMN LOAD - AXCLD' WORK 475
C                                                                               WORK 476
      IF(CLDMAX(N,M).GE.AXCLD(N,M))  GO TO 540                                  WORK 477
      CLDMAX(N,M)=AXCLD(N,M)                                                    WORK 478
      TMXCLD(N,M)=TIM1                                                          WORK 479
      GO TO 545                                                                 WORK 480
540   CONTINUE                                                                  WORK 481
      IF(CLDMIN(N,M).LE.AXCLD(N,M))  GO TO 545                                  WORK 482
      CLDMIN(N,M)=AXCLD(N,M)                                                    WORK 483
      TMNCLD(N,M)=TIM1                                                          WORK 484
545   CONTINUE                                                                  WORK 485
550   CONTINUE                                                                  WORK 486
C                                                                               WORK 487
      CALL MODE                                                                 WORK 488
C                                                                               WORK 489
      LWRITE = LWRITE + 1                                                       WORK 490
      IF(LWRITE.LT.LRTEST)  GO TO 560                                           WORK 491
      LWRITE = 0                                                                WORK 492
      WRITE(6,1077)  JJ,TIM1,DDY1(JJ),U(2),LL1                                  WORK 493
C                                                                               WORK 494
      WRITE(20)  JJ,LL1,L,SHEAR,BM,OTMOM,AXCLD,ACC,TACC,V,                      WORK 495
     2 DBU,DBUU,W,UUH,ALP,TIM1,DT,DDY1(JJ)                                      WORK 496
560   CONTINUE                                                                  WORK 497
C                                                                               WORK 498
C....... END OF TIME INCREMENT ..........................................WORK 499
C                                                                               WORK 500
      IF(ABS(TIM1-ZTIME).GT.2.5000002E-03.AND.TIM1.LT.TIMEND) GO TO 10          WORK 501
      ITIME = ITIME + 1                                                         WORK 502
      ZTIME = ITIME                                                             WORK 503
      CALL GONE                                                                 WORK 504
      WRITE(6,1001)                                                             WORK 505
C                                                                               WORK 506
      IF(TIM1.LT.TIMEND)  GO TO 10                                              WORK 507
C                                                                               WORK 508
      CALL GONE                                                                 WORK 509
      WRITE(6,1090)  JJ                                                         WORK 510
1090  FORMAT(10H0 JJEND =  ,I4)                                                 WORK 511
C                                                                               WORK 512
C....... TERMINATION OF RUN .............................................WORK 513
C                                                                               WORK 514
      RETURN                                                                    WORK 515
      END                                                                       WORK 516
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$IBFTC CONV    DECK      **MATRIX** **DU** **DBLE PRECISION**       Q56 CONV   1
      SUBROUTINE CONV                                                 CONV   2
C                                                                     CONV   3
C*********************************************************************CONV   4
C                                                                     CONV   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES   CONV   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.CONV   7
C                                                                     CONV   8
C     THE PURPOSE OF SUBROUTINE 'CONV' IS TO SOLVE THE EQUATIONS OF   CONV   9
C     MOTION FOR EACH TIME INTERVAL AND TO RETURN UPDATED RESPONSES TO CONV  10
C     SUBROUTINE 'WORK'.                                              CONV  11
C                          D O U B L E   P R E C I S I O N            CONV  12
C                                                                     CONV  13
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE CONV  14
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- CONV  15
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                       CONV  16
C                  LAST REVISION - 4 DECEMBER 1966                    CONV  17
C                                                                     CONV  18
C*********************************************************************CONV  19
C                                                                     CONV  20
C        NOTE ON PREFIX LETTERS..                                     CONV  21
C          'D'    INCREMENTAL VARIABLE                               CONV  22
C          'DB'   DOUBLE PRECISION VARIABLE                          CONV  23
C          'DBD'  DOUBLE PRECISION INCREMENTAL VARIABLE               CONV  24
C                                                                     CONV  25
C          NOTE.. 'COMMON/ACEWOR/' IS USED TO COMMUNICATE ONLY      CONV  26
C          THE VALUE OF 'DBDT' TO 'SUBR CONV' FROM 'SUBR ACEL'.     CONV  27
C          THIS MEANS (1) WHAT FOLLOWS 'DBDT' IN 'COMMON/ACEWOR/'   CONV  28
C          IS IMMATERIAL TO 'SUBR CONV' AND HENCE IS NOT USED.      CONV  29
C          (2) CONSEQUENTLY 'SUBR CONV' MUST BE LOADED AFTER ANY    CONV  30
C          OTHER SUBROUTINE THAT HAS THE COMPLETELY CORRECT          CONV  31
C          'COMMON/ACEWOR/' AS IT IS USED IN COMMUNICATING          CONV  32
C          BETWEEN 'SUBR ACEL' AND 'SUBR WORK'.                     CONV  33
C                                                                     CONV  34
C*********************************************************************CONV  35
C                                                                     CONV  36
C           VARIABLES USED HEREIN                                     CONV  37
C                                                                     CONV  38
C   U       HORIZONTAL DISPLACEMENT RELATIVE TO GROUND                CONV  39
C   V       VELOCITY RELATIVE TO GROUND                               CONV  40
C   ACC     ACCELERATION RELATIVE TO GROUND                           CONV  41
C   W       JOINT ROTATION                                            CONV  42
C   UU      INTERFLOOR DISPLACEMENT                                   CONV  43
C                                                                     CONV  44
C*********************************************************************CONV  45
C                                                                     CONV  46
      DOUBLE PRECISION DBTIME                                         CONV  47
C                                                                     CONV  48
      DOUBLE PRECISION    C(20,3,3),SINV(20,3,3),DR(21,3),DP(20,3),   CONV  49
     2 USEDI(3,3),USEDPI(3),CTSI(20,3,3),S(20,3,3),                   CONV  50
     3 AACC(22), BVEL(22),DBZM(22),DBH(22), DBDU(22),        DBDUJH(22),CONV  51
     4 DBUUH(22),DBV(22),DBACC(22),DBW(22,3),                         CONV  52
     5 DBLAMD,DBDT,C1,C2,C3,C4,C5,DBDDDY,   SS,SSINV,    DBU,DBUU    CONV  53
C                                                                     CONV  54
      COMMON/ACEWOR/DBTIME,DBDT                                       CONV  55
C                                                                     CONV  56
C...***. USE DUMMY NAME 'Z(8)' INSTEAD OF 'S(8)' IN 'COMMON/MNWCGN/'...*CONV  57
C                                                                     CONV  58
      COMMON/MNWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4),    CONV  59
     2 GA(22,3),CA(22,3),HT(22),H(22),ZM(22),A(5),Z(8),P,F,ZETA,FUNDFQ,CONV  60
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                   CONV  61
C                                                                     CONV  62
      COMMON/WKCONV/SCA(22,3),SCB(22,3),SCC(22,3),SCABH(22,3),        CONV  63
     2 SCBCH(22,3),SCDHH(22),SGA(22,3),SGB(22,3),SGC(22,3),           CONV  64
     3 DU(22),U(22),UU(22),DUUH(22),UUH(22),DV(22),V(22),             CONV  65
     4 DACC(22),ACC(22),DW(22,3),W(22,3),       DDDY,DT,KC,N22       CONV  66
C                                                                     CONV  67
      COMMON/CONMOD/ DBU(22),DBUU(22)                                 CONV  68
C                                                                     CONV  69
      COMMON/CONMIN/SS(3,3),SSINV(3,3),N                              CONV  70
C                                                                     CONV  71
      IF(LL1.EQ.1)  GO TO 1601                                        CONV  72
      LL1=1                                                           CONV  73
      WRITE(6,1600)                                                   CONV  74
1600  FORMAT(50H    'DU'   VIA MATRICES USING DOUBLE PRECISION      ) CONV  75
      DBLAMD = DBLE(BLAMDA)                                           CONV  76
      C1 = 3.0D+00                                                    CONV  77
      DO 690 N=2,21                                                   CONV  78
      DBZM(N) = DBLE(ZM(N))                                           CONV  79
      DBH(N) = DBLE(H(N))                                             CONV  80
      DBACC(N) = DBLE(ACC(N))                                         CONV  81
690   CONTINUE                                                        CONV  82
1601  CONTINUE                                                        CONV  83
      C2 = DBDT/2.0D+00                                               CONV  84
      C3 = 6.0D+00/DBDT                                               CONV  85
      C4 = C1/DBDT                                                    CONV  86
      C5 = C3/DBDT                                                    CONV  87
      DBDDDY = DBLE(DDDY)                                             CONV  88
      DO 700 N=1,20                                                   CONV  89
      N1=N+1                                                          CONV  90
      AACC(N1) =-C3*DBV(N1) - C1*DBACC(N1)                            CONV  91
      BVEL(N1) = -C1*DBV(N1) - C2*DBACC(N1)                           CONV  92
C                                                                     CONV  93
C..... EVALUATE 'S'  - THE STIFFNESS MATRICES ON THE MAIN DIAGONAL  ..CONV  94
C                                                                     CONV  95
      S(N,1,1) = DBLE(SCDHH(N) + SCDHH(N1)) + ( C5 + C4*DBLAMD)*DBZM(N1)CONV  96
      S(N,2,2) = DBLE( SCC(N,2) + SGA(N1,2) + SCA(N1,2) )             CONV  97
      S(N,3,3) = DBLE( SGC(N1,2) + SCC(N,3) + SGA(N1,3) + SCA(N1,3) ) CONV  98
      S(N,1,2) = DBLE( SCBCH(N,2) - SCABH(N1,2) )                     CONV  99
      S(N,1,3) = DBLE( SCBCH(N,3) - SCABH(N1,3) )                     CONV 100
      S(N,2,3) = DBLE ( SGB(N1,2) )                                   CONV 101
      S(N,2,1) = S(N,1,2)                                             CONV 102
      S(N,3,1) = S(N,1,3)                                             CONV 103
      S(N,3,2) = S(N,2,3)                                             CONV 104
C                                                                     CONV 105
C..... EVALUATE 'C'  - THE OFF DIAGONAL STIFFNESS MATRICES ..........CONV 106
C                                                                     CONV 107
      C(N,1,1) = DBLE( -SCDHH(N1) )                                   CONV 108
      C(N,2,2) = DBLE( SCB(N1,2) )                                    CONV 109
      C(N,3,3) = DBLE( SCB(N1,3) )                                    CONV 110
      C(N,1,2) = DBLE( -SCBCH(N1,2) )                                 CONV 111
      C(N,1,3) = DBLE( -SCBCH(N1,3) )                                 CONV 112
      C(N,2,1) = DBLE( SCABH(N1,2) )                                  CONV 113
      C(N,3,1) = DBLE( SCABH(N1,3) )                                  CONV 114
C                                                                     CONV 115
C..... EVALUATE 'DP' VECTOR OF EFFECTIVE INCREMENTAL FORCES ..........CONV 116
C                                                                     CONV 117
      DP(N,1) = -(DBZM(N1)*( DBDDDY    + AACC(N1) + DBLAMD*BVEL(N1) ) ) CONV 118
      DP(N,2) = 0.0D+00                                               CONV 119
      DP(N,3) = 0.0D+00                                               CONV 120
700   CONTINUE                                                        CONV 121
C                                                                     CONV 122
      DO 770 N=2,21                                                   CONV 123
      NM1=N-1                                                         CONV 124
      DO 720 I=1,3                                                    CONV 125
      DO 720 J=I,3                                                    CONV 126
720   SS(I,J) = S(NM1,I,J)*1.0D-06                                    CONV 127
C                                                                     CONV 128
      CALL MINV                                                       CONV 129
C                                                                     CONV 130
      DO 725 I=1,3                                                    CONV 131
      DO 725 J=I,3                                                    CONV 132
725   SINV(NM1,I,J) = SSINV(I,J)*1.0D-06                              CONV 133
      SINV(NM1,2,1) = SINV(NM1,1,2)                                   CONV 134
      SINV(NM1,3,1) = SINV(NM1,1,3)                                   CONV 135
      SINV(NM1,3,2) = SINV(NM1,2,3)                                   CONV 136
      DO 730 I=1,3                                                    CONV 137
      DO 730 J=1,3                                                    CONV 138
      CTSI(NM1,I,J) = 0.0                                             CONV 139
      DO 730 K=1,3                                                    CONV 140
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      CTSI(NM1,I,J) = CTSI(NM1,I,J) + (C(NM1,K,I)*SINV(NM1,K,J))        CONV 141
730   CONTINUE                                                          CONV 142
      IF(N.EQ.21)  GO TO 771                                            CONV 143
      DO 735 I=1,3                                                      CONV 144
      DO 735 J=1,3                                                      CONV 145
      USED1(I,J) = 0.0                                                  CONV 146
      DO 735 K=1,3                                                      CONV 147
735   USED1(I,J) = USED1(I,J) + (CTSI(NM1,I,K)*C(NM1,K,J))              CONV 148
      DO 740 I=1,3                                                      CONV 149
      DO 740 J=1,3                                                      CONV 150
740   S(N,I,J) = S(N,I,J) - USED1(I,J)                                  CONV 151
      DO 750 I=1,3                                                      CONV 152
      USEDP1(I) = 0.0                                                   CONV 153
      DO 750 J=1,3                                                      CONV 154
750   USEDP1(I) = USEDP1(I) + (CTSI(NM1,I,J)*DP(NM1,J))                 CONV 155
      DO 755 I=1,3                                                      CONV 156
755   DP(N,I) = DP(N,I) - USEDP1(I)                                     CONV 157
770   CONTINUE                                                          CONV 158
771   CONTINUE                                                          CONV 159
C                                                                       CONV 160
C..... BACK-SUBSTITUTE TO OBTAIN 'DR' VECTOR OF INCR DISP AND ROT'NS..CONV 161
C                                                                       CONV 162
      DO 805 I=1,3                                                      CONV 163
      DR(20,I) = 0.0                                                    CONV 164
      DO 805 J=1,3                                                      CONV 165
805   DR(20,I) = DR(20,I) + (SINV(20,I,J)*DP(20,J))                     CONV 166
      DO 840 NN=1,19                                                    CONV 167
      N = 20-NN                                                         CONV 168
      DO 820 I=1,3                                                      CONV 169
      USEDP1(I) = 0.0                                                   CONV 170
      DO 810 J=1,3                                                      CONV 171
810   USEDP1(I) = USEDP1(I) + (C(N,I,J)*DR(N+1,J))                      CONV 172
820   DP(N,I) = DP(N,I) - USEDP1(I)                                     CONV 173
      DO 835 I=1,3                                                      CONV 174
      DR(N,I) = 0.0                                                     CONV 175
      DO 835 J=1,3                                                      CONV 176
835   DR(N,I) = DR(N,I) + (SINV(N,I,J)*DP(N,J))                         CONV 177
840   CONTINUE                                                          CONV 178
C                                                                       CONV 179
      DO 850 N=2,21                                                     CONV 180
      NM1=N-1                                                           CONV 181
      DBDU(N) = DR(NM1,1)                                               CONV 182
      DBU(N) = DBU(N) + DBDU(N)                                         CONV 183
      DBW(N,2) = DBW(N,2) + DR(NM1,2)                                   CONV 184
      DBW(N,3) = DBW(N,3) + DR(NM1,3)                                   CONV 185
      DBACC(N) = DBACC(N) + AACC(N) + C5*DBDU(N)                        CONV 186
      DBV(N)  = DBV(N)  +BVEL(N) + C4*DBDU(N)                           CONV 187
      U(N) = SNGL(DBU(N))                                               CONV 188
      W(N,2) = SNGL(DBW(N,2))                                           CONV 189
      W(N,3) = SNGL(DBW(N,3))                                           CONV 190
      DW(N,2) = SNGL( DR(NM1,2) )                                       CONV 191
      DW(N,3) = SNGL( DR(NM1,3) )                                       CONV 192
      V(N) = SNGL(DBV(N))                                               CONV 193
      ACC(N) = SNGL(DBACC(N))                                           CONV 194
850   CONTINUE                                                          CONV 195
      DO 860 N=2,21                                                     CONV 196
      N1 = N+1                                                          CONV 197
      DBUU(N) = DBU(N) - DBU(N1)                                        CONV 198
      DBDUUH(N) = (DBDU(N) - DBDU(N1) )*DBH(N)                          CONV 199
      DBUUH(N) = DBUUH(N) + DBDUUH(N)                                   CONV 200
      DUUH(N) = SNGL(DBDUUH(N))                                         CONV 201
      UUH(N) = SNGL(DBUUH(N))                                           CONV 202
      UU(N) = SNGL(DBUU(N))                                             CONV 203
860   CONTINUE                                                          CONV 204
C                                                                       CONV 205
C..... END OF 'MATRIX SOLUTION' OF EQUATIONS OF MOTION FOR THIS 'DT' .CONV 206
C                                                                       CONV 207
      RETURN                                                            CONV 208
      END                                                               CONV 209
```

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$IBFTC MINV    DECK *INVERTS '3X3 SYMMETRIC' MATRIX, DOUBLE PREC*   Q56 MINV   1
      SUBROUTINE MINV                                                   MINV   2
C                                                                       MINV   3
C*********************************************************************MINV   4
C                                                                       MINV   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES     MINV   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.  MINV   7
C                                                                       MINV   8
C     THE PURPOSE OF SUBROUTINE 'MINV' IS TO INVERT A '3X3 SYMMETRIC'   MINV   9
C     MATRIX.  THE INPUT MATRIX IS 'SS' AND THE INVERSE MATRIX RE-      MINV  10
C     TURNED TO SUBROUTINE 'CONV' IS 'SSINV'.                           MINV  11
C     CAUTION.. 'SSINV' RETURNS WITH ONLY THE UPPER TRIANGLE FILLED     MINV  12
C        SINCE IT IS SYMMETRIC.                                         MINV  13
C                                                                       MINV  14
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE   MINV  15
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-   MINV  16
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                         MINV  17
C                 LAST REVISION - 4 DECEMBER 1966                       MINV  18
C                                                                       MINV  19
C*********************************************************************MINV  20
C                                                                       MINV  21
      DOUBLE PRECISION SS,SSINV,ADG,DETTT                               MINV  22
      COMMON/CONMIN/SS(3,3),SSINV(3,3),N                                MINV  23
      DIMENSION ADG(3,3)                                                MINV  24
      ADG(1,1) = (SS(2,2)*SS(3,3)) - (SS(2,3)*SS(2,3))                  MINV  25
      ADG(2,2) = (SS(1,1)*SS(3,3)) - (SS(1,3)*SS(1,3))                  MINV  26
      ADG(3,3) = (SS(1,1)*SS(2,2)) - (SS(1,2)*SS(1,2))                  MINV  27
      ADG(1,2) = (SS(1,3)*SS(2,3)) - (SS(1,2)*SS(3,3))                  MINV  28
      ADG(1,3) = (SS(1,2)*SS(2,3)) - (SS(1,3)*SS(2,2))                  MINV  29
      ADG(2,3) = (SS(1,2)*SS(1,3)) - (SS(1,1)*SS(2,3))                  MINV  30
      DETTT = SS(1,1) / ( (ADG(3,3)*ADG(2,2) ) - (ADG(2,3)*ADG(2,3) ) ) MINV  31
      IF(DABS(DETTT).LT.1.0D+06)  GO TO 44                              MINV  32
      WRITE(6,1004) DETTT,N                                             MINV  33
1004  FORMAT(1H1,10X,8H DETTT =           ,1PE15.7,10X,3H N=,I4)        MINV  34
      STOP                                                              MINV  35
44    CONTINUE                                                          MINV  36
      SSINV(1,1) = ADG(1,1)*DETTT                                       MINV  37
      SSINV(2,2) = ADG(2,2)*DETTT                                       MINV  38
      SSINV(3,3) = ADG(3,3)*DETTT                                       MINV  39
      SSINV(1,2) = ADG(1,2)*DETTT                                       MINV  40
      SSINV(1,3) = ADG(1,3)*DETTT                                       MINV  41
      SSINV(2,3) = ADG(2,3)*DETTT                                       MINV  42
      RETURN                                                            MINV  43
      END                                                               MINV  44
```

```
$IBFTC GONE     DECK      **WRITES OUT RESULTS OF PROGRAM**          Q56 GONE   1
      SUBROUTINE GONE                                                 GONE   2
C                                                                     GONE   3
C*********************************************************************GONE   4
C                                                                     GONE   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES   GONE   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.GONE   7
C                                                                     GONE   8
C     THE PURPOSE OF THIS SUBROUTINE IS TO EVALUATE THE DUCTILITY     GONE   9
C     FACTORS, TO WRITE OUT THE MOST POSITIVE AND MOST NEGATIVE VALUES GONE  10
C     OF THE VARIOUS RESPONSES ALONG WITH THE TIMES AT WHICH THEY     GONE  11
C     OCCUR, AND TO PLOT OUT THE DISPLACEMENT ENVELOPE AND THE COLUMN GONE  12
C     AND GIRDER DUCTILITY FACTORS.                                   GONE  13
C                                                                     GONE  14
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE GONE  15
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- GONE  16
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                       GONE  17
C                 LAST REVISION - 11 JANUARY 1967                     GONE  18
C                                                                     GONE  19
C*********************************************************************GONE  20
C                                                                     GONE  21
      DIMENSION DUCTIL(22,3,4)                                        GONE  22
C                                                                     GONE  23
      COMMON/MNWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4),    GONE  24
     2 GA(22,3),CA(22,3),HT(22),H(22),ZM(22),A(5),S(8),P,F,ZETA,FUNDFQ, GONE 25
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                   GONE  26
C                                                                     GONE  27
      COMMON/WRKGON/UMAX(22),UMIN(22),TMXU(22),TMNU(22),              GONE  28
     2 VMAX(22),VMIN(22),TMXV(22),TMNV(22),                           GONE  29
     3 UUMAX(22),UUMIN(22),TMXUU(22),TMNUU(22),                       GONE  30
     4 SRMAX(22),SRMIN(22),TMXSR(22),TMNSR(22),                       GONE  31
     5 WMAX(22,3),WMIN(22,3),TMXW(22,3),TMNW(22,3),                   GONE  32
     6 ALPMAX(22,3,4),ALPMIN(22,3,4),TMXALP(22,3,4),TMNALP(22,3,4),   GONE  33
     7 BMMAX(22,3,4),BMMIN(22,3,4),TMXBM(22,3,4),TMNBM(22,3,4),       GONE  34
     8 TACCMX(22),TACCMN(22),TMXTAC(22),TMNTAC(22),                   GONE  35
     9 GSRMX(22,3),GSRMN(22,3),TMXGSR(22,3),TMNGSR(22,3),            GONE  36
     1 CLDMAX(22,3),CLDMIN(22,3),TMXCLD(22,3),TMNCLD(22,3),          GONE  37
     2 OTMMX(22),OTMMN(22),TMXOTM(22),TMNOTM(22),                    GONE  38
     3 ABSBMT(22,3),TABSBM(22,3),ABSSRT(22),TABSSR(22)               GONE  39
C                                                                     GONE  40
      COMMON/MNGONE/ YIELD(22,3,4)                                    GONE  41
C                                                                     GONE  42
      COMMON/WKGNIN/LLX,LLI,LLB,LLS,LLD,LLP, LGD(2),LINE(129)         GONE  43
C                                                                     GONE  44
C .......  DUCTILITY BY GIBERSON ....................................GONE  45
C                                                                     GONE  46
      DO 600 M=2,3                                                    GONE  47
      DO 595 J=1,4                                                    GONE  48
      DO 590 N=2,22                                                   GONE  49
      IF((-ALPMIN(N,M,J) + ALPMAX(N,M,J)).GT.0.0)  GO TO 570          GONE  50
      IF(-BMMIN(N,M,J).GT.BMMAX(N,M,J))  GO TO 550                    GONE  51
      DUCTIL(N,M,J)=BMMAX(N,M,J)/BMA(N,M,J)                           GONE  52
      GO TO 590                                                       GONE  53
550   DUCTIL(N,M,J)=BMMIN(N,M,J)/BMB(N,M,J)                           GONE  54
      GO TO 590                                                       GONE  55
C                                                                     GONE  56
570   CONTINUE                                                        GONE  57
      IF(-ALPMIN(N,M,J).GT.ALPMAX(N,M,J))  GO TO 580                  GONE  58
      DUCALP = ALPMAX(N,M,J)                                          GONE  59
      GO TO 585                                                       GONE  60
580   DUCALP = -ALPMIN(N,M,J)                                         GONE  61
585   DUCTIL(N,M,J) = 1.0 + DUCALP/(YIELD(N,M,J) + (P*DUCALP)/(1.0-P)) GONE 62
C                                                                     GONE  63
590   CONTINUE                                                        GONE  64
595   CONTINUE                                                        GONE  65
600   CONTINUE                                                        GONE  66
C                                                                     GONE  67
C.........WRITE OUT MAXIMUM AND MINIMUM VALUES AND CORRESPONDING TIMES .GONE 68
C.............. 'MAXIMUM'  CORRESPONDS TO  'MOST POSITIVE VALUE'  .........GONE 69
C.............. 'MINIMUM'  CORRESPONDS TO  'MOST NEGATIVE VALUE'  .........GONE 70
C                                                                     GONE  71
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                 GONE  72
1001  FORMAT(1H1 ,10A6,15H,BILINEARITY =  ,F5.3,16H,FRAC CR DAMP =   ,  GONE 73
     2 F7.5,13H,FUND FREQ =  ,F7.5, 8H RAD/SEC  ,//,                  GONE  74
     3 5X, 15HUSING THE FIRST ,F10.5, 15H SECONDS OF THE , 13A6,2X,//) GONE 75
      WRITE(6,1300)                                                   GONE  76
1300  FORMAT(20H UMAX,TMXU,UMIN,TMNU,55X,27HTACCMX,TMXTAC,TACCMN,TMNTAC)GONE 77
      NN = 21                                                         GONE  78
      DO 710 N=2,22                                                   GONE  79
      NN = NN - 1                                                     GONE  80
      WRITE(6,1311) NN,UMAX(N),TMXU(N),UMIN(N),TMNU(N),TACCMX(N),     GONE  81
     2 TMXTAC(N),TACCMN(N),TMNTAC(N)                                  GONE  82
1311  FORMAT((1X,I3,4(7X,1PE15.7,2X,0PF8.5) ))                        GONE  83
710   CONTINUE                                                        GONE  84
C                                                                     GONE  85
      WRITE(6,1002)                                                   GONE  86
1002  FORMAT(1H0)                                                     GONE  87
      WRITE(6,1320)                                                   GONE  88
1320  FORMAT(24H UUMAX,TMXUU,UUMIN,TMNUU,51X,23HSRMAX,TMXSR,SRMIN,TMNSR)GONE 89
      NN = 21                                                         GONE  90
      DO 720 N=2,22                                                   GONE  91
      NN = NN - 1                                                     GONE  92
      WRITE(6,1311) NN,UUMAX(N),TMXUU(N),UUMIN(N),TMNUU(N), SRMAX(N),  GONE 93
     2 TMXSR(N),SRMIN(N),TMNSR(N)                                     GONE  94
720   CONTINUE                                                        GONE  95
C                                                                     GONE  96
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                 GONE  97
      WRITE(6,1330)                                                   GONE  98
1330  FORMAT(40H (WMAX,  TMXW,  WMIN,  TMNW,  M=2,3)      )           GONE  99
      NN = 21                                                         GONE 100
      DO 730 N=2,22                                                   GONE 101
      NN = NN - 1                                                     GONE 102
      WRITE(6,1311) NN,(WMAX(N,M),TMXW(N,M),WMIN(N,M),TMNW(N,M),M=2,3) GONE 103
730   CONTINUE                                                        GONE 104
C                                                                     GONE 105
      WRITE(6,1002)                                                   GONE 106
      WRITE(6,1340)                                                   GONE 107
1340  FORMAT(30H  VMAX,  TMXV,  VMIN,  TMNV         )                 GONE 108
      NN = 21                                                         GONE 109
      DO 740 N=2,22                                                   GONE 110
      NN = NN - 1                                                     GONE 111
      WRITE(6,1345) NN,VMAX(N),TMXV(N),VMIN(N),TMNV(N)                GONE 112
1345  FORMAT((1X,I3,2(7X,1PE15.7,2X,0PF8.5) ))                        GONE 113
740   CONTINUE                                                        GONE 114
C                                                                     GONE 115
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                 GONE 116
      WRITE(6,1350)                                                   GONE 117
1350  FORMAT(36H   ((ALPMAX(N,M,J),J=1,4),M=2,3)              )       GONE 118
      NN = 21                                                         GONE 119
      DO 750 N=2,22                                                   GONE 120
      NN = NN - 1                                                     GONE 121
      WRITE(6,1312) NN,((ALPMAX(N,M,J),J=1,4),M=2,3)                  GONE 122
1312  FORMAT((1X,I3,2(4X,1P4E15.7) ))                                 GONE 123
750   CONTINUE                                                        GONE 124
C                                                                     GONE 125
      WRITE(6,1002)                                                   GONE 126
      WRITE(6,1360)                                                   GONE 127
1360  FORMAT(36H  ((TMXALP(N,M,J),J=1,4),M=2,3)              )        GONE 128
      NN = 21                                                         GONE 129
      DO 760 N=2,22                                                   GONE 130
      NN = NN - 1                                                     GONE 131
      WRITE(6,1313)  NN,((TMXALP(N,M,J),J=1,4),M=2,3)                 GONE 132
1313  FORMAT((1X,I3,2(4F15.7,4X) ))                                   GONE 133
760   CONTINUE                                                        GONE 134
C                                                                     GONE 135
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                 GONE 136
      WRITE(6,1370)                                                   GONE 137
1370  FORMAT(36H  ALPMIN(N,M,J),J=1,4),M=2,3)          )              GONE 138
      NN = 21                                                         GONE 139
      DO 770 N=2,22                                                   GONE 140
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      NN = NN - 1                                                       GONE 141
      WRITE(6,1312)  NN,((ALPMIN(N,M,J),J=1,4),M=2,3)                   GONE 142
770   CONTINUE                                                          GONE 143
C                                                                       GONE 144
      WRITE(6,1002)                                                     GONE 145
      WRITE(6,1380)                                                     GONE 146
1380  FORMAT(36H  TMNALP(N,M,J),J=1,4),M=2,3)                 1         GONE 147
      NN = 21                                                           GONE 148
      DO 780 N=2,22                                                     GONE 149
      NN = NN - 1                                                       GONE 150
      WRITE(6,1313)  NN,((TMNALP(N,M,J),J=1,4),M=2,3)                   GONE 151
780   CONTINUE                                                          GONE 152
C                                                                       GONE 153
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 154
      WRITE(6,1390)                                                     GONE 155
1390  FORMAT(50H  ((GSRMX,  TMXGSR,  GSRMN,  TMNGSR),  M=2,3)          )GONE 156
      NN = 21                                                           GONE 157
      DO 790 N=2,22                                                     GONE 158
      NN = NN - 1                                                       GONE 159
      WRITE(6,1311) NN,(GSRMX(N,M),TMXGSR(N,M),GSRMN(N,M),TMNGSR(N,M),  GONE 160
     2 M=2,3)                                                           GONE 161
790   CONTINUE                                                          GONE 162
C                                                                       GONE 163
      WRITE(6,1002)                                                     GONE 164
      WRITE(6,1400)                                                     GONE 165
1400  FORMAT(50H  ((CLDMAX,  TMXCLD,  CLDMIN,  TMNCLD), M=2,3)          )GONE 166
      NN = 21                                                           GONE 167
      DO 800 N=2,22                                                     GONE 168
      NN = NN - 1                                                       GONE 169
      WRITE(6,1311) NN,(CLDMAX(N,M),TMXCLD(N,M),CLDMIN(N,M),TMNCLD(N,M),GONE 170
     2  M=2,3)                                                          GONE 171
800   CONTINUE                                                          GONE 172
C                                                                       GONE 173
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 174
      WRITE(6,1410)                                                     GONE 175
1410  FORMAT(36H  ((BMMAX(N,M,J),J=1,4),M=2,3)          )               GONE 176
      NN = 21                                                           GONE 177
      DO 810 N=2,22                                                     GONE 178
      NN = NN - 1                                                       GONE 179
      WRITE(6,1312) NN,((BMMAX(N,M,J),J=1,4),M=2,3)                     GONE 180
810   CONTINUE                                                          GONE 181
C                                                                       GONE 182
      WRITE(6,1002)                                                     GONE 183
      WRITE(6,1420)                                                     GONE 184
1420  FORMAT(36H  ((TMXBM(N,M,J),J=1,4),M=2,3)                1         GONE 185
      NN = 21                                                           GONE 186
      DO 820 N=2,22                                                     GONE 187
      NN = NN - 1                                                       GONE 188
      WRITE(6,1313) NN,((TMXBM(N,M,J),J=1,4),M=2,3)                     GONE 189
820   CONTINUE                                                          GONE 190
C                                                                       GONE 191
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 192
      WRITE(6,1430)                                                     GONE 193
1430  FORMAT(36H  ((BMMIN(N,M,J),J=1,4),M=2,3)                         ) GONE 194
      NN = 21                                                           GONE 195
      DO 830 N=2,22                                                     GONE 196
      NN = NN - 1                                                       GONE 197
      WRITE(6,1312) NN,((BMMIN(N,M,J),J=1,4),M=2,3)                     GONE 198
830   CONTINUE                                                          GONE 199
C                                                                       GONE 200
      WRITE(6,1002)                                                     GONE 201
      WRITE(6,1440)                                                     GONE 202
1440  FORMAT(36H  ((TMNBM(N,M,J),J=1,4),M=2,3)                        ) GONE 203
      NN = 21                                                           GONE 204
      DO 840 N=2,22                                                     GONE 205
      NN = NN - 1                                                       GONE 206
      WRITE(6,1313) NN,((TMNBM(N,M,J),J=1,4),M=2,3)                     GONE 207
840   CONTINUE                                                          GONE 208
C                                                                       GONE 209
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 210
      WRITE(6,1445)                                                     GONE 211
1445  FORMAT(20X,80H SUM-CHECK OF ALL OF THE FORCES ACTING ON EACH FLOORGONE 212
     2 AND THE TIME OF OCCURRENCE        )                              GONE 213
      NN=21                                                             GONE 214
      DO 845 N=2,22                                                     GONE 215
      NN=NN-1                                                           GONE 216
      WRITE(6,1345) NN,ABSSRT(N),TABSSR(N)                              GONE 217
845   CONTINUE                                                          GONE 218
C                                                                       GONE 219
      WRITE(6,1002)                                                     GONE 220
      WRITE(6,1450)                                                     GONE 221
1450  FORMAT(15X,90H SUM-CHECK OF THE BENDING MOMENTS AT EACH  'JOINT(N,GONE 222
     2M)'  AND THE TIME OF OCCURRENCE                    )              GONE 223
      NN = 21                                                           GONE 224
      DO 850 N=2,22                                                     GONE 225
      NN = NN - 1                                                       GONE 226
      WRITE(6,1345) NN,(ABSBMT(N,M),TABSBM(N,M),M=2,3)                  GONE 227
850   CONTINUE                                                          GONE 228
C                                                                       GONE 229
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 230
      WRITE(6,1460)                                                     GONE 231
1460  FORMAT(10X,100H OVERTURNING MOMENTS IN 'INCH LBS' AND THE TIME OF GONE 232
     2OCCURRENCE .. OTMMX,TMXOTM,   OTMMN,TMNOTM            )          GONE 233
      NN=21                                                             GONE 234
      DO 855 N=2,22                                                     GONE 235
      NN=NN-1                                                           GONE 236
      WRITE(6,1345)  NN,OTMMX(N),TMXOTM(N),OTMMN(N),TMNOTM(N)           GONE 237
855   CONTINUE                                                          GONE 238
C                                                                       GONE 239
      WRITE(6,1002)                                                     GONE 240
      WRITE(6,1465)                                                     GONE 241
1465  FORMAT(42H  ((DUCTIL(N,M,J),J=1,4),M=2,3) ,GIBERSON     )        GONE 242
      NN = 21                                                           GONE 243
      DO 860 N=2,22                                                     GONE 244
      NN = NN - 1                                                       GONE 245
      WRITE(6,1313) NN,((DUCTIL(N,M,J),J=1,4),M=2,3)                    GONE 246
860   CONTINUE                                                          GONE 247
C                                                                       GONE 248
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 249
      WRITE(6,1470)                                                     GONE 250
1470  FORMAT(40X,65H MOST NEGATIVE AND MOST POSITIVE DISPLACEMENTS IN INGONE 251
     2CHES            )                                                GONE 252
      WRITE(6,1480)                                                     GONE 253
1480  FORMAT(132H -16 -15 -14 -13 -12 -11 -10  -9  -8  -7  -6  -5  -4  -GONE 254
     2 3  -2  -1   0   1   2   3   4   5   6   7   8   9  10  11  12  13 GONE 255
     3 14  15  16    )                                                  GONE 256
      WRITE(6,1485)                                                     GONE 257
1485  FORMAT(33(3X,1H+))                                                GONE 258
      NN=21                                                             GONE 259
      DO 610 N=2,21                                                     GONE 260
      NN=NN-1                                                           GONE 261
      LINE(65)=LLP                                                      GONE 262
      IF(UMIN(N).LE.-16.0)  GO TO 605                                   GONE 263
      I=IFIX(4.0*UMIN(N)  -0.5) + 65                                    GONE 264
      LINE(I)=LLD                                                       GONE 265
605   IF(UMAX(N).GT.16.0)  GO TO 608                                    GONE 266
      J=IFIX((4.0*UMAX(N)) + 0.5) + 65                                  GONE 267
      LINE(J)=LLD                                                       GONE 268
608   WRITE(6,1490) NN,LINE                                             GONE 269
1490  FORMAT(1X,I2,129A1)                                               GONE 270
      LINE(I)=LLB                                                       GONE 271
      LINE(J)=LLB                                                       GONE 272
610   CONTINUE                                                          GONE 273
      LINE(65)=LLP                                                      GONE 274
      WRITE(6,1495) LGD,LINE                                            GONE 275
1495  FORMAT(1X,2A1,129A1)                                              GONE 276
      LINE(65)=LLB                                                      GONE 277
      WRITE(6,1485)                                                     GONE 278
C                                                                       GONE 279
      WRITE(6,1002)                                                     GONE 280
```

```
      WRITE(6,1497)                                                     GONE 281
1497  FORMAT(45X,18HSTORY  UMIN-INCHES, 10X, 12H UMAX-INCHES      )     GONE 282
      NN=21                                                             GONE 283
      DO 611 N=2,21                                                     GONE 284
      NN=NN-1                                                           GONE 285
      WRITE(6,1498)  NN,UMIN(N),JMAX(N)                                 GONE 286
1498  FORMAT(47X,I2, 2X,F12.6,10X,F12.6  )                              GONE 287
611   CONTINUE                                                          GONE 288
C                                                                       GONE 289
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 290
      WRITE(6,1500)                                                     GONE 291
1500  FORMAT(20X,78H  COLUMN DUCTILITY FACTORS,      X = EXT COL,  I = IGONE 292
     2NT COL, GIBERSON              ,//)                               GONE 293
      WRITE(6,1520)                                                     GONE 294
1520  FORMAT(       132H   0                   1                   2    GONE 295
     2              3                   4                   5         GONE 296
     3       6                          )                              GONE 297
      WRITE(6,1525)                                                     GONE 298
1525  FORMAT(3X,1H+,6(19X,1H+))                                         GONE 299
      NN=21                                                             GONE 300
      DO 620 N=2,22                                                     GONE 301
      NN=NN-1                                                           GONE 302
      IF(N.EQ.2)  GO TO 616                                             GONE 303
      LINE(21)=LLP                                                      GONE 304
      IF(DUCTIL(N,2,2).GT.6.4)  GO TO 612                               GONE 305
      I= DUCTIL(N,2,2)*20.0 +1.5                                        GONE 306
      LINE(I)=LLX                                                       GONE 307
612   IF(DUCTIL(N,3,2).GT.6.4)  GO TO 615                               GONE 308
      J= DUCTIL(N,3,2)*20.0 + 1.5                                       GONE 309
      LINE(J)=LLI                                                       GONE 310
      IF(N.EQ.22)  GO TO 622                                            GONE 311
615   WRITE(6,1490) NN,LINE                                             GONE 312
      LINE(I)=LLB                                                       GONE 313
      LINE(J)=LLB                                                       GONE 314
616   LINE(21)=LLP                                                      GONE 315
      IF(DUCTIL(N,2,4).GT.6.4)  GO TO 618                               GONE 316
      I= DUCTIL(N,2,4)*20.0 +1.5                                        GONE 317
      LINE(I)=LLX                                                       GONE 318
618   IF(DUCTIL(N,3,4).GT.6.4)  GO TO 619                               GONE 319
      J= DUCTIL(N,3,4)*20.0 +1.5                                        GONE 320
      LINE(J)=LLI                                                       GONE 321
619   WRITE(6,1490) NN,LINE                                             GONE 322
      LINE(I)=LLB                                                       GONE 323
      LINE(J)=LLB                                                       GONE 324
620   CONTINUE                                                          GONE 325
622   WRITE(6,1495) LGD,LINE                                            GONE 326
      LINE(I) = LLB                                                     GONE 327
      LINE(J) = LLB                                                     GONE 328
      LINE(21)=LLB                                                      GONE 329
      WRITE(6,1525)                                                     GONE 330
      WRITE(6,1520)                                                     GONE 331
C                                                                       GONE 332
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 333
      WRITE(6,1528)                                                     GONE 334
1528  FORMAT(114H MAXIMUM VALUE OF COLUMN DUCTILITY FACTORS BETWEEN ADJAGONE 335
     2CENT FLOORS,   X = EXT COL,  I = INT COL,  GIBERSON           )  GONE 336
      WRITE(6,1520)                                                     GONE 337
      WRITE(6,1526)                                                     GONE 338
1526  FORMAT(3X,1H+,6(19X,1H+),//)                                      GONE 339
      NN=21                                                             GONE 340
      DO 660 N=2,21                                                     GONE 341
      NN=NN-1                                                           GONE 342
      N1=N+1                                                            GONE 343
      LINE(21)=LLP                                                      GONE 344
      IF(DUCTIL(N,2,4).GT.DUCTIL(N1,2,2))  GO TO 635                    GONE 345
      I=(DUCTIL(N1,2,2)*20.0) + 1.5                                     GONE 346
      GO TO 640                                                         GONE 347
635   I=(DUCTIL(N ,2,4)*20.0) + 1.5                                     GONE 348
640   IF(DUCTIL(N,3,4).GT.DUCTIL(N1,3,2))  GO TO 645                    GONE 349
      J=(DUCTIL(N1,3,2)*20.0) + 1.5                                     GONE 350
      GO TO 650                                                         GONE 351
645   J=(DUCTIL(N,3,4)*20.0) + 1.5                                      GONE 352
650   LINE(I)=LLX                                                       GONE 353
      LINE(J)=LLI                                                       GONE 354
      WRITE(6,1540) NN,LINE                                             GONE 355
      LINE(I)=LLB                                                       GONE 356
      LINE(J)=LLB                                                       GONE 357
660   CONTINUE                                                          GONE 358
      WRITE(6,1525)                                                     GONE 359
      WRITE(6,1520)                                                     GONE 360
C                                                                       GONE 361
      WRITE(6,1001)  NAMEST,P,ZETA,FUNDFQ,TIM1,NAMEQK                   GONE 362
      WRITE(6,1530)                                                     GONE 363
1530  FORMAT(132HEXTERIOR AND INTERIOR GIRDER DUCTILITY FACTORS,  X = EXGONE 364
     2T GIR AT EXT COL, . = EXT GIR AT INT COL, I = INT GIR AT INT COL, GONE 365
     3 GIBERSON     )                                                  GONE 366
      WRITE(6,1520)                                                     GONE 367
      WRITE(6,1526)                                                     GONE 368
      NN=21                                                             GONE 369
      DO 700 N=2,21                                                     GONE 370
      NN=NN-1                                                           GONE 371
      LINE(21)=LLP                                                      GONE 372
      IF(DUCTIL(N,2,3).GT.6.4)  GO TO 680                               GONE 373
      I= DUCTIL(N,2,3)*20.0 + 1.5                                       GONE 374
      LINE(I)=LLX                                                       GONE 375
680   IF(DUCTIL(N,3,1).GT.6.4)  GO TO 690                               GONE 376
      J= DUCTIL(N,3,1)*20.0 + 1.5                                       GONE 377
      LINE(J)=LLD                                                       GONE 378
690   IF(DUCTIL(N,3,3).GT.6.4)  GO TO 695                               GONE 379
      K= DUCTIL(N,3,3)*20.0 + 1.5                                       GONE 380
      LINE(K)=LLI                                                       GONE 381
695   WRITE(6,1540) NN,LINE                                             GONE 382
1540  FORMAT(1X,I2,129A1,//)                                            GONE 383
      LINE(I)=LLB                                                       GONE 384
      LINE(J)=LLB                                                       GONE 385
      LINE(K)=LLB                                                       GONE 386
700   CONTINUE                                                          GONE 387
      LINE(21)=LLP                                                      GONE 388
      WRITE(6,1550)  LGD,LINE                                           GONE 389
1550  FORMAT(1X,2A1,129A1,//)                                           GONE 390
      WRITE(6,1525)                                                     GONE 391
      WRITE(6,1520)                                                     GONE 392
      LINE(21)=LLB                                                      GONE 393
C                                                                       GONE 394
      RETURN                                                            GONE 395
      END                                                               GONE 396
```

```
$IBFTC MODE    DECK **MODAL ANALYSIS - DBLE PREC**                Q56 MODE   1
      SUBROUTINE MODE                                                 MODE   2
C                                                                     MODE   3
C*********************************************************************MODE   4
C                                                                     MODE   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES   MODE   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION.MODE   7
C                                                                     MODE   8
C     THE PURPOSE OF SUBROUTINE 'MODE' IS TO MAKE A MODAL ANALYSIS    MODE   9
C     AFTER EACH TIME INCREMENT OF THE HORIZONTAL DISPLACEMENT VECTOR MODE  10
C     (20X1) WITH RESPECT TO THE MODES OF THE LINEAR STRUCTURE WITH   MODE  11
C     FLEXIBLE GIRDERS.  THE EIGENVECTOR CORRESPONDING TO THE LARGEST MODE  12
C     EIGENVALUE IS READ IN FIRST.  THEY DIAGONALIZE THE TRANSFORMED  MODE  13
C     STIFFNESS MATRIX  M**(-1/2) * K * M**(-1/2)  AND ARE NORMALIZED MODE  14
C     SUCH THAT  (EIGENVECTOR TRANSPOSED)*(EIGENVECTOR) = I .         MODE  15
C                                                                     MODE  16
C                       D O U B L E   P R E C I S I O N               MODE  17
C     DOUBLE PRECISION PREFIXES ARE NOT GENERALLY USED HEREIN.        MODE  18
C                                                                     MODE  19
C     AN OPTIONAL DUMMY SUBROUTINE 'MODE' IS AVAILABLE WHEN A MODAL   MODE  20
C     ANALYSIS IS NOT DESIRED.                                        MODE  21
C                                                                     MODE  22
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE MODE  23
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI- MODE  24
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                       MODE  25
C                 LAST REVISION - 4 DECEMBER 1966                     MODE  26
C                                                                     MODE  27
C*********************************************************************MODE  28
C                                                                     MODE  29
      DOUBLE PRECISION                  AIDA(20), ZMHAF(20),EVALU(20),MODE  30
     2 EVECT(20,20),CKVECT(20),UM(4,20),  UUM(4,20),TUM(4),   TUUM(4),MODE  31
     3 AMEV(20,20),DMEV(20,20),EVTM(20,20),          ZZM,ZMH,ABSAMV   MODE  32
     4  ,DBU,DBUU                                                     MODE  33
C                                                                     MODE  34
      DIMENSION SEVECT(20)                                            MODE  35
C                                                                     MODE  36
      COMMON/MNWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4),    MODE  37
     2 GA(22,3),CA(22,3),HT(22),H(22),ZM(22),A(5),S(8),P,F,ZETA,FUNDFQ,MODE  38
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                   MODE  39
C                                                                     MODE  40
C                                                                     MODE  41
      COMMON/CONMOD/ DBU(22),DBUU(22)                                 MODE  42
C                                                                     MODE  43
C..... INITIALIZE THE MODAL ANALYSIS MATRICES (TO STATEMENT 669) .....MODE  44
C                                                                     MODE  45
      IF(LL1.EQ.1)  GO TO 669                                         MODE  46
      LL1 = 1                                                         MODE  47
      DO 605  N=1,20                                                  MODE  48
      ZZM= DBLE(1.0/ZM(N+1))                                          MODE  49
      ZMHAF(N) = DSQ(ZZM)                                             MODE  50
      NN = 21-N                                                       MODE  51
C                                                                     MODE  52
C..... READ IN EIGENVALUES AND EIGENVECTORS OF LINEAR STRUCTURE ......MODE  53
C                                                                     MODE  54
      READ(5,1210) EVAL                                               MODE  55
      EVALU(NN) = DBLE(EVAL)                                          MODE  56
      READ(5,1210) (SEVECT(J),J=1,20)                                 MODE  57
1210  FORMAT(6D12,8X)                                                 MODE  58
      DO 603 J=1,20                                                   MODE  59
      EVECT(J,NN) = DBLE(SEVECT(J))                                   MODE  60
603   CONTINUE                                                        MODE  61
605   CONTINUE                                                        MODE  62
      DO 640 I=1,20                                                   MODE  63
      ZMH =  ZMHAF(I)                                                 MODE  64
      DO 630 J=1,20                                                   MODE  65
      AMEV(I,J) = ZMH*EVECT(I,J)                                      MODE  66
630   CONTINUE                                                        MODE  67
640   CONTINUE                                                        MODE  68
C                                                                     MODE  69
C..... WRITE OUT SELECTED (ARBITRARY) MODES AS A CHECK ...............MODE  70
C                                                                     MODE  71
      DO 645 J=1,17,8                                                 MODE  72
      ABSAMV= 0.0D+00                                                 MODE  73
      DO 643 K=1,20                                                   MODE  74
      IF(ABSAMV.GE.DABS(AMEV(K,J)))  GO TO 642                        MODE  75
      ABSAMV =DABS(AMEV(K,J))                                         MODE  76
642   CONTINUE                                                        MODE  77
643   CONTINUE                                                        MODE  78
      DO 644  K=1,20                                                  MODE  79
      CKVECT(K) = AMEV(K,J)/ABSAMV                                    MODE  80
644   CONTINUE                                                        MODE  81
      WRITE(6,1215) J,(K,CKVECT(K),K=1,20)                            MODE  82
1215  FORMAT( 9H1  MODE    ,I4,/,(5X,I5,5X,1PE15.7))                  MODE  83
645   CONTINUE                                                        MODE  84
C                                                                     MODE  85
C..... EVALUATE (TRANSPOSED EIGENVECTOR)*(MASS**(1/2)) ...............MODE  86
C                                                                     MODE  87
      DO 660  J=1,20                                                  MODE  88
      ZMH =  ZMHAF(J)                                                 MODE  89
      DO 650  I=1,20                                                  MODE  90
      EVTM(I,J) = EVECT(J,I)/ZMH                                      MODE  91
650   CONTINUE                                                        MODE  92
660   CONTINUE                                                        MODE  93
C                                                                     MODE  94
C..... EVALUATE THE MODAL PARTICIPATION FACTOR 'AIDA' ................MODE  95
C                                                                     MODE  96
669   CONTINUE                                                        MODE  97
      DO 680  I=1,20                                                  MODE  98
      AIDA(I) = 0.0D+00                                               MODE  99
      DO 670  J=1,20                                                  MODE 100
      AIDA(I) = AIDA(I) + EVTM(I,J)*DBU(J+1)                          MODE 101
670   CONTINUE                                                        MODE 102
680   CONTINUE                                                        MODE 103
      WRITE(6,1220) AIDA                                              MODE 104
1220  FORMAT(7H0 AIDA  ,/,(4X,1PE17.7))                               MODE 105
C                                                                     MODE 106
C..... EVALUATE THE MODAL CONTRIBUTIONS TO THE DISPLACEMENTS AND .....MODE 107
C..... INTER-FLOOR DISPLACEMENTS OF SELECTED (ARBITRARY) FLOORS ... MODE 108
C                                                                     MODE 109
      JN = 0                                                          MODE 110
      DO 690 I = 2,17,5                                               MODE 111
      JN=JN+1                                                         MODE 112
      TUM(JN)= 0.0D+00                                                MODE 113
      TUUM(JN) = 0.0D+00                                              MODE 114
      DO 685  J=1,20                                                  MODE 115
      UM(JN,J)= AMEV(I,J)*AIDA(J)                                     MODE 116
      UUM(JN,J) = (AMEV(I,J) - AMEV(I+1,J))*AIDA(J)                   MODE 117
      TUM(JN) = TUM(JN) + UM(JN,J)                                    MODE 118
      TUUM(JN) = TUUM(JN) + UUM(JN,J)                                 MODE 119
685   CONTINUE                                                        MODE 120
690   CONTINUE                                                        MODE 121
      WRITE(6,1230) (N,(UM(JN,N),  UUM(JN,N),JN=1,4),N=1,20)          MODE 122
1230  FORMAT(( 2X,I2, 4(2X,1P2E15.7)  ))                              MODE 123
      N = 21                                                          MODE 124
      WRITE(6,1230) N,(TUM(JN),  TUUM(JN),JN=1,4)                     MODE 125
      WRITE(6,1230) N, (DBU(J),DBUU(J),J=3,18,5)                      MODE 126
      WRITE(6,1001)                                                   MODE 127
1001  FORMAT(1H1)                                                     MODE 128
      RETURN                                                          MODE 129
      END                                                             MODE 130
```

```
$IBFTC MODE    DECK *THIS IS A DUMMY SUBROUTINE*                    Q56 DUMY   1
      SUBROUTINE MODE                                                  DUMY   2
C                                                                      DUMY   3
C*********************************************************************DUMY   4
C                                                                      DUMY   5
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES    DUMY   6
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION. DUMY   7
C                                                                      DUMY   8
C     DUMMY SUBROUTINE - USED WHEN A MODAL ANALYSIS IS -NOT- WANTED.   DUMY   9
C                                                                      DUMY  10
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE  DUMY  11
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-  DUMY  12
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                        DUMY  13
C                                                                      DUMY  14
C*********************************************************************DUMY  15
C                                                                      DUMY  16
      RETURN                                                           DUMY  17
      END                                                              DUMY  18
```

```
$IBFTC ZERO    DECK *INITIALIZING BY 'BLOCK DATA'*                  Q56 ZERO   1
C                                                                      ZERO   2
C*********************************************************************ZERO   3
C                                                                      ZERO   4
C     THE PURPOSE OF THIS PROGRAM IS TO CALCULATE VARIOUS RESPONSES    ZERO   5
C     OF NONLINEAR TALL STRUCTURES SUBJECTED TO EARTHQUAKE EXCITATION. ZERO   6
C                                                                      ZERO   7
C     THE PURPOSE OF 'BLOCK DATA' IS TO PLACE PRESCRIBED NON-ZERO      ZERO   8
C     VALUES IN CERTAIN MEMORY CELLS AT THE TIME OF LOADING.           ZERO   9
C                                                                      ZERO  10
C     THIS PROGRAM INCLUDING ALL SUBROUTINES WAS WRITTEN BY MELBOURNE  ZERO  11
C     F. GIBERSON, DEPARTMENT OF APPLIED MECHANICS, CALIFORNIA INSTI-  ZERO  12
C     TUTE OF TECHNOLOGY, PASADENA, CALIFORNIA.                        ZERO  13
C                  LAST REVISION - 4 DECEMBER 1966                     ZERO  14
C                                                                      ZERO  15
C*********************************************************************ZERO  16
C                                                                      ZERO  17
      BLOCK DATA                                                       ZERO  18
C                                                                      ZERO  19
      COMMON/WKGNIN/LLX,LLI,LL3,LLS,LLD,LLP, LGD(2),LINE(129)          ZERO  20
C                                                                      ZERO  21
      COMMON/MNWCGN/NAMEST(10),NAMEQK(13),BMA(22,3,4),BMB(22,3,4),     ZERO  22
     2 GA(22,3),GA(22,3),HT(22),H(22),ZM(22),A(5),S(8),P,F,ZETA,FUNDFQ, ZERO  23
     3 ZL,AL,TIMEND,ALAMDA,BLAMDA,IELPL,TIM1,LRTEST                    ZERO  24
C                                                                      ZERO  25
      DATA LLX,LLI,LLB,LLS,LLD,LLP/1HX,1HI,1H ,1H*,1H.,1H+/  ,         ZERO  26
     2  LGD(1), LGD(2)/1HG,1HO/  ,  (LINE(I),I=1,129) /129*1H  /     , ZERO  27
     3 (((BMA(N,M,J),N=1,22 ),J=1,4),M=1,3)/264*1.0/                   ZERO  28
      END                                                              ZERO  29
```

THE FOLLOWING IBMAP 'TAPE CONTROL' PROGRAM IS USED BY PROGRAM 'Q56'.

```
$IBMAP U20     DECK                                                    U20    1
       ENTRY   .UN20.                                                 U20    2
.UN20. PZE     X                                                       U20    3
X      FILE    RESULT TAPE,6(1),HOLD,BLK=256,BIN,MOUNT,OUTPUT         U20    4
       END                                                            U20    5
```

REFERENCES

1. Clough, R. W., K. L. Benuska and T. Y. Lin & Associates. FHA Study of Seismic Design Criteria for High-Rise Buildings, (HUD TS-3), Federal Housing Administration, Washington, D.C., (August 1966).

2. T. Y. Lin & Associates. A Computer Program to Analyze the Dynamic Response of High Rise Buildings to Nuclear Blast Loading, Vol. I: Elastic Analysis, Vol. II: Dynamic Nonlinear Response Analysis. (OCD-OS-63-44), Office of the Secretary of the Army, Department of the Army, Washington, D.C., (October 1963).

3. Sano, R., and K. Muto. Earthquake and Typhoon Resistant Construction, (Japanese), Tokyo, Japan, 1935.

4. Suyehiro, K. "Engineering Seismology," ASCE, Proceedings, v.58, no.5, pt.2, (May 1932), 110p.

5. Biot, M. A. "Theory of Elastic Systems Vibrating Under Transient Impulse With an Application to Earthquake Proof Buildings," National Academy of Science, Proceedings, v.19 (1933) 262-268.

6. Biot, M. A. "Analytical and Experimental Methods in Engineering Seismology," ASCE, Transactions, v.108 (1943) 262-268.

7. Freeman, J. R. Earthquake Damage and Earthquake Insurance, N.Y., McGraw-Hill, 1932.

8. Hudson, D. E. "Response Spectrum Techniques in Engineering Seismology." In World Conference on Earthquake Engineering, Berkeley, California, 1956, Proceedings. San Francisco, Earthquake Engineering Research Institute, 1956, Paper No. 4.

9. Housner, G. W. "Vibration of Structures Induced by Seismic Waves, Part I: Earthquakes," (Chapter 50). In Shock and Vibration Handbook, v.3. Edited by Cyril M. Harris and Charles E. Crede. N.Y., McGraw-Hill, 1961.

10. Caughey, T. K. "Random Excitation of a System with Bilinear Hysteresis," Journal of Applied Mechanics, v.27, no.4 (December 1960) 649-652.

11. Berg, G. V. and S. S. Thomaides. "Energy Consumption by Structures in Strong Motion Earthquakes." In World Conference on Earthquake Engineering, 2d, Tokyo and Kyoto, Japan 1960, Proceedings. Tokyo, Science Council of Japan, 1960.

12. Penzien, J. "Dynamic Response of Elasto-Plastic Frames," ASCE, Structural Division, Journal, v.86, no.ST7 (July 1960) 81-94.

13. Iwan, W. D. The Dynamic Response of Bilinear Hysteretic Systems, Earthquake Engineering Research Laboratory (Ph.D. Thesis), California Institute of Technology, 1961.

14. Jennings, Paul C. Response of Simple Yielding Structures to Earthquake Excitation, Earthquake Engineering Research Laboratory (Ph. D. Thesis), California Institute of Technology, 1963.

15. Chelapati, C. V. Response of Simple Inelastic Systems to Ground Motion, Ph.D. Thesis, University of Illinois, Urbana, Illinois, 1962.

16. Lutes, L. D. Stationary Random Response of Bilinear Hysteretic Systems, Earthquake Engineering Research Laboratory (Ph.D. Thesis), California Institute of Technology, 1967.

17. Jacobsen, L. S. and R. S. Ayre. "Experimentally Determined Dynamic Shears in a Fifteen Story Model," Seismological Society of America, Bulletin, v.28, no.4 (October 1938) 269-311.

18. Tung, T. P. and N. M. Newmark. "Numerical Analysis of Earthquake Response of a Tall Structure," Seismological Society of America, Bulletin, v.45, no.4 (October 1955) 269-278.

19. Jennings, Richard L. The Response of Multistoried Structures to Strong Ground Motion, M.S. Thesis, University of Illinois, Urbana, Illinois, 1958.

20. Jennings, Richard L. and N. M. Newmark. "Elastic Response of Multi-story Shear Beam Type Structures Subjected to Strong Ground Motion." In World Conference on Earthquake Engineering, 2d, Tokyo and Kyoto, Japan, 1960, Proceedings, Tokyo, Science Council of Japan, 1960, v.2, 699-718.

21. Clough, R. W. "On the Importance of Higher Modes of Vibration in the Earthquake Response of a Tall Building," Seismological Society of America, Bulletin, v.45, no.4 (October 1955) 289-301.

22. Blume, J. A. "Structural Dynamics in Earthquake-Resistant Design," ASCE, Structural Division, Journal, v.84, no.ST4 (July 1958) Proc. Paper 1695.

23. Blume, J. A., L. Corning and N. M. Newmark. Design of Multi-Story Reinforced Concrete Buildings for Earthquake Motions. Chicago, Illinois, Portland Cement Association, 1961.

24. Goldberg, J. E., J. L. Bogdanoff and Z. L. Moh. "Forced Vibration and Natural Periods of Tall Building Frames," Seismological Society of America, Bulletin, v.49, no.1 (1959) 33-48.

25. Penzien, J. "Elasto-Plastic Response of Idealized Multi-Story Structures." In World Conference on Earthquake Engineering, 2d, Tokyo and Kyoto, Japan, 1960, Proceedings, Tokyo, Science Council of Japan, 1960, v.2, Session II, 739-760.

26. Bustamante, J. I. "Seismic Shears and Overturning Moments in Buildings." In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.3, Session IV, 144-160.

27. Umemura, H., Y. Osawa and A. Shibato. "Study on Shearing Forces in Structures Caused by Medium Earthquakes Recorded in Japan." In World Conference on Earthquake Engineering, 3d, 1965, Auckland and Wellington, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.3, Session IV, 539-571.

28. Berg, G. V. "Response of Multi-Story Structures to Earthquakes," ASCE, Engineering Mechanics Division, Journal, v.87, no.EM2 (April 1961) 1-16.

29. Muto, K. "Recent Trends in High Rise Building Design in Japan." (Special Address) In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.1, 119-146.

30. Westergaard, H. M. "Earthquake Shock Transmission in Tall Buildings," Engineering News-Record, v.111 (November 30, 1933) 654-656.

31. Jacobsen, L. S. "Vibrational Transfer from Shear Buildings to Ground," ASCE, Engineering Mechanics Division, Journal, v.90, no.EM3 (June 1964) 21-38.

32. Kanai, K. "Some New Problems of Seismic Vibrations of a Structure." In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.2, Session II, 260-275.

33. Bycroft, G. N. "Yield Displacements in Multi-Story Aseismic Design," Seismological Society of America, Bulletin, v.50, no.3 (July 1960) 441-453.

34. Bycroft, G. N. "Effect of Stiffness Taper in Aseismic Design," Seismological Society of America, Bulletin, v.50, no.4 (October 1960) 537-552.

35. Murphy, M. J., G. N. Bycroft and L. W. Harrison, "Electrical Analog for Earthquake Shear Stresses in a Multi-Story Building." In World Conference on Earthquake Engineering, Berkeley, California, USA, 1956, Proceedings, San Francisco, Earthquake Engineering Research Institute, 1956, Paper No.9.

36. Clough, R. W. "Earthquake Analysis by Response Spectrum Superposition," Seismological Society of America, Bulletin, v.52, no.3 (July 1962) 647-660.

37. Clough, R. W., I. P. King and E. L. Wilson. "Large Capacity Multistory Frame Analysis Programs," ASCE, Structural Division, Journal, v.89, no.ST4 (August 1963) 179-204.

38. Clough, R. W., I. P. King and E. L. Wilson. "Structural Analysis of Multistory Buildings," ASCE, Structural Division, Journal, v.90, no.ST3 (June 1964) 19-34.

39. Clough, R. W., K. L. Benuska and E. L. Wilson. "Inelastic Earthquake Response of Tall Buildings." In World Conference on Earthquake Engineering, 3d, Auckland and Wellington, New Zealand, 1965, Proceedings, Wellington, New Zealand National Committee on Earthquake Engineering, 1965, v.2, Session II, 68-89.

40. Clough, R. W. and E. L. Wilson. "Dynamic Response by Step-by-Step Matrix Analysis," Symposium on the Use of Computers in Civil Engineering, Lisbon, Portugal, 1962.

41. Richard, R. M. and J. E. Goldberg. "Analysis of Nonlinear Structures, Force Method," ASCE, Structural Division, Journal, v.91, no.ST6 (December 1965) 33-48.

42. Melin, J. "Numerical Integration by the Beta Method." American Society of Civil Engineers, Conference on Electronic Computation, 1st, 1958, Kansas City, Missouri, 227-236.

43. Newmark, N. M. "A Method of Computation for Structural Dynamics," ASCE, Engineering Mechanics Division, Journal, v.85, no.EM3 (July 1959) 67-94.

44. O'Kelley, M. E. J. Vibration of Viscously Damped Linear Dynamic Systems, Dynamics Laboratory (Ph.D. Thesis), California Institute of Technology, 1964.

Several textbooks on the design of indeterminant structures such as those modeled in this report are available. Among them are the following:

45. Ferguson, P. M. Reinforced Concrete Fundamentals, (1st ed., 1958), (2d ed., rev. 1965), N.Y., J. Wiley.

46. Hurty, W. C. and M. F. Rubenstein. Dynamics of Structures, Englewood Cliffs, N.J., Prentice-Hall, 1964.

47. Timoshenko, S. and D. H. Young. Theory of Structures, (2nd ed.), N.Y., McGraw-Hill, 1965.

Two bibliographies of research work covering the entire earthquake engineering field are noted:

48. Hollis, E. P. Bibliography of Engineering Seismology (2nd ed.), San Francisco, Earthquake Engineering Research Institute, 1966.

49. Rosenblueth, E. "Earthquake Resistant Design," Applied Mechanics Review, v.14, no.12 (December 1961) 923-926.